STABILITY OF CYLINDRICAL SHELLS UNDER THERMAL SHOCK

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Abstract

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Let a thin circular cylindrical shell of finite length be subjected to the action of an axisymmetric thermal pulse in a certain zone located near one of the ends.

As the heat flow propagates along the structure, part of the shell length will be subjected to dynamic compression. This effect will be especially strong in the case of thermal shock. Under known conditions, buckling will occur in the compressed zone of the shell, accompanied by snapping; this phenomenon is dangerous for the structure. To describe the behavior of the shell in the governing equations, it is necessary to take into account the inertia forces corresponding not only to deflection but also to displacements in the middle surface. Since such a problem proves to be very complicated, we shall conventionally assume that the variation of thermoelastic strain along the shell length does not depend on the deflection and is determined from the solution of the corresponding one-dimensional problem ([1]). As for the buckling phenomenon, we shall assume that the deflections produced in this process are comparable with the shell thickness and are obtained from the solution of a geometrically nonlinear problem.

Let us write the system of differential equations describing the displacements of points of the middle surface along the shell length ([1])

$$\theta_{,t} - \chi\theta_{,xx} + \beta' u_{,xt} = 0, \tag{1}$$

$$u_{,xx} - \frac{1}{c^2} u_{,tt} - \alpha\theta_{,x} = 0 \tag{2}$$

and large deflections ([2])

$$\frac{D}{h}\nabla^2\nabla^2(w - w_0) = L(w, \Phi) + \frac{1}{R}\Phi_{,xx} - \frac{\gamma}{g} w_{,tt}, \tag{3}$$

$$\frac{1}{E}\nabla^2\nabla^2\Phi = -\frac{1}{2}\left[L(w,w) - L(w_0,w_0)\right] - \frac{1}{R}(w - w_0)_{,xx}. \tag{4}$$

Here θ is temperature; χ is the coefficient of thermal diffusivity; β' is a coefficient accounting for the reverse effect of elastic deformation; x, y are coordinates measured along the generator and along the arc; t is time; u is the displacement of points of the middle surface along the axis; α is the coefficient of linear expansion; c is the velocity of propagation of a longitudinal elastic wave in the material; w is the total deflection; w_0 is the initial deflection; Φ is the stress function in the middle surface; R is the radius of curvature of the middle surface; h is the shell thickness; D is the cylindrical stiffness; γ is the specific weight of the material. Subscripts after a comma denote differentiation with respect to the corresponding variable; ∇^2 is the Laplace operator; L is a known bilinear operator.

Assume that the shell has an initial imperfection. We approximate the total and initial deflections by means of an expression of the type $w = f(\sin\alpha_0 x \sin\beta_0 y + \psi\sin^2\alpha_0 x + \varphi)$, $\alpha_0 = m\pi/l$; $\beta_0 = n/R$; m is the number of half-waves along the shell generator; n is the number of waves around the circumference; l is the shell length. Substitute this expression into the right-hand side of equation (4) and

after integration, we find the function Φ. In this case the expression for Φ contains the term $(-py^2/2)$, where $p = p(t)$ is the intensity of the compressive forces in the selected section of the shell, determined by the formula $p = E(\partial u/\partial x - \alpha\theta)$. To find the relation between the parameters of the shell deflection and the compressive forces varying in time, we apply the Bubnov-Galerkin method to equation (3). As a result, we arrive at a nonlinear ordinary differential equation of the second order with respect to the deflection amplitude f.

We shall investigate the case in which sharp local heating of one of the shell ends occurs; the remaining surface will be taken as thermally insulated. The solution of the problem is reduced to the integration of the following system of resolving equations:

$$\theta^*_{,\tau} - d\theta^*_{,x^*x^*} + \alpha\beta' u^*_{,x^*\tau} = 0, \tag{5}$$

$$u^*_{,x^*x^*} - u^*_{,\tau\tau} - \theta^*_{,x^*} = 0, \tag{6}$$

$$\xi_{,\tau\tau} - s\Bigg\{\left[p^* - \frac{1}{16}\frac{(1+\rho^4)}{\rho^2}\eta(\xi^2-\xi_0^2)\right]\xi - \frac{1}{12(1-\mu')}\frac{(1+\rho^?)^2}{\rho^2}\eta(\xi+\xi_0)$$
$$-\psi^2\eta\rho^2\xi(\xi^2-\xi_0^2)\left[\frac{1}{(1+\rho^2)^2}+\frac{1}{(1+9\rho^2)^2}\right]+\frac{1}{4\rho^2}\psi\xi(\xi-\xi_0)\left[1+\frac{4\rho^2}{(1+\rho^?)^2}\right]$$
$$+\psi\frac{\rho^2}{(1+\rho^2)^?}(\xi^2-\xi_0^2)-\frac{\rho^2}{\eta(1+\rho^2)^2}(\xi-\xi_0)\Bigg\}=0. \tag{7}$$

Here the following dimensionless parameters have been introduced:

$$x^* = x/l, \quad \tau = tc/l, \quad \theta^* = \theta\alpha, \quad u^* = u/l, \quad v^* = v/i, \quad d = \chi/cl, \quad \lambda = l/i,$$

$$p^* = pR/Eh, \quad \xi = f/h, \quad \xi_0 = f_0/h, \quad \eta = n^2h/R, \quad \rho = m\pi R/nl, \quad s = (l/R)^2\eta\rho^2,$$

where $i = R/\sqrt{2}$ is the radius of inertia of the shell section, and λ is the overall flexibility of the structure. The value of ψ is conventionally taken from the solution of the static problem for an ideal shell.

The solution of equations (5) and (6) was obtained in closed form by successive application to them of Fourier and Laplace integral transforms under the following boundary and initial conditions:

$$\theta^*_{,x^*} = u^* = 0 \quad \text{for } x^* = 0 \text{ and } x^* = 1;$$

$$u^* = u^*_{,\tau} = 0; \qquad \theta^* = \frac{Q\alpha}{c_v\gamma}\delta(x^*) \quad \text{for } \tau = 0,$$

where Q is the amount of heat energy supplied; c_v is the specific heat capacity of the material; $\delta(x^*)$ is the Dirac function.

Equation (7) was integrated by the Runge-Kutta method with the aid of the BESM-2M digital computer, starting from the initial conditions

$$\xi - \xi_0 = \xi_{,\tau} = 0.$$

In the course of solving the problem, the coefficient of dynamicity was determined, representing the ratio of the value of the maximum compressive force to the upper critical value p^*/p^*_{B}, and the number of waves at which the violent buckling process begins.

The results of the computations are presented in Fig. 1 in the form of curves characterizing the rate of increase of the deflection ξ in the shell in the section $x = l/2$ as a function of the dimensionless time τ for various parameters of the thermal pulse θ. They refer to a shell with the ratio $R/h = 300$, the amplitude

Fig. 1

Figure 1: Fig. 1

of the initial deflection $\xi_0 = 0.001$, and the wave-formation parameter $\rho = 3$; $L/R = 3.75$. As the critical time, the instant τ_{cr} corresponding to the front of the violent increase in deflections was conventionally taken. As we see, in the case $\theta = 5$ the most probable form of shell buckling corresponds to the number of waves $n = 54$, while the critical time parameter is $\tau \approx 0.09$; analogous results are shown in the graph for the values $\theta = 1$ and $\theta = 0.3$.

Thus, in the present work a mathematical model is proposed for describing the process of buckling of a cylindrical shell under thermal

Fig. 1

shock. The calculations showed that the effect of dynamicity is especially pronounced for thinner shells, at a significant ratio R/h.

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Note: Figure translations are in progress. See original paper for figures.