SIMILARITY THEORY FOR LARGE-SCALE MOTIONS OF PLANETARY ATMOSPHERES

GEOPHYSICS

1970

SovietRxiv

Abstract

Full Text

UDC 551.513.1:523.1

GEOPHYSICS

G. S. GOLITSYN

SIMILARITY THEORY FOR LARGE-SCALE MOTIONS OF PLANETARY ATMOSPHERES

(Presented by Academician A. N. Kolmogorov, June 3, 1969)

1. One of the most important problems of modern meteorology—the theory of the general circulation of the atmosphere—is at present also becoming of interest for planetary physics in connection with the achievements of modern space technology. Attempts to calculate the characteristics of atmospheric circulation, so far only for Mars, have already been undertaken on the basis of setting up the corresponding numerical experiments. However, such an approach requires very laborious and extensive computations and very specific assumptions about a number of characteristics of the atmosphere (the absorptive capacities of optically active admixtures, knowledge of their relative concentrations, etc.), which we do not at present possess. Another approach is to obtain certain quantitative estimates on the basis of energetic and thermodynamic considerations and the methods of similarity theory.

One attempt along these lines was undertaken in ([1]), where a formula was introduced for the mean rate of generation (and dissipation) of the kinetic energy of large-scale motions per unit mass

$$\varepsilon = (k\delta T/T_1)(q_A/4M), \tag{1}$$

where k is a numerical coefficient of order 0.1 (according to actual data) for the terrestrial atmosphere; δT is the characteristic temperature difference over the surface or in the planet' s atmosphere; T_1 is the temperature of the most heated regions; $q_A = q(1-A)$; q is the solar constant for the planet; A is its albedo; $M = p_s/g$ is the mass of a unit atmospheric column. As the characteristic scale L of synoptic processes in ([1]), for rapidly rotating planets the Obukhov scale ([2]) $L_0 = c/l \approx c/\Omega$ was adopted, where c is the speed of sound and l is the Coriolis parameter, or the radius for slowly rotating planets. For the terrestrial planets, characteristic velocities $U \approx (\varepsilon L)^{1/3}$ and times $\tau \approx L^{2/3}\varepsilon^{-1/3}$ were estimated. In doing so, it was assumed that $k = 0.1$ for all planets, while the values of δT and T_1 were taken from observations (see ([3])).

However, in any closed theory of the general circulation the quantities δT and T_1 must be determined, not specified in advance. We shall now proceed to an attempt to construct such a theory, using the methods of similarity and dimensional analysis ([4]).

2. The following physical ideas underlie it: the intensity of the circulation and the temperature difference that causes it are interrelated and mutually consistent and are determined by the influx of heat to the atmosphere, its mass, and its thermal properties. The heat balance is achieved by the radiation of long-wave radiation into space, the atmosphere being assumed gray, i.e., the temperature of thermal radiation is determined by the influx q_A. For simplicity we do not take into account absorption of direct radiation by the atmosphere itself, which is legitimate for atmospheres that are not very thick*.

* Generally speaking, it is necessary to take into account the dimensionless parameter—the ratio l/H, where l is the characteristic mean free path of a photon and H is the effective thickness of the atmosphere. It may influence, for example, the magnitude of k in (1), but, taking into account that the requirements of thermodynamics impose certain general limits independent of the nature of the substance ($k < 1$!), it may be hoped that the estimates obtained will be sufficiently universal. Indirectly, these effects will be considered in Sec. 5 in discussing the intensity of the circulation on Venus.

Let us first consider a nonrotating planet. The determining parameters in the equations of atmospheric dynamics, averaged over height, are the surface-mean influx of energy $q_A/4$ ($\mathrm{g \cdot sec^{-3}}$), the heat capacity of the atmosphere per unit mass c_p ($\mathrm{cm^2 \cdot sec^{-2} \cdot deg^{-1}}$), the mass of an atmospheric column M ($\mathrm{g \cdot cm^{-2}}$), the radius of the planet r (cm), and the constant in the Stefan-Boltzmann radiation law $\sigma = 5.67 \cdot 10^{-5}\ \mathrm{g \cdot sec^{-3} \cdot deg^{-4}}$. From these 5 parameters only one dimensionless combination can be formed:

