SPHERICAL WAVE OF A SCALAR PLANKEON

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Abstract

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SPHERICAL WAVE OF A SCALAR PLANKEON

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The basic equation for studying a spherical wave corresponding to a "scalar" plankeon has the form (^1)

$$(g^{ik}\hat{p}_i\hat{p}_k + m^2c^2 + \hbar^2 R/6)\,\psi = 0. \tag{1}$$

The interval corresponding to the "Einstein universe" is

$$-ds^2 = -c^2 d\tau^2 + \frac{dr^2}{1 - r^2/a^2} + r^2 d\Omega^2. \tag{2}$$

In this case

$$R = 6/a^2. \tag{3}$$

Since for the scalar wave function

$$g^{ik}\hat{p}_i\hat{p}_k = -\hbar^2 g^{ik}\nabla_i\nabla_k = -\hbar^2 g^{ik}\left(\frac{\partial^2}{\partial x^i \partial x^k} - \Gamma^m_{ik}\frac{\partial}{\partial x^m}\right),$$

equation (1) takes the form

$$\left[g^{ik}\left(\frac{\partial^2}{\partial x^i \partial x^k} - \Gamma^m_{ik}\frac{\partial}{\partial x^m}\right) - \left(\frac{m^2c^2}{\hbar^2} + \frac{1}{a^2}\right)\right]\psi = 0. \tag{4}$$

For the metric (2), the nonzero Christoffel symbols have the form

$$\Gamma^1_{11} = \frac{r}{a^2(1 - r^2/a^2)}, \qquad \Gamma^1_{22} = r\left(1 - \frac{r^2}{a^2}\right), \qquad \Gamma^1_{33} = r\sin^2\theta\left(1 - \frac{r^2}{a^2}\right),$$

$$\Gamma^2_{12} = \Gamma^3_{13} = 1/r, \qquad \Gamma^2_{33} = -\sin\theta\cos\theta, \qquad \Gamma^3_{23} = \operatorname{ctg}\theta.$$

Now (4) can be written in explicit form

$$\left[-\frac{\partial^2}{c^2\partial\tau^2}+\left(1-\frac{r^2}{a^2}\right)\left(\frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r}\right)-\frac{r}{a^2}\frac{\partial}{\partial r}+\frac{1}{r^2}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]-\left(\frac{m^2c^2}{\hbar^2}+\frac{1}{a^2}\right)\right]\psi = \tag{5}$$

Since

$$\frac{\partial^2\psi}{\partial r^2}+\frac{2}{r}\frac{\partial\psi}{\partial r}=\frac{1}{r}\frac{\partial^2(r\psi)}{\partial r^2},\qquad r\frac{\partial\psi}{\partial r}=\frac{\partial(r\psi)}{\partial r}-\psi,$$

then (5) takes the form

$$\left[-\frac{\partial^2(r\psi)}{c^2\partial\tau^2}+\left(1-\frac{r^2}{a^2}\right)\frac{\partial^2(r\psi)}{\partial r^2}-\frac{r}{a^2}\frac{\partial(r\psi)}{\partial r}+\frac{r\psi}{a^2}+\frac{1}{r^2}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial(r\psi)}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2(r\psi)}{\partial\varphi^2}\right]-\left(\frac{m^2c^2}{\hbar^2}+\frac{1}{a^2}\right.$$

or

$$\left[-\frac{\partial^2}{c^2\partial\tau^2}+\left(1-\frac{r^2}{a^2}\right)\frac{\partial^2}{\partial r^2}-\frac{r}{a^2}\frac{\partial}{\partial r}+\frac{1}{r^2}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right)-\frac{m^2c^2}{\hbar^2}\right]\varphi=0, \tag{6}$$

where $\varphi=\psi r$.

