# ON THE DEPENDENCE OF THE REGIME OF GENERATION OF INDUCED RADIATION ON THE SPATIAL DISTRIBUTION OF ENERGY

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Fig. 1

Figure 1: Fig. 1

**Abstract**

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## PHYSICS

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# ON THE DEPENDENCE OF THE REGIME OF GENERATION OF INDUCED RADIATION ON THE SPATIAL DISTRIBUTION OF ENERGY

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In paper ([1]) it was noted that the motion of a dielectric during the generation of induced radiation will lead to a modulation of the generation process. Subsequently, A. T. Tursunov ([2]) experimentally discovered and investigated this modulation. The present work is devoted to studying the influence of the energy distribution of a standing wave on the regime of generation of induced radiation.

Let the dielectric move along the $z$-axis with velocity $v$ (Fig. 1). For sufficiently small $v$ (see below) one may consider the problem to be quasistationary; in this approximation the electromagnetic field of the resonator is determined by the instantaneous position of the system, and first one should consider a resonator with an immobile dielectric.

**Fig. 1**

The resonator with a dielectric (region 2) is shown in Fig. 1; from the figure the meaning of the quantities $z_0, a, d, L$ is clear. Let the system be unbounded in the plane perpendicular to the $z$-axis; we shall consider field oscillations for which the wave vector coincides with the $z$-axis. In this one-dimensional case the natural oscillations of the resonator field are described by a system of standing waves (longitudinal modes). Let $k_i = 2\pi\lambda_i^{-1}$ be the wave vector of the $i$-th mode, $\lambda_i$ the wavelength in regions 1 and 3, $\mu$ the refractive index of the dielectric, and $i$ the ordinal number of the mode. Using the conditions of continuity of the tangential components of the electric and magnetic fields at the dielectric boundaries, we obtain a system of equations determining the possible values of the wave vector $k_i$, the ratios of the maximum amplitudes, and also the phase of the standing wave in the dielectric.

In determining $k_i$, the boundary conditions lead to a transcendental equation; its consideration shows that among the resonator modes there may be "immobile" modes, for which $k_i$ does not depend on $z_0$ (for these modes $k_i d = p\pi$, $p$ an integer), while the remaining modes are "sliding" modes, for which $k_i$ depends on $z_0$ (with period $\lambda_i/2$). The mechanism that ensures the change in the frequencies of the "sliding" modes upon displacement of region 2 is associated with the Doppler effect at the dielectric boundaries.

Taking $k_i$ as known, one can find the ratios $W_1^i, W_2^i, W_3^i$—the energy densities of the $i$-th mode in regions 1, 2, and 3 respectively:

$$\eta_1(k_i z_0) = \frac{W_1^i}{W_2^i} = \left[\mu^2 - (\mu^2 - 1)\cos^2 k_i z_0\right]^{-1};$$

$$\eta_2(k_i z_0) = \frac{W_3^i}{W_2^i} = \left[\mu^2 - (\mu^2 - 1)\cos^2 k_i (d - z_0)\right]^{-1}. \tag{1}$$

Let the total energy of the $i$-th mode be $\varepsilon^i$; then

$$W_1^i = \eta_1 \eta_2 \frac{\varepsilon^i}{a}; \qquad W_2^i = \eta_2 \frac{\varepsilon^i}{a}; \qquad W_3^i = \eta_3 \eta_2 \frac{\varepsilon^i}{a};$$

$$\eta_2(k_i, z_0) = \left(1 + \frac{z_0}{a}\eta_1 + \frac{d - z_0}{a}\eta_3\right)^{-1}. \tag{2}$$

As we see, $W_1^i, W_2^i$, and $W_3^i$ depend on $z_0$ periodically with period $\lambda_i/2$. For the "sliding" modes this periodicity is approximate.

When the dielectric moves, $z_0 = vt$, and the redistribution of energy over the regions will occur with frequency $f_i = (\tau_i)^{-1} = 2v\lambda_i^{-1}$.

The exact solution of Maxwell' s equations, in addition to standing waves, must also contain a certain traveling wave that ensures redistribution of energy over regions 1, 2, and 3. One can estimate $W_b$—the mean energy density associated with this traveling wave: $W_b c\tau_i \sim (W_2^{i\,\max} - W_2^{i\,\min}) < W_2^{i\,\max} L$. In order that, in determining the distribution of energy over the regions, the contribution of the traveling wave can be neglected, it is necessary to require that the condition $W_b/W_2^{i\,\max} \ll 1$ be satisfied, whence $v \ll \dfrac{\lambda_i}{L} c$, which is the condition of quasistationarity.

