AN R-OPERATION WITH FULL DEPTH OF CHAINS OVER SYSTEMS OF SETS OF CARDINALITY τ

1970

SovietRxiv

Abstract

Full Text

UDC 519.52

Z. I. KOZLOVA

AN R-OPERATION WITH FULL DEPTH OF CHAINS OVER SYSTEMS OF SETS OF CARDINALITY τ

(Presented by Academician M. A. Lavrent' ev on 12 I 1970)

Let V be the totality of all ordinal numbers; K_I the totality of indefinite ordinal numbers, K_{II} the limiting ordinal numbers; $I = \{i \mid i \in V \ \& \ i < \omega_\nu\}$, $\tau = \aleph_\nu$ a strongly inaccessible cardinal number; $Y_\gamma = \langle i_0, \dots, i_a, \dots \mid \gamma \rangle$ a tuple of rank γ, where $\gamma < \omega_\nu$ and $(\forall a < \gamma)[i_a \in I]$; $W = \{Y_\gamma \mid \gamma < \omega_\nu\}$ the space of tuples $(\operatorname{Card} W = \tau)$; $\eta \subseteq I$, $\vartheta \subseteq W$ chains; $\Xi = \{\eta \mid \eta \subseteq I\}$, $\Xi^* = \{\vartheta \mid \vartheta \subseteq W\}$ spaces of chains; $N \subseteq \Xi$, $M \subseteq \Xi^*$ bases; $\mathfrak{R}$ the basic space, whose subsets are studied.

Put $Y_\gamma,\ i_\gamma = \langle i_0, \dots, i_a, \dots, i_\gamma \rangle$, if $Y_\gamma = \langle i_0, \dots, i_a, \dots \mid \gamma \rangle$;

$$Y_\gamma Y_{\gamma'} = \langle i_0, \dots, i_a, \dots, i_{\gamma'}, \dots, i^\alpha, \dots \mid \gamma + \gamma' \rangle,$$

if $Y_\gamma = \langle i_0, \dots, i_a, \dots \mid \gamma \rangle$,

$$Y_{\gamma'} = \langle i_\gamma, \dots, i_{\gamma+\alpha}, \dots \mid \gamma' \rangle.$$

We shall say that Y is subordinate to Y', and write $Y' < Y$, if

$$Y = \langle i_0, \dots, i_a, \dots \mid \gamma \rangle, \qquad Y' = \langle i_0, \dots, i_a, \dots \mid \gamma' \rangle$$

and $\gamma' < \gamma$; $Y' \leq Y$, if $Y' < Y$ or $Y' = Y$. The rank of Y is denoted by $\rho(Y)$.

A. A. Lyapunov ([1]) considered an R-process of transfinite transformation of sets of a given family of arbitrary cardinality, starting from a given system of $\Delta\Sigma$-operations. The depth of the chains of this R-process is equal to ω. We shall study an R-process of transfinite transformation of sets of a given family of cardinality τ, starting from a given system of $\Delta\Sigma$-operations, when the depth of the chains of the R-process is full, i.e. equal to ω_ν.

Let

$$\mathfrak{R} = (N_Y)_{Y \in W}, \qquad N_Y \subseteq \Xi,$$

$(E_Y)_{Y \in W}$ be an arbitrary family of sets of the space $\mathfrak{R}$, in which

$$E_{Y_\gamma} = \bigcap_{Y' < Y_\gamma} E_{Y'} \quad \text{for } \gamma \in K_{\mathrm{II}}.$$