Set $r=a\sin\chi$; then (6) takes the form

$$\left[-\frac{\partial^2}{c^2\partial\tau^2}+\frac{1}{a^2}\frac{\partial^2}{\partial\chi^2}+\frac{1}{a^2\sin^2\chi}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right)-\frac{m^2c^2}{\hbar^2}\right]\varphi=0. \tag{7}$$

The operator

$$\frac{1}{\sin\theta}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)\right]+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}=-\hat{l}^2=-l(l+1),$$

where $l=0,1,2,...$ is the angular momentum (orbital quantum number).

Therefore (7) takes the form:

$$\left[-\frac{\partial^2}{c^2\partial\tau^2}+\frac{1}{a^2}\frac{\partial^2}{\partial\chi^2}-\left(\frac{m^2c^2}{\hbar^2}+\frac{l(l+1)}{a^2\sin^2\chi}\right)\right]\varphi=0. \tag{8}$$

Let

$$\varphi = A(\chi) e^{-\frac{i}{\hbar} E\tau} \mathcal{Y}_{lm}(\theta; \varphi), \tag{9}$$

where

$$\mathcal{Y}_m = \theta_{lm} e^{im\varphi}; \tag{10}$$

$$\theta_{lm} = \frac{(-1)^{l+m}}{2^l l!} \sqrt{\frac{(2l+1)(l-m)!}{2(l+m)!}} \sin^m \theta \frac{d^{l+m} \sin^{2l} \theta}{(d \cos \theta)^{l+m}},$$

$m = 0, \pm 1, \pm 2, ...$ is the magnetic quantum number.

Substituting (9) into (8), we obtain

$$\left[\left(\frac{E^2}{c^2} - m^2 c^2 \right) \frac{a^2}{\hbar^2} - \frac{l(l+1)}{\sin^2 \chi} \right] A + A_{\chi\chi} = 0. \tag{11}$$

Denote

$$(E^2/c^2 - m^2 c^2) a^2/\hbar^2 = \xi^2 \tag{12}$$

and instead of $l(l+1)$ introduce $n(n-1) = l(l+1)$, whence $n_1 = l+1,\ n_2 = -l$; then (11) assumes the standard form

$$\sin^2 \chi A_{\chi\chi} + \left[\xi^2 \sin^2 \chi - n(n-1) \right] A = 0. \tag{13}$$

The general solution of equation (13), as is known, has the form ([2])

$$A = \sin^n \chi \left(\frac{1}{\sin \chi} D \right)^n \left(\bar{A}_1 e^{-i\xi\chi} + \bar{A}_2 e^{i\xi\chi} \right), \tag{14}$$

where

$$\left(\frac{1}{\sin \chi} D \right)^n = \frac{1}{\sin \chi} \frac{d}{d\chi} \left[\frac{1}{\sin \chi} \frac{d}{d\chi} \left(\frac{1}{\sin \chi} \frac{d}{d\chi} (\cdots) \right) \right]. \tag{15}$$

Thus we find that

$$\psi = \frac{\bar{A}_1}{r} \theta_{lm} e^{-\frac{i}{\hbar} E\tau + im\varphi} \sin^{l+1} \chi \left(\frac{1}{\sin \chi} D \right)^{l+1} e^{-i\xi\chi} = \frac{\varphi}{r},$$

$$\psi^* = \frac{\bar{A}_2}{r}\theta^*_{lm} e^{\frac{i}{\hbar}E\tau - im\varphi}\sin^{-l}\chi\left(\frac{1}{\sin\chi}D\right)^{-l} e^{-i\xi\chi} = \frac{\varphi^*}{r}, \tag{16}$$

where $\xi = pa/\hbar,\ p = \sqrt{E^2/c^2 - m^2c^2}$ is the momentum.

