INVESTIGATION OF STRESSES

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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INVESTIGATION OF STRESSES

IN THE BUCKLING OF A SPHERICAL SHELL

As shown in the author' s book ([1]), the elastic deformation of a shell under a considerable deviation from its initial form is close to a certain isometric transformation of its middle surface. The stresses caused by strong local bending at the boundary of the buckling region are determined by the formula

$$\sigma = c' E \frac{\delta^{1/2} \alpha^{1/2}}{\rho^{1/2}}. \tag{1}$$

Here 2α is the angle between the tangent planes along the edge of the isometric transformation; ρ is the radius of curvature of the edge; δ is the thickness of the shell; E is the modulus of elasticity of the material; c' is a certain constant. For a shell in the form of a spherical segment under axisymmetric buckling, the values of the parameters α and ρ are shown in Fig. 1.

The purpose of the present work is an experimental verification of formula (1) for spherical segments. In the experiment, it was not the stresses themselves that were determined, but the radius of curvature $\widetilde{R}$ of the shell at the boundary of buckling (at the point P, Fig. 2). After this, the stresses were determined by the formula

$$\sigma = \frac{E\delta}{2} \frac{1}{\widetilde{R}}. \tag{2}$$

We take the curvature $1/\widetilde{R}$ instead of the change in curvature in view of the considerable bending deformation.

Fig. 1

Fig. 2

Figure 2: Fig. 2

Fig. 2

For the case of spherical segments, formula (1) can be transformed to a simpler form if, instead of the parameters α and ρ, one introduces the deflection H at the center of the segment. Noting that $\alpha \simeq \rho/R$, $2HR \simeq \rho^2$, we obtain

$$\sigma = c' E \sqrt{\delta H}/R, \tag{3}$$

where R is the radius of curvature of the segment. Comparing formulas (2) and (3) for the stresses σ, we obtain the following relation between the radius of curvature $\widetilde{R}$ and the coefficient c' in the formula for σ:

$$c' = \frac{1}{2}\frac{R}{\widetilde{R}}\sqrt{\frac{\delta}{H}}. \tag{4}$$

Shells in the form of a spherical segment were made of copper by vacuum deposition onto a perfect spherical form. The radius of curvature of the shells was $R = 100$ mm. The thickness of the individual specimens ranged from 0.03 to 0.05 mm. A thin scratch was applied to the inner surface of the specimen

with thickness of the order of 1μ in the plane of the normal section passing through the apex of the segment.

The shell was clamped in a special fixture between two steel rings ground to a spherical surface with curvature equal to the curvature of the shell. Thus ideal conditions for fixing the shell along the edge were ensured. Buckling of the shell was produced by a concentrated force from a special mushroom-shaped punch with a spherical surface of the head of radius 60 mm, which was applied to the shell by a micrometer screw.

The fixture with the shell was rigidly fastened to the table of a UIM-21 universal measuring microscope, so that the plane of the normal section marked by a line was inclined to the plane of the table at an angle of 45°. In the field of view of the microscope, the line drawn on the surface of the shell was observed, and the parameters of its deformation during buckling of the shell were measured. The accuracy of the measurements was within 1μ.

Under the microscope the deflection H at the center of the segment and the chord b were measured (Fig. 2). The shift h was constant, equal to 50μ. The radius of curvature $\widetilde{R}$ at the point P was determined by the formula

$$\widetilde{R} = \frac{(b/2)^2}{2h\sqrt{2}}.$$

(The appearance of the factor $\sqrt{2}$ is connected with the inclination of the plane of the section under consideration to the plane of the table.) The value $\widetilde{R}$ was determined for deflections $H = 0.3 \div 0.7$ mm. Such deformation must be considered significant for the given shell thicknesses, and therefore formula (1) for σ is quite applicable.

Using the experimentally found values of $\widetilde{R}$, the coefficient c' was determined by formula (4). It turned out that, for the indicated deflections at the center and the indicated thicknesses of the specimens, the value of the coefficient $c' \simeq 0.7$ and differs from this mean value by no more than 0.05. This value of the coefficient c' differs from the theoretical value found in (1), equal to 0.9. The discrepancy is explained as follows.

The determination of the coefficient c' in (1) is connected with the solution of a certain variational problem for the functional J, which specifies the deformation energy of the shell. The value $c' = 0.9$ corresponds to the first approximate solution of this problem. The refined solution of the problem, contained in ([1], p. 57), gives the value 0.8 for the coefficient c'.

The remaining discrepancy between the theoretical and experimental values of c' is apparently explained by the weak sensitivity of the functional J to the shape of the shell near the equilibrium state, which cannot be said about the influence of this shape on the coefficient c'. Thus, for example, in the indicated variant of the solution of the variational problem ([1], p. 58), a deviation in the shape of the shell that gives a change in energy of only 1.5% entails a change in the coefficient c' of 10%.

The experiment described was carried out by the author jointly with collaborators G. P. Kropachev and A. N. Pedorenko.

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1. A. V. Pogorelov, *Geometric Methods in the Nonlinear Theory of Elastic Shells*, "Nauka," 1967.

Note: Figure translations are in progress. See original paper for figures.