By the ν-conjunctive extension of $\Delta\Sigma$-operations with bases of the family $\mathfrak{R}$, where $\nu = \omega^\lambda \leq \omega_\nu$, we call the family of $\Delta\Sigma$-operations whose bases

$$(M_Y^\lambda)_{Y\in W} = \mathfrak{M}(\nu)$$

are defined as follows:

$$\mu \in M_Y^\lambda \equiv (\rho(Y) \in K_I \& Y \in \mu \vee \rho(Y) \in K_{\mathrm{II}} \& (\forall Y' < Y)[Y' \in \mu]) \&$$
$$\& (\forall YY' \in \mu)(\exists \eta \in N_{YY'})(\forall i \in \eta)[YY', i \in \mu] \&$$
$$\& (\forall \gamma < \nu)(\forall Y' < Y_\gamma)[\gamma \in K_{\mathrm{II}} \& YY' \in \mu \to$$
$$\to (\exists \eta \in N_{YY_\gamma})(\forall i \in \eta)[YY_\gamma, i \in \mu]].$$

Put

$$E_Y^0 = E_Y, \qquad E_Y^1 = R_Y, \qquad \mathfrak{M}\{E_{Y'}\} = \Phi_{M_Y^1}\{E_{Y'}\}.$$

If the derived sets $(E_Y^\alpha)_{Y\in W}$ are defined, then we construct the family of bases $(N_Y^\alpha)_{Y\in W}$ in the following way: let

$$Z_Y^\alpha(\eta) = \{Y_{\omega_\alpha} \mid (\forall \beta < \omega^\alpha)[Y_\beta < Y_{\omega_\alpha} \to YY_\beta \in \eta]\},$$
$$N_Y^\alpha(\eta) = \{\xi \mid (\exists Y_{\omega_\alpha} \in Z_Y^\alpha(\eta))(\exists \eta_{Y_{\omega_\alpha}} \in N_{YY_{\omega_\alpha}}) \times$$
$$\times [\xi = (YY_{\omega_\alpha}, i)_{i \in \eta_{Y_{\omega_\alpha}},\ Y_{\omega_\alpha} \in Z_Y^\alpha(\eta)}]\};$$

then

$$N_Y^\alpha = \{\eta \mid (\exists \mu \in M_Y^\alpha)(\exists \xi \in N_Y^\alpha(\mu))[\eta = \mu + \xi]\}.$$

Put

$$E_Y^{\alpha+1} = R_{Y;\{N_Y^\alpha\}}\{E_Y^\alpha\}.$$

If the derivative sets $(E_Y^\alpha)_{Y\in W}$ are defined for all $\alpha < \varkappa < \omega_\nu,\ \varkappa \in K_{\mathrm{II}}$, then put $E_Y^\varkappa = \bigcap_{\alpha<\varkappa} E_Y^\alpha$.

Construct a family of bases $(N_Y^\varkappa)_{Y\in W}$ and put

$$E_Y^{\varkappa+1} = R_{Y;\{N_Y^\varkappa\}}\{E_Y^\varkappa\}.$$

Thus, we shall construct derivative sets $(E_Y^\alpha)_{Y\in W}$ for all $\alpha < \omega_\nu$. Put

$$E_Y^{\omega_\nu} = \bigcap_{\alpha<\omega_\nu} E_Y^\alpha.$$

Repeating the process of differentiating sets, starting from the family of sets $(E_Y^{\omega_\nu})_{Y\in W}$ as the initial family, we obtain derivative sets $(E_Y^{\omega_\nu\cdot 2})_{Y\in W}$. By further repetition of this process we obtain derivative sets $(E_Y^{\omega_\nu\cdot\beta})_{Y\in W}$ for $\beta < \omega_{\nu+1}$, while taking

$$E_Y^{\omega_\nu\cdot\varkappa} = \bigcap_{\alpha<\varkappa} E_Y^{\omega_\nu\cdot\alpha},$$

if $\varkappa \in K_{\mathrm{II}}$. Let
$$\vartheta^{\omega_\nu \cdot \alpha}(x) = \{Y \mid x \in E_Y^{\omega_\nu \cdot \alpha}\}.$$
Since the sequence of chains
$$\vartheta^{\omega_\nu}(x) \supseteq \vartheta^{\omega_\nu \cdot 2}(x) \supseteq \cdots \supseteq \vartheta^{\omega_\nu \cdot \alpha}(x) \supseteq \cdots$$
does not increase, $\operatorname{Card} W = \aleph_\nu$, and $\operatorname{Card} \omega_{\nu+1} = \aleph_{\nu+1}$, it follows that
$$(\exists \alpha_0 < \omega_{\nu+1})(\forall \alpha > \alpha_0) \times [\vartheta^{\omega_\nu \cdot \alpha_0}(x) = \vartheta^{\omega_\nu \cdot \alpha}(x)].$$
The stabilized chain $\vartheta^{\omega_\nu + \alpha_0}(x)$ we shall denote by $\vartheta^{\omega_{\nu+1}}(x)$.