If the system under consideration serves for the generation of stimulated radiation, then a positive population inversion is created in the dielectric. The generation regime is described by the kinetic equations (3), applicable in the case when the mirrors are adjacent to the dielectric. For the composite resonator (Fig. 1) these equations must be modified:

a) The energy losses of the $i$-th mode will now depend on the position of the dielectric

$$\gamma(k_i, z_0) = \frac{\eta_2}{a}\{[\gamma_1^s\eta_1 + \gamma_2^s + \gamma_3^s\eta_3] + [\gamma_1^0\eta_1 z_0 + \gamma_2^0 a + \gamma_3^0\eta_3(d - z_0)]\}; \quad (3)$$

here the first square bracket takes into account losses proportional to $W_1^i$, $W_2^i$, and $W_3^i$ (for example, losses at the mirrors); $\gamma_1^s, \gamma_2^s, \gamma_3^s$ are the coefficients of surface losses in the corresponding regions. The second square bracket takes into account losses uniformly distributed over the volume (for example, scattering); $\gamma_1^0, \gamma_2^0, \gamma_3^0$ are the coefficients of volume losses (in $\gamma^s$ and $\gamma^0$ the indices $i$ are omitted). $\gamma(k_i, z_0) = -N_i^{-1}dN_i/dt$ is the specific (per photon per second) loss of the system in the absence of generation; $N_i$ is the number of photons in the $i$-th mode. When the dielectric moves, $\eta_1, \eta_2, \eta_3$ and $z_0 = vt$ explicitly depend on time and, consequently, $\gamma(k_i, z_0)$ also depends on time, this dependence being approximately periodic with period $\tau_i$.

b) Since the probability of stimulated emission is proportional to the energy in region 2 ($aW_2^i$), this probability also changes with period $\tau_i$, for $aW_2^i = \eta_2 N_i \hbar k_i c$.

Thus, both the losses of the optical generator and the probability of stimulated emission vary in time with period $\tau_i$; this will lead to the fact that the radiation intensity of such a system will also vary with period $\tau_i$. Note that for $d \gg a$ the probability of stimulated emission changes strongly, while the depth of modulation of the losses at the mirrors is then small; for $d \ll a$, conversely, the probability of stimulated emission changes little, but the losses at the mirrors are strongly modulated.

If the characteristic time for establishing the stationary generation regime (the relaxation time) is substantially less than the period $\tau_i$, then the generation problem is "quasistationary," and one may use the results obtained for stationary generation of an immobile dielectric.

If the mirrors are adjacent to the dielectric, then in the spatially homogeneous case (rapid migration of the population inversion) stationary generation (for a homogeneously broadened luminescence line) is possible only on one mode, located closest to the maximum of the function $g(k)$, which gives the intensity distribution in the luminescence line. In the case of a composite resonator with a moving dielectric, in "quasistationary" generation there will be the mode for which $\Omega(k_i, z_0) = g\eta_2\gamma^{-1}$ has a maximum. As the dielectric moves, the maximum of $\Omega(k_i, z_0)$ falls at different times on different $k_i$, and generation will terminate on one mode and start on another. The generation pattern will repeat after the time $\tau_i$. At low pump intensities it may turn out that during some part of the period $\tau_i$ generation does not occur at all.

Fig. 2

Figure 2: Fig. 2

In cases where the spatial inhomogeneity of the mode field plays a substantial role (solid-state lasers), several modes may be present simultaneously in steady-state generation; their number depends on the magnitude of the pumping. The multimode nature of steady-state generation is connected with the fact that the regions of effective depletion of the inverse population by different modes are spatially separated [(3)]. If the mirrors are located at the boundaries of the dielectric, then the numbers of the modes entering generation are arranged consecutively [(4)]. In a composite resonator with a stationary dielectric, in steady-state generation there will be many modes arranged not consecutively in $i$, but separated by intervals in which there are many (tens of) modes that do not enter generation [(1)]. This is due to the fact that, as the pump intensity increases, the first to enter generation is the mode for which $\Omega(k_i, z_0)$ has a maximum, the second is usually the mode closest to it in magnitude of $\Omega(k_i, z_0)$ (but not in the number $i$), and so on. Since $\Omega(k_i, z_0)$ is not a smooth, but an approximately periodic function of $k$ (with a period comparable with $k_i - k_{i-1}$), the sequence of decreasing values of $\Omega(k_i, z_0)$ does not correspond to modes arranged consecutively in $i$. When the dielectric moves, the sequence of decreasing values of $\Omega(k_i, vt)$ will correspond to the sequence of $i$ arranged consecutively near the maximum of the luminescence line; to be sure, $\Omega(k_i, vt)$ will attain its maximum values for different $k_i$ at different times. Thus, when region 2 moves, a sufficiently complete depletion of the inverse population can be ensured by modes arranged consecutively in $i$. Consequently, the mode frequencies in the generation spectrum will be pulled toward the center of the luminescence line, and although the number of modes may change little, the spectral interval occupied by them will decrease. This effect of narrowing of the generation spectrum will increase as the velocity $v$ increases and as the pump power decreases.

