STATICO-GEOMETRIC ANALOGY AND THE METHOD OF COMPLEX TRANSFORMATION IN THE LINEAR THEORY OF ELASTIC SHELLS OF TIMOSHENKO TYPE

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Abstract

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STATICO-GEOMETRIC ANALOGY AND THE METHOD OF COMPLEX TRANSFORMATION IN THE LINEAR THEORY OF ELASTIC SHELLS OF TIMOSHENKO TYPE

(Presented by Academician V. V. Novozhilov, 15 XII 1969)

Up to the present time V. V. Novozhilov's method of complex transformation has been applied only within the framework of the classical Kirchhoff–Love theory of thin shells ([1–3]). In the present work, on the basis of the established statico-geometric analogy, this method is extended to the theory of elastic shells of Timoshenko type, constructed with allowance for transverse shear deformations ([4–6]).

The initial equations of the theory of elastic shells of Timoshenko type are represented as follows: the components of the strain tensors $\varepsilon_{\alpha\beta}$, $\chi_{\alpha\beta}$ and of the kinematic vector (the vector of shear deformations) χ_α

$$\varepsilon_{\alpha\beta} = {}^1/_2 \left(\nabla_\alpha v_\beta + \nabla_\beta v_\alpha - 2b_{\alpha\beta} w\right),$$
$$\chi_{\alpha\beta} = {}^1/_2 \left(\nabla_\alpha \varphi_\beta + \nabla_\beta \varphi_\alpha\right), \qquad \chi_\alpha = \varphi_\alpha + \nabla_\alpha w + b_{\alpha\beta} v^\beta; \tag{1}$$

equilibrium equations

$$\nabla_\alpha T^{\alpha\beta} - b_\alpha^\beta N^\alpha + p^\beta = 0,$$
$$\nabla_\alpha N^\alpha + b_{\alpha\beta} T^{\alpha\beta} + p = 0,$$
$$\nabla_\alpha M^{\alpha\beta} - N^\beta + m^\beta = 0; \tag{2}$$

elasticity relations

$$T^{\alpha\beta} = BE^{\alpha\beta\gamma\delta}\varepsilon_{\gamma\delta}, \qquad N_\alpha = 2k'Gh\chi^\alpha, \qquad M^{\alpha\beta} = DE^{\alpha\beta\gamma\delta}\chi_{\gamma\delta}, \tag{3}$$

where v^α, w are the components of the vector of elastic displacement of points of the middle surface of the shell; φ_α are the angles of rotation of the normal; ∇_α is the symbol of covariant differentiation; $T^{\alpha\beta}$, $M^{\alpha\beta}$, N^{α} are the components of the tensors of tangential forces, moments, and the vector of transverse forces; p^{β}, p, m^{β} are the components of the prescribed vectors of external forces and moments; $a_{\alpha\beta}$, $b_{\alpha\beta}$ are the tensors of the first and second fundamental forms of the middle surface; $c^{\alpha\beta}$ are the components of the discriminant tensor; $B = 2Eh/(1-\nu^2)$, $D = 2Eh^3/3(1-\nu^2)$, $E^{\alpha\beta\gamma\delta} = a^{\alpha\gamma}a^{\beta\delta} + \nu c^{\alpha\gamma}c^{\beta\delta}$; G and k' are the shear modulus and coefficient; $2h$ is the thickness of the shell.

The compatibility relations for the strains were obtained within the framework of the theory of Timoshenko type in ([7,8]). After some transformations these relations may be represented in the form

$$\begin{aligned}
\nabla_\alpha \left(c^{\alpha\gamma}c^{\beta\delta}\mu_{\gamma\delta}\right) + b^{\beta}_{\alpha}\left(c^{\alpha\beta}\eta_\beta\right) &= 0,\\
-\nabla_\alpha \left(c^{\alpha\beta}\eta_\beta\right) + b_{\alpha\beta}\left(c^{\alpha\gamma}c^{\beta\delta}\mu_{\gamma\delta}\right) &= 0,\\
-\nabla_\alpha \left(c^{\alpha\gamma}c^{\beta\delta}\varepsilon_{\gamma\delta}\right) + c^{\alpha\beta}\eta_\beta &= 0,
\end{aligned} \tag{4}$$

where

$$\mu_{\alpha\beta} = \chi_{\alpha\beta} - {}^{1}/{}_{2}\left(\nabla_\beta\chi_\alpha + \nabla_\alpha\chi_\beta\right), \qquad \eta_\beta = \zeta_\beta - c^{\alpha\gamma}b_{\beta\gamma}\chi_\alpha. \tag{5}$$