We shall call the operation
$$(\omega_\nu) R_{Y;\mathfrak{m}}$$
over a family of sets $[E_Y)_{Y \in W}$ the $(\omega_\nu) R_{Y;\mathfrak{m}}$-operation
$$(\omega_\nu) R_{Y;\mathfrak{m}}\{E_Y\} = \bigcap_{\alpha < \omega_{\nu+1}} E_Y^{\omega_\nu \cdot \alpha} = E_Y^{\omega_{\nu+1}}.$$

The family of $\Delta\Sigma$-operations
$$((\omega_\nu) R_{Y;\mathfrak{m}})_{Y \in W}$$
is the ω_ν-conjunctive extension of $\Delta\Sigma$-operations with bases of the family $\mathfrak{M}$.

Put
$$(\nu) R_{Y;\mathfrak{m}} = \Phi_{M_Y^\lambda},$$
where $M_Y^\lambda \in \mathfrak{M}(\nu)$, $\nu = \omega^\lambda \leqslant \omega_\lambda$. Let $\mathfrak{C} = (P_Y)_{Y \in W}$, where $P_Y \subseteq \Xi$, and let $\theta \subseteq \Xi^*$, $Y \in \vartheta \in \theta$. We replace the tuple Y by the family of tuples
$$\lambda_Y^1 = \{Y;\ i_0 \mid (i_0) = \xi \in P_Y\}.$$
If Y is replaced by λ_Y^α, and at the same time at least one of the tuples
$$YY_\alpha = Y\langle i_0, \dots, i_{\alpha'}, \dots \mid \alpha\rangle \in \lambda_Y^\alpha$$
is replaced by
$$\lambda_{YY_\alpha} = \{Y\langle i_0, \dots, i_\alpha\rangle \mid (i_\alpha) = \xi \in P_{YY_\alpha}\},$$
then we say that Y is replaced by $\lambda_Y^{\alpha+1}$. If Y is replaced by λ_Y^α for all $\alpha < \gamma$, then we replace the tuple
$$YY_\gamma = Y\langle i_0, \dots, i_\alpha, \dots \mid \gamma\rangle,$$
where
$$Y\langle i_0, \dots, i_{\alpha'}, \dots \mid \alpha\rangle \in \lambda_Y^\alpha \quad \text{for } \alpha < \gamma,$$
by the family of tuples
$$\lambda_{Y\langle i_0, \dots, i_\alpha, \dots \mid \gamma\rangle} = \{Y\langle i_0, \dots, i_\gamma\rangle \mid (i_\gamma) = \xi \in P_{YY_\gamma}\},$$

and say that Y is replaced by the family of tuples $\lambda_Y^{\gamma+1}$.

By the ν-disjunctive extension of $\Delta\Sigma$-operations with bases of the family $\mathfrak{C}$ we shall mean the family of $\Delta\Sigma$-operations whose bases $(\theta_Y)_{Y\in W} = \mathfrak{C}(\nu)$ are defined as follows:

$$\vartheta \in \theta_Y \equiv \rho(Y) \in K_{\mathrm{II}} \,\&\, (\exists Y' < Y)[\vartheta = \{Y'\}] \vee \rho(Y) \in K_I \& \vartheta = \{Y\} \vee (\exists \vartheta' \in \theta_Y)(\exists Y' \in \vartheta')(\exists \gamma < \nu)(\exists \lambda_{Y'}^{\gamma+1})[\vartheta =$$

The operations supplementary to the $(\omega_\nu)R_{Y;\mathfrak{m}}$-operations are

$$(\omega_\nu)R^c_{Y;\mathfrak{m}^c}$$

-operations, whose bases are the bases of the ω_ν-disjunctive extension of $\Delta\Sigma$-operations with bases of the family

$$\mathfrak{M}^c = (N^c_Y)_{Y\in W}.$$

They can also be obtained by means of the R^c-process, starting from operations with bases of the family $\mathfrak{M}^c$.