$$\mathcal{M} = \sigma^{3/8} c_p^{-5/2} (q_A/4)^{5/8} r M^{-1}. \tag{2}$$

Under the adopted assumptions and restrictions, motions in the atmospheres of planets with the same number $\mathcal{M}$ must be similar. The values of the number $\mathcal{M}$ for terrestrial-type planets are $1.1 \cdot 10^{-3}$ (Earth), $3.4 \cdot 10^{-2}$ (Mars, $p_s = 5$ mb), $1.3 \cdot 10^{-5}$ (Venus, $p_s \approx 90$ atm), i.e., they turn out to be small. Consequently, the dependence on $\mathcal{M}$ should not be significant. The smallness of $\mathcal{M}$ is equivalent to the fact that M is large; therefore, in the first approximation the mass M may be disregarded. This means that, with respect to a number of global characteristics of circulation, planetary atmospheres turn out to be mass-autonomous.

From the remaining 4 parameters one can form combinations with the dimensions of velocity, time, and energy. The first has the form

$$w^{1/2} = c_p^{1/2} \sigma^{-1/8} (q_A/4)^{1/8} = c_e (\varkappa - 1)^{-1/2}. \tag{3}$$

Recalling that for an ideal gas $c_p = \varkappa(\varkappa-1)^{-1}R/\mu$, where $\varkappa = c_p/c_v$, R is the universal gas constant, μ is molecular weight, and $(q_A/4\sigma)^{1/4} = T_e$ is the effective radiation temperature of the atmosphere, we see that $w = c_e^2(\varkappa-1)^{-1}$ is the enthalpy, while $c_e = (\varkappa-1)^{1/2}w^{1/2}$ is the speed of sound (accurate to a certain factor that takes into account the difference between T_e and the mean temperature of the atmosphere).

The constant with the dimension of time

$$\tau_e = r/c_e = r(\varkappa-1)^{-1/2}c_p^{-1/2}(4\sigma/q_A)^{1/8} \tag{4}$$

has the meaning of the relaxation time of pressure or density perturbations of the atmosphere on the global scale.

The constant with the dimension of energy has the form

$$E = B\sigma^{-1/8}c_p^{-1/2}(q_A/4)^{7/8}r^3, \tag{5}$$

where B is a certain numerical coefficient. It may vary somewhat from planet to planet, if only because there are a number of additional factors not taken into account here that influence the intensity of atmospheric processes. Taking (3) and (4) into account, we write (5) as

$$E = [B(\varkappa-1)^{1/2}/4\pi]q_A\pi r^2 \cdot r/c_e = [B(\varkappa-1)^{1/2}/4\pi]Q_A\tau_e. \tag{6}$$

Thus, accurate to a numerical factor, the energy E is equal to the product of the total power of solar radiation Q_A entering the planet by the relaxation time τ_e.

The total enthalpy of the atmosphere must obviously depend on M. It is natural to assume that (5) determines the total kinetic energy of the atmosphere. The value E/B, according to (5), for the Earth is $1.12 \cdot 10^{27}$ erg, and for Mars $0.97 \cdot 10^{26}$ erg. The magnitude of the total kinetic energy of the terrestrial atmosphere fluctuates from season to season and is (5) $(6 \div 9) \cdot 10^{27}$ erg. According to calculations (6), the total kinetic energy of the atmosphere of Mars is $(1.2 \div 1.6) \cdot 10^{26}$ erg. Thus, accurate to a factor on the order of unity, (5) does indeed determine the total kinetic energy. From this follows a number of consequences: 1) the kinetic energy per unit volume $\rho U^2/2$ does not depend on M or p_s; 2) the mean velocity of atmospheric motions is equal to

$$U = (E/2\pi r^2 M)^{1/2} = (B/2\pi)^{1/2}\sigma^{1/16}c_p^{-1/4}(q_A/4)^{7/16}(r/M)^{1/2}; \tag{7}$$

3) the dimensionless parameter $\mathcal{M}$, to within a numerical factor of order unity, turns out to be equal to the square of the Mach number Ma:

$$\mathcal{M} = [2\pi(\chi - 1)/B]U^2/c_e^2 \approx \mathrm{Ma}^2; \tag{8}$$

4) the time scale of atmospheric motions is, in order of magnitude, equal to

$$\tau_U \approx r/U \approx (2\pi/B)^{1/2} c_p^{1/4} \sigma^{-1/16} (q_A/4)^{-7/16} (rM)^{1/2}; \tag{9}$$