For $l = 0,\ m = 0\ (n = 0)$ we shall have

$$\psi = \frac{A_1}{r}e^{-\frac{i}{\hbar}(E\tau + pa\chi)}, \qquad \psi^* = \frac{A_2}{r}e^{\frac{i}{\hbar}(E\tau + pa\chi)}. \tag{17}$$

The trace of the energy-momentum tensor corresponds to the given wave equation (1) and has the form ([3])

$$-T = \frac{m^2c^2}{\hbar^2}\langle\psi\psi^*\rangle. \tag{18}$$

If we choose $A_1A_2 = \beta\frac{m^2c^3a^2}{\hbar}$, where $\beta = \text{const}$ is determined below, then $\nabla\psi\nabla\psi^*$ will have the dimension of energy density (as it should). In this case

$$\langle\psi\psi^*\rangle = \frac{8\pi\beta m^2c^3a^2}{\pi^2a^3\hbar}\int_0^a \frac{dr}{\sqrt{1 - r^2/a^2}} = \frac{8m^2c^3\beta}{\pi\hbar}\chi\Big|_0^{\pi/2} = \frac{4\beta m^2c^3}{\hbar}; \tag{19}$$

$$-T = 4\beta m^4c^5/\hbar^3; \tag{20}$$

$$R = 6/a^2 = -\chi T = \frac{32\pi\beta m^4c}{\hbar^3}G. \tag{21}$$

If we define $-T = 2\varepsilon$ (with $3p + \varepsilon = 0$), which holds for the Einstein space with metric (2), then we find

$$-T = 3mc^2/2\pi a^3. \tag{22}$$

From (20) and (22) we have $mca = \hbar(3/8\pi\beta)^{1/3}$. Put $\beta = 3/8\pi$; then

$$mca = \hbar. \tag{23}$$

From (21) and (23) we have

$$m = \sqrt{\frac{c\hbar}{2G}}, \qquad a = L = \sqrt{\frac{2G\hbar}{c^3}} = 2Gm/c^2 = r_g, \tag{24}$$

i.e., for m and a we obtain the Planck values (the values of the mass and size of the planckeon), with $a = L = r_g$–the gravitational radius, as must be the case for an Einstein universe. At the same time, the result obtained checks the calculations performed.

Denote

$$\frac{1}{\hbar}[E\tau + pa\chi] = \frac{1}{\hbar}\left[E\tau + pa \arcsin\frac{r}{a}\right] = \alpha,$$

then (17), for $m = 0,\ l = 0$ ($n = 0$), takes the form

$$\psi = \frac{A_1}{r}e^{-i\alpha}. \tag{25}$$

The de Broglie wave will propagate according to the law:

$$E\tau + pa \arcsin\frac{r}{a} = \alpha\hbar,$$

or

$$r = a\sin\frac{E\tau - \alpha\hbar}{pa} = a\sin\frac{E(\tau-\tau_0)}{pa} = a\sin\left[\frac{E}{pc}\frac{c(\tau-\tau_0)}{a}\right]. \tag{26}$$

Hence the phase velocity is

$$\frac{V_{\text{ph}}}{c} = \sqrt{1+\frac{m^2c^2}{p^2}}\cos\left[\sqrt{1+\frac{m^2c^2}{p^2}}\,\frac{c(\tau-\tau_0)}{a}\right],$$

the group velocity is

$$\frac{V_{\text{gr}}}{c} = \frac{1}{\sqrt{1+m^2c^2/p^2}}.$$

Expression (26) shows that the de Broglie wave corresponding to the planckeon is localized in the region $0 \le r \le a = r_g = L$ and that the wave packet, held by its own gravitational field, is stable and does not spread.

For values $c\tau/a \ll 1$,

$$r = c(\tau-\tau_0)E/pc = E(\tau-\tau_0)/p = c(\tau-\tau_0)\sqrt{1+m^2c^2/p^2}, \tag{27}$$

i.e., for $a \to \infty$, when gravity is switched off, the usual-

···situation of spreading of the wave packet, since as $\tau \to \infty$, $r \to \infty$. The conclusion regarding the stability of the wave packet in its own gravitational field of a definite energy is fundamental, although quite expected. This conclusion is significant in studying the structure of elementary particles in their own gravitational field.

Earlier M. A. Markov [6] proposed that the maximon (a particle equivalent to the planckeon) is described by a particle-like solution of the Einstein and Dirac equations in the form of a limiting state of a wave packet whose energy is gravitationally closed in a region of size L.

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.