For an $(\omega_\nu)R_{Y;\mathfrak{m}}$-operation one may define external indices:

$$(\omega_\nu)R_{Y;\mathfrak{m}}\,\mathrm{Ind}(x \mid \{E_Y\}) = \begin{cases} \omega_{\nu+1}, & \text{if } x \in E_Y^{\omega_{\nu+1}}, \\ \beta, & \text{if } x \in \bigcap\limits_{\alpha<\beta} E_Y^\alpha \setminus E_Y^\beta \quad (\beta < \omega_{\nu+1}). \end{cases}$$

The internal index of an $(\omega_\nu)R_{Y;\mathfrak{m}}$-operation is defined at points $x \in E_Y^{\omega_{\nu+1}}$. It is equal to the least of the numbers β such that the structure of the family of sets $(E_Y^\alpha)_{Y\in W}$ remains unchanged for all $\alpha \geqslant \beta$.

The external indices $(\omega_\nu)R_{\mathfrak{M}}$-operations satisfy the principle of comparison of indices:

Let $\mathfrak{M}_1 = (N_Y)_{Y\in W}$, $\mathfrak{M}_2 = (P_Y)_{Y\in W}$ be families of bases, where $N_Y \subseteq \Xi$, $P_Y \subseteq \Xi$; $(E_Y)_{Y\in W}$, $(H_Y)_{Y\in W}$ are arbitrary families of sets; $(E_Y^\alpha)_{Y\in W,\alpha<\omega_{\nu+1}}$, $(H_Y^\alpha)_{Y\in W,\alpha<\omega_{\nu+1}}$ are the derived sets of these families;

$$\Phi^{\alpha+1}_{M_Y} \equiv R_{Y;\{N_Y^\alpha\}}; \qquad \Phi^{\alpha+1}_{Q_Y} \equiv R_{Y;\{P_Y^\alpha\}} \quad \text{for } \alpha < \omega_\nu; \qquad U_{YY'} = E_Y \cup CH_{Y'},$$

$$\Phi^{\alpha+1}_{L_{YY'}}\{U_{YY'}\} = \Phi^{-\alpha+1,c}_{Y_1',Q_{Y'}}\{\Phi^{-\alpha+1}_{Y_1,M_Y}\{U_{Y_1Y_1'}\}\},$$

where

$$\Phi_{\overline{M_Y}^{\alpha+1}}\{E_{Y'}\} = E_Y \cap \Phi^{\alpha+1}_{M_Y}\{E_{Y'}\}, \qquad \Phi_{\overline{Q_Y}^{\alpha+1}}\{E_{Y'}\} = E_Y \cap \Phi^{\alpha+1}_{Q_Y}\{E_{Y'}\}.$$

Then the R-iteration with variable bases

$$\mathfrak{L} = (L^{\alpha+1}_{YY'})_{Y,Y'\in W,\alpha<\omega_{\nu+1}},$$

performed over the family of sets $(U_{YY'})$, gives, for any $\alpha < \omega_{\nu+1}$,

$$U^{\alpha}_{YY'} = \bigcap_{\beta\leq\alpha}(E^{\beta}_{Y} \cup CH^{\beta}_{Y'}),$$

$$(\omega_\nu)R_{\mathfrak{D}}\{U_{YY'}\} = \{x \mid (\omega_\nu)R_{\mathfrak{M}_1}\operatorname{Ind}(x \mid \{E_Y\}) \geq (\omega_\nu)R_{\mathfrak{M}_2}\operatorname{Ind}(x \mid \{H_Y\})\}.$$