**Fig. 2**

Let us note some features of the generation process connected with the redistribution of energy over regions 1, 2, and 3:

1) If, in the spiking regime of generation, a spike proves to be shorter in time than $\tau_i$, then the redistribution of energy affects the time regime of generation only indirectly. If the duration of a spike is greater than $\tau_i$, then interruptions of generation as a result of energy redistribution will lead to the spike being unable to develop normally, and the generation will be stretched out in time; at a sufficiently high modulation frequency, the spikes stretched out in time will merge into a single sequence in which the generation pattern is repeated (in frequency and intensity) after a time $\tau_i$, i.e., a spike-free periodic regime (modulated with period $\tau_i$) takes place. Since the modulation on different modes is shifted in time, the total radiation intensity can give the appearance of "continuous" generation in

time.

2) When light falls on the boundary of the dielectric at the Brewster angle (Fig. 2), when there is no reflection, a simple consideration shows that all longitudinal modes ($k_i$ in the gaps perpendicular to the mirrors) are "stationary," i.e., their frequencies do not depend on the position of the dielectric; moreover, these modes are equidistant in frequency, $k_i = \pi\Delta^{-1}i$ ($\Delta$ is the optical length of the resonator). In addition, independently of the position of the dielectric, the energy density in the dielectric and in the gaps is the same and, consequently, no modulation should be observed, which is confirmed by experiment [(2)]. Since in the resonator of Fig. 2 mode discrimination due to the different distribution of energy over the regions of the resonator does not occur, then (as the pump intensity increases) with a stationary dielectric the modes will enter generation consecutively in the index $i$. We note that the resonator shown in Fig. 2 is convenient for experiments whose results are compared with theories that do not take into account the distribution of energy over regions 1, 2, 3 as a function of the position of the dielectric (for example, the theory of a "travelingrayele

medium" (4)), and with theories for which the equidistance of the frequencies of the longitudinal modes is essential.

3) Modulation should also be observed when the active region is stationary, but a moving dielectric with boundaries parallel to the mirrors has been additionally introduced into the resonator. The results of such experiments and of the experiments mentioned in item 2) make it possible to determine: (a) which features of generation are connected only with the redistribution of energy, and (b) which features of generation are connected only with the motion of active centers relative to the nodes and antinodes of standing waves (4). Experiments with a moving active region in the resonator shown in Fig. 1, however, do not provide such a possibility of separating effects (a) and (b).

4) In generation on "creeping" modes $W_1^i \neq W_3^i$, and, consequently, through the right and left mirrors (for equal transparency) different radiation powers will emerge at the given frequency. The energy distributions in the generation spectrum will be different for radiation passing through the right and left mirrors.

5) The case in which the dielectric boundaries make an arbitrary angle $\alpha \neq 0$ with the mirrors has not been considered in this work; however, it may be expected that, for sufficiently small angles of inclination, the transverse structure of the mode will become more complicated—the intensity will depend periodically on $y$. When the dielectric moves along the $z$ axis, this structure will move along the $y$ axis with velocity $v \operatorname{ctg} \alpha$.

6) A change in the optical length of the resonator over a cross section perpendicular to the $z$ axis (owing to inhomogeneity of the dielectric) will lead to the energy density being different at different points of the cross

section (and, therefore, the generation conditions will be different), which will strengthen the influence of defects of the dielectric on the transverse structure of the mode and complicate this structure. When the dielectric moves, this structure will change periodically with period $\tau_i$.

7) Additional inhomogeneity (in the distribution of energy over the cross section) can affect not only the transverse structure of the mode, but also the temporal regime (when generation arises, the inhomogeneity increases). When light is incident on the boundary of the dielectric at the Brewster angle (Fig. 2), there will be no strengthening of the influence of inhomogeneities on the generation regime due to different conditions at the end face of the dielectric. In particular, the system shown in Fig. 2 will be more stable in generation under thermal expansion or under longitudinal mechanical oscillations of the active region than the system shown in Fig. 1. If the spikiness of the temporal regime is connected with the presence of inhomogeneities, then their partial elimination may lead to a more regular, and even continuous, generation regime with a stationary dielectric. This makes it possible, by introducing bodies with different inhomogeneities into the gaps (regions 1, 3), to investigate experimentally their influence on the generation regime.

8) In the case of an inhomogeneously broadened luminescence line, when the dielectric moves, not one but (depending on the pumping) several groups of closely spaced modes (a fork) may form in the generation spectrum.

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*Note: Figure translations are in progress. See original paper for figures.*

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