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Abstract

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CYBERNETICS AND CONTROL THEORY

I. M. MAKAROV, V. Z. RAKHMANKULOV

ON THE USE OF DISCONTINUOUS SLIDING HYPERPLANES IN SYSTEMS WITH VARIABLE STRUCTURE

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At the present time, in solving a number of important problems of automatic control of nonstationary plants, systems with variable structure have become widely used. As is known ^(1,2), in implementing control algorithms for VSS, in many cases the switching function is formed as a continuous function of the error coordinate and its $(n-1)$ derivatives. Obtaining exact values of high-order derivatives is a difficult technical problem; therefore an effective means of eliminating repeated differentiation of the error signal is the method of introducing information about the internal coordinates of the system into the control law.

However, the use of VSS algorithms with information about internal coordinates for controlling plants whose parameters vary over wide ranges usually leads to a violation of the weak dependence, inherent in VSS, of the dynamic properties of the system on changes in the plant parameters. In such cases special measures must be provided to eliminate the influence of the plant nonstationarity on the dynamics of the VSS.

It is shown below that, by introducing a logical law for changing the switching function, it is possible to weaken the influence of changes in the plant parameters on the dynamics of a VSS with information about internal coordinates in the control law.

Let the differential equation describing the motion of an automatic-control system with variable structure have the form

$$dx_i/dt = x_{i+1}, \quad i = 1, 2, \dots, (n-1),$$

$$dx_n/dt = -\sum_{i=1}^n a_i x_i - \sum_{i=1}^{n-1} \psi_i x_i, \quad (1)$$

where x_i is the error coordinate; a_i are constant or varying, within a bounded range, parameters of the control plant; $\sum_{i=1}^{n-1} \psi_i x_i$ is the discontinuous control,

$$\psi_i = \begin{cases} \omega_i, & \text{for } x_i S > 0, \\ \lambda_i, & \text{for } x_i S < 0; \end{cases}$$

it is assumed that $|\psi_i| \leq A_i$; ω_i, λ_i are constants; A_i is a positive constant quantity;

$$S = \sum_{i=1}^k r_i x_i + \sum_{l=k+1}^n r_l Z_l = 0$$

is the sliding hyperplane, $k < n$; r_i, r_l are constants; Z_l is an internal coordinate of the system, which is a linear combination of the x_i .

The motion in the sliding mode is completed in accordance with (3) and is described by the linear homogeneous differential equation:

$$(c_{i(t)}, X) = 0, \quad i = 1, 2, \dots, n, \quad c_{i(t)} = f(r_i, r_l, t), \quad c_{i(t)} > 0. \quad (2)$$

The choice of the coefficients $c_i(t)$ from the set $c_i(t) > 0$ is subject to the restriction imposed by the condition $|\psi_i| \leq A_i$.

The use, in the control law, of information about the internal coordinates Z_l causes an arbitrary change in the position of the sliding hyperplane S in the phase space X , and consequently an arbitrary change in the roots of the characteristic polynomial of equation (2). As a result of an arbitrary change in the position of the switching surface, the character of the transient processes in the system changes in an uncontrollable way and may become unsatisfactory (for example, when complex-conjugate pairs of roots appear in (2)).

Assuming that the rate of change of the parameters of the controlled plant is small in comparison with the rate of the transient process, one can choose two fixed positions of the switching surface in the phase space X . As a result, we obtain two equations:

$$(b', X) = 0, \quad (b'', X) = 0, \quad (3)$$

where the vector $b' = \begin{pmatrix} c'_1 \\ \vdots \\ c'_n \end{pmatrix}$, and the vector $b'' = \begin{pmatrix} c''_1 \\ \vdots \\ c''_n \end{pmatrix}$.

Let, among the eigenvalues λ'_i corresponding to equation (3) with the components of the vector b' , there be complex-conjugate pairs, while the eigenvalues

λ_i'' corresponding to equation (3) with the components of the vector b'' are only negative real ones, and let the characteristic equation of system (1)

$$p^n + a_n p^{n-1} + \sum_{i=1}^{n-1} (a_i + b\omega_i) p^{i-1} = 0 \quad (4)$$

for $\psi_i = \omega_i$ have no positive real roots, and let $b\omega_i \geq -a_{i \min}$, $b\lambda_i \leq -a_{i \max}$, with $|\omega_i| \leq A_i$, $|\lambda_i| \leq A_i$.