Let $\mathfrak{M} = (N_\alpha)_{\alpha\in I\cup W_{\mathfrak{M}}}$, where $N_\alpha \subseteq \Xi$, and let $W_{\mathfrak{M}}$ be the set of all coordinated sequences $(i_\alpha)_\gamma = Y_\gamma$, for $\gamma \in K_{\text{II}}$, $\gamma < \omega_\nu$, in which $i_{\alpha+1}$, i_χ for $\chi \in K_{\text{II}}$ are immediately subordinate, respectively, to $i_\alpha, (i_\alpha)_\chi$. An $(\omega_\nu)T_{\mathfrak{M}}$-operation over a family of sets $(E_i)_{i\in I}$ is called a $\Delta\Sigma$-operation whose base M is the base of an ω_ν-conjunctive extension of $\Delta\Sigma$-operations with bases of the family $\mathfrak{M}$, i.e.

$$\mu \in M \equiv 0 \in \mu \ \&\ (\forall i \in \mu)(\exists \eta \in N_i)[\eta \subset \mu]\ \&$$

$$\&\ (\forall (i_\alpha)_\gamma \in W_{\mathfrak{M}})[(i_\alpha)_\gamma \subset \mu \to (\exists \eta \in N_{(i_\alpha)_\gamma})[\eta \subset \mu]].$$

The base of the complementary operation $(\omega_\nu)T^{c}_{\mathfrak{M}^c}$ is the base of an ω_ν-disjunctive extension of $\Delta\Sigma$-operations with bases of the family

$$\mathfrak{M}^c = (N^c_\alpha)_{\alpha\in I\cup W_{\mathfrak{M}}}.$$

The operations $(\omega_\nu)T_{\mathfrak{M}}$ and $(\omega_\nu)T^{c}_{\mathfrak{M}^c}$ are called operations without interlocking if there exists a family of pairwise nonintersecting chains $(\eta^\alpha)_{\alpha\in I\cup W_{\mathfrak{M}}}$ having the properties:

$$(\text{H})\ \forall\alpha\,[\eta^\alpha \sim I \ \&\ 0 \notin \eta^\alpha \ \&\ (\alpha \in I \to \alpha \notin \eta^\alpha)\ \&\ (\alpha \in W_{\mathfrak{M}} \to (\forall i \in \alpha)[i \notin \eta^\alpha])].$$

$$(\text{H}^{c}_{\mathfrak{M}})\ \forall\alpha\ (\forall \eta \in N_\alpha)\ [\eta \subseteq \eta^\alpha].$$

Every $(\omega_\nu)T$-operation without interlocking is equivalent to some $(\omega_\nu)R$-operation, and conversely. Let the base N satisfy the conditions

$$1^\circ)\quad (\omega_\nu)T_N \succ \bigcup_{\tau},\quad (\omega_\nu)T_N \succ \bigcap_{\tau},\quad (\omega_\nu)T_N \succ \Phi_{N^c},\quad (\omega_\nu)T_N \succ \Phi_{N\alpha c}\quad \text{for } \alpha < \omega_\nu,$$

$$[1^{00}) \quad (\omega_\nu)T_N \succ \bigcup_\tau, \quad (\omega_\nu)T_N \succ \bigcap_\tau, \quad (\omega_\nu)T_N \succ \Phi_{N^c}].$$

$$2^\circ) \quad (\Phi_N, d) \prec \Phi_N, \quad (\Phi_{N^c}, d) \prec \Phi_{N^c}.$$

Condition 2° may be replaced by the condition

$$2') \quad (\omega_\nu)T_{T_N} \prec (\omega_\nu)T_N, \qquad ((\omega_\nu)T_N^c, d) \prec (\omega_\nu)T_N^c.$$