5) the order of the total rate of generation (dissipation) of kinetic energy in the entire atmosphere of the planet is equal to

$$\mathcal{E} \approx E/\tau_U \approx (B/2\pi)^{3/2} \sigma^{3/16} c_p^{-3/4} (q_A/4)^{21/16} r^{5/2} M^{-1/2}, \tag{10}$$

whence, in calculation per unit mass, we have

$$\varepsilon \approx \mathcal{E}/4\pi r^2 M \approx \frac{1}{2}(B/2\pi)^{3/2} \sigma^{3/16} c_p^{-3/4} (q_A/4)^{21/16} r^{1/2} M^{-3/2}; \tag{11}$$

6) comparing (11) with (1), the efficiency of the atmosphere can be written in the form

$$\eta = k\delta T/T_1 \approx \frac{1}{2}(B/2\pi)^{3/2} \sigma^{3/16} c_p^{-3/4} (q_A/4)^{5/16} (r/M)^{1/2} \approx \mathrm{Ma}. \tag{12}$$

Taking $T_1 \approx T_e$, one can estimate the characteristic temperature difference

$$\delta T \approx \eta T_e/k \approx (2k)^{-1} (B/2\pi)^{3/2} \sigma^{-1/16} c_p^{-3/4} (q_A/4)^{9/16} (r/M)^{1/2}. \tag{13}$$

Sometimes one has to introduce $\alpha = T_e/T_1$; then in (13) k must be replaced by $k\alpha$. Note that (1) is not a consequence of the similarity theory developed here, but follows from other considerations ([1]). The same applies to (13).

3. Let us include among the determining parameters the angular velocity of the planet' s proper rotation Ω (sec^{-1}). The number $\mathcal{M} \approx \mathrm{Ma}^2$ will still be considered small. Then from the parameters σ, c_p, $q_A/4$, r, and Ω, only one dimensionless combination can be formed,

$$\lambda = (\chi - 1)^{-1/2} c_p^{-1/2} (4\sigma/q_A)^{1/8} \Omega r = \Omega r/c_r = \Omega \tau_e = r/L_0. \tag{14}$$

Formula (5) must now be multiplied by some function $f(\lambda)$, with $f(0) = 1$. The dependences $U \sim M^{-1/2}$, $\tau_U \sim M^{1/2}$, $\varepsilon \sim M^{-3/2}$, $\eta \sim M^{-1/2}$, $\delta T \sim M^{-1/2}$ are preserved here as well, just as is the independence of $\frac{1}{2}\rho U^2$ from M. In the general case, when $\mathcal{M} \gtrsim 1$, formula (5) must be multiplied by a function of two dimensionless parameters, $f_1(\mathcal{M}, \lambda)$. From the standpoint of similarity this case is the most complicated, and it is difficult here to obtain any even qualitative results.

4. Let us indicate another derivation of the relations of Sec. 2, approximately taking rotation into account, on the basis of the ideas developed in ([1]). Let us write the heat-balance equation, averaged over height:

$$Mc_p u_i \partial T/\partial x_i = \sigma T_e^4. \tag{15}$$

In order of magnitude, the advection $u_i \partial T/\partial x_i \approx U\delta T/(\pi r/2)$, where $U \approx (\varepsilon L)^{1/3}$. Introducing $\lambda_1 = \pi r/2L_0$ (for slowly rotating planets $\lambda_1 = 1$), and taking (1) into account, from (15) we obtain

$$\delta T \approx (\pi/2)^{1/2}(\lambda_1/k)^{1/4}\sigma^{-1/16}c_p^{-3/4}(q_A/4)^{9/16}(r/M)^{1/2}. \tag{16}$$

Substituting δT into (1), we determine ε, then U and E, the structure of all the resulting formulas proving to be the same as in Sec. 2. Comparing the formulas for E, we find

$$B \approx \pi^2 k^{1/2}\lambda_1^{-1/2}, \tag{17}$$

which unifies the similarity theory and the ideas of ([1]).

It follows from this that

$$f(\lambda) \approx \lambda_1^{-1/2} = (\pi\lambda/2)^{-1/2} \qquad \text{for } \lambda \gg 1. \tag{18}$$

Let us also give a formula relating k and δT. It follows from (16):

$$k \approx (\pi^2/4)\lambda_1(\delta T)^{-4}\sigma^{-1/4}c_p^{-3}(q_A/4)^{9/4}(r/M)^2. \tag{19}$$

Thus, in the present formulation all quantities are determined to within one empirical parameter B, or k, or δT.