The introduction of a logical law for switching the constant set of coefficients c'_i , which determine the components of the vector b' , to the constant set of coefficients c''_i , which determine the components of the vector b'' , during the control process makes it possible to ensure, in the final stage of the transient process, the motion of the representing point along a sliding trajectory along the hyperplane $(b'', X) = 0$. But the direct "joining" of sliding trajectories on the hyperplane $(b', X) = 0$ with sliding trajectories on the hyperplane $(b'', X) = 0$ is usually prevented by the condition $|\psi_i| \leq A_i$. Therefore, at the instant of switching the coefficients, the representing point with coordinates $(x'_1, x'_2, \dots, x'_n)$ does not belong to the hyperplane $(b'', X) = 0$, and its distance from $(b'', X) = 0$ is characterized by the quantity

$$T' = \sum_{i=1}^n c''_i x'_i, \quad \text{where} \quad \sum_{i=1}^n c''_i{}^2 = 1. \quad (5)$$

If conditions are ensured for the representing point to reach the hyperplane $(b'', X) = 0$ from the position $(x'_1, x'_2, \dots, x'_n)$ in a finite interval of time, then the motion as a whole can be given the desired character by appropriately choosing the instant of switching of the function S during the transient process.

Let us consider motion along the sliding hyperplane $(b'', X) = 0$ with an aperiodic character of the trajectories of the representing point. If $\lambda''_1, \dots, \lambda''_n$ are the roots of the equation $(b'', \dot{X}) = 0$, arranged in decreasing order of their values, then to the hyperplane

$$x_2 - \lambda''_1 x_1 = 0, \quad (b'', X) = 0 \quad (6)$$

asymptotically approach all sliding trajectories on the switching surface $(b'', X) = 0$, lying in the region F bounded by the hyperplanes

$$x_i = 0, \quad (b'', X) = 0;$$

$$\sum_{i=1}^n a_i x_i + \sum_{i=1}^{n-1} \psi_i x_i + \lambda''_n x_n = 0, \quad (b'', X) = 0 \quad (7)$$

and including the hyperplane (6).

Hence, if the vector $X''_\tau = \{x''_{1\tau}, \dots, x''_{n\tau}\}$, obtained as a result of the linear transformation of the vector $x'_0 = \{x'_1, \dots, x'_n\}$ after switching the function S in accordance with the equation

$$X''_\tau = W_\tau X'_0, \quad (8)$$

where $W_\tau = De^{\xi\tau}D^{-1}$; the matrix

$$D = \left\| \begin{array}{cccc} 1 & \cdot & \cdot & 1 \\ \xi_1 & \cdot & \cdot & \xi_n \\ \cdot & \cdot & \cdot & \cdot \\ \xi_1^{n-1} & \cdot & \cdot & \xi_n^{n-1} \end{array} \right\|;$$

the matrix

$$e^{\xi\tau} = \left\| \begin{array}{cccc} e^{\xi_1\tau} & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & e^{\xi_n\tau} \end{array} \right\|;$$

D^{-1} is the inverse matrix; ξ_1, \dots, ξ_n are the roots of the characteristic equation (4); τ is the time of the linear transition of the representative point from $(b', X) = 0$ to $(b'', X) = 0$, and if it lies inside the region F on the sliding hyperplane $(b'', X) = 0$, then it may be asserted that the representative point has been transferred to an acceptable sliding trajectory on the hyperplane $(b'', X) = 0$, moving along which it reaches the origin without crossing the plane $X_1 = 0$.

The obtained conditions must be satisfied for any mutual positions of the sliding hyperplanes $(b', X) = 0$ and $(b'', X) = 0$, determined by variation of the object parameters within the prescribed range.

Thus, the logical law for correcting the switching function makes it possible to ensure a weak dependence of the control processes on changes in the object parameters in a variable-structure system with information on the internal coordinates of the system in the control algorithm.

Institute of Automation and Telemechanics
(Technical Cybernetics)
Moscow

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.

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