Denote by $(\omega_\nu)\mathfrak{B}_N$ the class of $\Delta\Sigma$-operations possessing the following properties: 1) the trivial operations $\bigcap, \bigcup, \Phi_N, \Phi_{N^c}, \Phi_N^\alpha, \Phi_N^{\alpha c}, T_{\{N^\alpha\}}, T_{\{N^{\alpha c}\}}^c$, for all $\alpha < \omega_\nu$, belong to the class $(\omega_\nu)\mathfrak{B}_N$; 2) if $\Phi_L \in (\omega_\nu)\mathfrak{B}_N$, $\Phi_{M_i} \in (\omega_\nu)\mathfrak{B}_N$ for $i \in I$, then $\Phi_L\{\Phi_{M_i}\} \in (\omega_\nu)\mathfrak{B}_N$; 3) the class $(\omega_\nu)\mathfrak{B}_N$ is closed with respect to shifts of bases; 4) $(\omega_\nu)\mathfrak{B}_N$ is the smallest class of sets satisfying conditions 1)–3). By $(\omega_\nu)T\mathfrak{B}_N$ we denote the class of ω_ν-conjunctive extensions of $\Delta\Sigma$-operations of the class $(\omega_\nu)\mathfrak{B}_N$. Under the given conditions, the operations belonging to the classes $(\omega_\nu)\mathfrak{B}_N$ and $(\omega_\nu)T\mathfrak{B}_N$ are no stronger than operations of type $(\omega_\nu)T_N$.

For each $(\omega_\nu)T$-operation one can construct equivalent operations leading to an increase of the index of the initial operation by one and to a doubling of the index.

Let $N \subseteq \Xi$. Put $\Phi_{N_0} \equiv \Phi_N$. The type of a $\Delta\Sigma$-operation $(\omega_\nu)T_N = \Phi_N$ will be denoted by $(\omega_\nu)T_N^1$. If the type $(\omega_\nu)T_N^\alpha$ is defined, then by $(\omega_\nu)T_N^{\bar\alpha}$ we denote the type of supplementary operations to operations of type $(\omega_\nu)T_N^\alpha$. The type

$$(\omega_\nu)T_N^{\alpha+1} \equiv (\omega_\nu)T_{\{N_{\bar\alpha}^c\}} \equiv \Phi_{N_{\alpha+1}},$$

where

$$\Phi_{N_{\bar\alpha}^c} \equiv (\omega_\nu)T_N^\alpha.$$

If $\Delta\Sigma$-operations Φ_{N_α} are defined for all $\alpha < \varkappa < \omega_{\nu+1}$, $\varkappa \in K_{\mathrm{II}}$, then let $(\beta_j) \to \varkappa$, $\beta_j < \varkappa$;

$$I = \bigcup_j \eta^j$$

be a decomposition of the space I such that $\forall j[\eta^j \sim I]$;

$$\varphi_j(t) = t : I \to \eta^j$$

is a bijection. Then

$$\Phi_{N_\varkappa}\{E_i\} = \bigcap_{(\beta_j)\to\varkappa} \Phi_{N_{\varphi_j}}\{E_{\varphi_j(i)}\}.$$

The type of $\Delta\Sigma$-operation

$$\Phi_{N_{\varkappa+1}} \equiv (\omega_\nu) T_{\{N_\varkappa\}}$$

will be denoted by $(\omega_\nu)T_N^{\varkappa+1}$. The strengthening of types of $\Delta\Sigma$-operations can also be continued to transfinite numbers α, where $\omega_{\nu+1} < \alpha < \omega_{\nu+2}$, as was done in work $(^2)$, Chapter IV. If the constructed operations are applied to some class of sets K, then we obtain an $(\omega_\nu)R_N$- or $(\omega_\nu)T_N$-classification of sets. This classification is monotone if the base N satisfies condition 1^0. If the base N satisfies conditions 1^0 and 2^0, then the operations $(\omega_\nu)T_N^{\alpha+1}$ are normal,

$$(\omega_\nu)T_{N_\alpha^c} \equiv (\omega_\nu)T_{N_\alpha^c}$$

with respect to the class of sets $K \supset \emptyset, \mathfrak{N}$. If N satisfies condition 1^0, then each of the bases $N_{\alpha+1}^c, N_\varkappa$, for $\varkappa \in K_{\mathrm{II}}$, satisfies this condition.