Since, according to the second law of thermodynamics, $k < 1$ (see (1)), from (19) we also have the inequality

$$\delta T > (\pi/2)^{1/2}\sigma^{-1/16}c_p^{-3/4}\lambda_1^{1/4}(q_A/4)^{9/16}(r/M)^{1/2}. \tag{20}$$

5. Let us see what the formulas obtained here give for terrestrial-type planets. For the Earth $k \approx 0.1$ and $\lambda_1 = 3$. Then $B \approx 2$ and $E \approx 2.3 \cdot 10^{27}$ erg, whence $U \approx 10$ m/sec. According to (16), $\delta T \approx 25^\circ$. Both values are almost twice smaller than those actually observed. The underestimate is most likely connected with the neglect of the clearly expressed zonality of the real wind, which impedes heat exchange in the meridional direction, thereby increasing δT, and consequently ε and U. However, the orders of magnitude are correct.

Taking for Mars $k = 0.1$ and $\lambda_1 = 1.5$, we obtain $B \approx 2.6$ and $E \approx 2.5 \cdot 10^{26}$ erg. Hence, for $p_s = 5$ mb, $g = 370$ cm/sec^2, and $r = 3400$ km, we have $U \approx 50$ m/sec, and $\delta T \approx 100^\circ$ K. These values are close to those obtained in (6) by numerical modeling of the circulation of the Martian atmosphere.

Let us turn to the atmosphere of Venus, taking $M = 10^5$ g/cm^2. For it $\lambda_1 = 1$, and it is meaningful to take into account $a = T_e/T_1 \approx 240^\circ/700^\circ \approx 1/3$, i.e., the constant k must be reduced threefold in comparison with the value 0.1 adopted in (1). Then $B = 1.8$ and $E \approx 1.3 \cdot 10^{27}$ erg. In this case $U \approx 70$ cm/sec, $\tau_U \approx 8 \cdot 10^6$ sec. ≈ 3 months, $\varepsilon \approx 3 \cdot 10^{-4}$ cm^2/sec^3. The characteristic temperature difference is of the order of only 2° K. This is two orders of magnitude smaller than was determined in (7), but agrees with the data of (8). The estimates of ε, U, and τ_U carried out in (1) were based on the results of (7). For $M = 10^5$ g/cm^2 the estimates of (1) give $\varepsilon \approx 0.05$ cm^2/sec^3, $U \approx 3$ m/sec. From the point of view of the ideas developed here, it is obvious that with such large velocities the advection of heat toward the poles should lead to a rapid equalization of temperature, even if a difference of 200° K between the equator and the poles could exist at some moment.

If it turns out that all, or almost all, direct solar radiation is absorbed by the atmosphere of Venus itself, without reaching the planet' s surface, then, drawing an analogy with the ocean (1), one may think that this will lead to a decrease in the value of k by 2-3 orders of magnitude. In this case the velocities will decrease by a factor of 3-5, while τ_U and δT will correspondingly increase.

In conclusion, I express my deep gratitude to A. M. Obukhov for numerous useful suggestions and discussions.

Institute of Atmospheric Physics
Academy of Sciences of the USSR
Moscow

Received
31 V 1969

CITED LITERATURE

1. G. S. Golitsyn, *Izv. AN SSSR, Physics of the Atmosphere and Ocean*, **4**, No. 11 (1968).

2. A. M. Obukhov, *Izv. AN SSSR, Ser. Geography and Geophysics*, **13**, No. 4 (1949).

3. V. I. Moroz, *The Physics of the Planets*, "Nauka," 1967.

4. L. I. Sedov, *Similarity and Dimensional Methods in Mechanics*, 6th ed., "Nauka," 1967.

5. E. P. Borisenkov, *Proceedings of the Arctic and Antarctic Institute*, No. 253 (1963).

6. S. V. Leovy, Y. Mintz, RM-5110-NASA, The RAND Corp., 1966.

7. A. D. Kuzmin, B. J. Clark, *Astronomical Journal*, **42**, No. 4 (1965).

8. G. L. Berge, Abstracts of Reports of the International Symposium, *Physics of the Moon and Planets*, Kiev, Oct. 1968.

Note: Figure translations are in progress. See original paper for figures.