Comparing the homogeneous equilibrium equations (2) and relations (4), we note a correspondence between the physical and geometrical quantities

$$T^{\alpha\beta} \leftrightarrow c^{\alpha\gamma}c^{\beta\delta}\mu_{\gamma\delta}, \qquad M^{\alpha\beta} \leftrightarrow -c^{\alpha\gamma}c^{\beta\delta}\varepsilon_{\gamma\delta}, \qquad N^{\alpha} \leftrightarrow -c^{\alpha\beta}\eta_\beta. \tag{6}$$

The established relation represents a **statico-geometric analogy** in the Timoshenko-type theory of elastic shells.

Let us now subject the original equations of the Timoshenko-type theory to a complex transformation, analogous to what was done by V. V. Novozhilov and K. F. Chernykh ([1–3]) within the Kirchhoff–Love theory.

Introduce complex forces and moments by the formulas

$$\begin{aligned}
\widetilde{T}^{\alpha\beta} &= T^{\alpha\beta} - 2iEhc\cdot c^{\alpha\gamma}c^{\beta\delta}\mu_{\gamma\delta},\\
\widetilde{M}^{\alpha\beta} &= M^{\alpha\beta} + 2iEhc\cdot c^{\alpha\gamma}c^{\beta\delta}\varepsilon_{\gamma\delta},\\
\widetilde{N}^{\alpha} &= N^{\alpha} + 2iEhc\cdot c^{\alpha\beta}\eta_\beta.
\end{aligned} \tag{7}$$

Here $i = \sqrt{-1}$, $c = h/\sqrt{3(1-\nu^2)}$.

Using the concept introduced in (3) of stress functions for the inhomogeneous problem of shell theory, we represent (7) in the form

$$\widetilde{T}^{\alpha\beta} = T^{\alpha\beta *} - 2iEhc \cdot c^{\alpha\gamma} c^{\beta\delta} \tilde{\mu}_{\gamma\delta},$$
$$\widetilde{M}^{\alpha\beta} = M^{\alpha\beta *} + 2iEhc \cdot c^{\alpha\gamma} c^{\beta\delta} \tilde{\varepsilon}_{\gamma\delta}, \quad (8)$$
$$\widetilde{N}^{\alpha} = N^{\alpha *} + 2iEhc \cdot c^{\alpha\beta} \tilde{\eta}_{\beta},$$

where $T^{\alpha\beta *}, M^{\alpha\beta *}, N^{\alpha *}$ are a certain solution of the inhomogeneous system of equilibrium equations (2); $\tilde{\mu}_{\alpha\beta} = \mu_{\alpha\beta} + i\overline{\mu}_{\alpha\beta}$, $\tilde{\varepsilon}_{\alpha\beta} = \varepsilon_{\alpha\beta} + i\overline{\varepsilon}_{\alpha\beta}$, $\tilde{\eta}_{\beta} = \eta_{\beta} + i\overline{\eta}_{\beta}$ are the corresponding complex expressions (5), constructed from complex combinations of displacements and stress functions:

$$\tilde{v}^{\beta} = v^{\beta} + i\overline{v}^{\beta}, \qquad \widetilde{w} = w + i\overline{w}.$$

The systems of equilibrium equations (2) and compatibility equations (4) are written as one system in complex forces—moments:

$$\nabla_{\alpha} \widetilde{T}^{\alpha\beta} - b_{\alpha}^{\beta} \widetilde{N}^{\alpha} + p^{\beta} = 0,$$
$$\nabla_{\alpha} \widetilde{N}^{\alpha} + b_{\alpha\beta} \widetilde{T}^{\alpha\beta} + p = 0, \quad (9)$$
$$\nabla_{\alpha} \widetilde{M}^{\alpha\beta} - \widetilde{N}^{\beta} + m^{\beta} = 0.$$

On the basis of equations (7)—(9), as well as (1) and (3), the problem of a complex formulation of the basic resolving equations of the Timoshenko-type theory can be solved. The resulting systems of resolving equations in complex forces or in complex displacements are, in order, two units higher (in real calculation) than the corresponding systems in the Kirchhoff-Love theory ([1−3]).

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Note: Figure translations are in progress. See original paper for figures.