If the class of sets $K \supset \emptyset, \mathfrak{N}$ is closed with respect to the operations of complementation and $\bigcup_n$ for $n < \omega$, and the base N satisfies conditions $1^0, 2^0$ or $2'$, then the class of external indices of operations of type $(\omega_\nu)T_N^{\alpha+1}$ for $\alpha < \omega_{\nu+1}$ is completely regular, and for the classes $(\omega_\nu)T_N^{\alpha+1}(K)$ the first and second separation laws and the multiple separation law hold with respect to operations $\Phi_{\mathfrak{M}}$ such that the class $(\omega_\nu)T_N^{\alpha+1}(K)$ is closed with respect to the operations $\Phi_{\mathfrak{M}i}$ for $i \in I$. These include the operations $(\omega_\nu)T_N^\beta$ for $\beta \leq \alpha+1$, $(\omega_\nu)T_{N^{\beta c}}$ for $\beta \leq \alpha$.

If the base N satisfies conditions $1^0, 2^0$ or $2'$ and condition 3^0. If $L \subseteq \mathfrak{M}_{\tau^*}$, where

$$\mathfrak{M} = \{N\} \cup \{N^i\}_{i\in I}, \qquad \tau^* \leq \aleph_{\nu+1},$$

and $\mathfrak{M}_{\tau^*}$ is the totality of intersections of $< \tau^*$ sets of the family $\mathfrak{M}$, then

$$\Phi_L < \Phi_N^*$$

or

$$\Phi_L < T_N;$$

then for the class $(\omega_\nu)T_N^{\varkappa+1}(K)$, $K \supset \emptyset, \mathfrak{N}$, and the properties H_p for $2 \leq p < \omega$, the covering theorems and expressions 4, 5 of work $(^3)$ hold.

We carry out the realization of the $(\omega_\nu)R_N$-classification of sets in the Baire space J^ω, starting from the operation

$$\Phi_N \equiv \bigcup_\tau$$

and the class of open-closed sets of this space. We obtain the classes of $(\omega_\nu)R$-sets

$$(\omega_\nu)R_\alpha, \quad (\omega_\nu)CR_\alpha, \quad (\omega_\nu)BR_\alpha = (\omega_\nu)R_\alpha \cap (\omega_\nu)CR_\alpha.$$

Each of the classes $(\omega_\nu)R_\alpha,\ (\omega_\nu)CR_\alpha$ has a universal set, is topologically invariant, and the classification of sets is essentially monotone. The question of the coincidence of the class

$$(B_1) = (\omega_\nu)BR_1$$

with the class of B-sets of this space remains open; here $(B) \subset (B_1)$. The question of the approximability of sets is open. The base N of the operation $\bigcup_\tau$ satisfies conditions $1^0, 2', 3^0$.

Complete bases of operations

$$(\omega_\nu)T^\alpha \equiv (\omega')T_N^\alpha$$

for

$$\Phi_N \equiv \bigcup_\tau$$

belong to the class (B_2) of projective sets of the space J^{ω_ν}. The class of $(\omega_\nu)R$-sets also belongs to the class (B_2).

Volgograd Pedagogical Institute
named after A. S. Serafimovich

Received
30 XII 1969

CITED LITERATURE

1. A. A. Lyapunov, *Algebra and Logic*, Seminar, **2**, 2, 47 (1963).

2. A. A. Lyapunov, *Proceedings of the V. A. Steklov Mathematical Institute, Academy of Sciences of the USSR*, **40**, 3 (1953).

3. Z. I. Kozlova, *Doklady Akademii Nauk*, **188**, No. 5, 1001 (1969).

Note: Figure translations are in progress. See original paper for figures.