# STARK BROADENING OF HYDROGEN SPECTRAL LINES IN A TURBULENT PLASMA

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## Abstract

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# STARK BROADENING OF HYDROGEN SPECTRAL LINES IN A TURBULENT PLASMA

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One of the principal tasks of plasma physics is the measurement of the energy density contained in electrostatic oscillations that arise as a result of the turbulent character of dissipative processes ([1]). Stark broadening of the Balmer lines of hydrogen in a plasma may serve this purpose. Such a possibility was first considered by Woolf ([2]) even before the development of a consistent theory ([3,4]) of Stark broadening of hydrogen lines in an equilibrium plasma. Subsequently, the Stark effect in high-frequency stochastic fields was discussed within the framework of the adiabatic approximation in ([5]). However, the adiabatic approximation is unsuitable for describing high-frequency fields ([3]). In the present work a theory of Stark broadening of hydrogen spectral lines in a turbulent plasma is developed, taking into account the nonadiabatic effect of Langmuir oscillations on the atom, as well as the contribution of one-particle broadening mechanisms.

1. The profiles of hydrogen spectral lines are formed in a plasma under the action of the electric microfields of ions and electrons. The low-frequency ($\omega_i \lesssim v_T N^{1/3}$) component $\mathbf{E}_{\text{ion}}$ of the electric microfield, caused by ions, determines the Stark splitting of the levels $\Delta\omega_\alpha = \dfrac{3}{2} n(n_1 - n_2)\dfrac{ea_0}{\hbar}E_{\text{ion}}$ and the precession frequency $\omega_E = {}^3/_2 n\dfrac{a_0 e}{\hbar}E_{\text{ion}}$ of the dipole moment $\mathbf{d}$ of the atom about the direction of the field $\mathbf{E}_{\text{ion}}$ (here $\alpha$ denotes the entire set of quantum numbers $n, n_1, n_2, m$, which determine the state of the atom in the parabolic coordinate system ([4]), and $a_0$ is the Bohr radius). Changes in the field $\mathbf{E}_{\text{ion}}$ occur adiabatically ($\omega_i < \omega_E$), and the projection $\mathbf{d}$ on the direction $\mathbf{E}_{\text{ion}}$ is conserved. Changes in its projection $d_E$ occur under the influence of the high-frequency ($\omega_e > \omega_{pe} \gg \omega_E$) component of the electric microfield $\mathbf{E}_{\text{el}}$, caused by plasma electrons. In an equilibrium plasma, nonadiabatic effects of electron fields are usually taken into account within the framework of impact theory ([3,4]). They determine the lifetime $\tau_\alpha^{\text{imp}} = 1/\gamma_\alpha^{\text{imp}}$ of the atom on a given Stark sublevel. The mechanism of Stark broadening of hydrogen lines in a plasma admits an especially simple interpretation when $\tau_\alpha^{\text{imp}}$ is found to be considerably shorter than the period of varia-

tion of the ionic field ([6,7]). In this case the ion field may be regarded as quasistatic, and the profile of each Stark component as Lorentzian with a half-width equal to $\gamma_\alpha^{\text{imp}}$. The resulting profile is obtained by averaging over all possible values of the field strength $\mathbf{E}_{\text{ion}}$.

Depending on the pattern of Stark splitting, the broadening of hydrogen spectral lines has a different character. The half-widths of lines with a strong central component, such as $Ly_\alpha, H_\alpha, P_\alpha$, are practically entirely determined by electron impact broadening of the unshifted components: $\Delta\omega_{1/2} \approx \gamma_\alpha$. Conversely, the half-widths of lines without a central (unshifted) component, such as $Ly_\beta, Ly_\delta, H_\beta, H_\delta$, are determined ···

are determined by the linear Stark effect in the mean quasistatic field of the ions

$$\Delta\omega_{1/2} \approx 12.5\,(n^2 - n'^2)N^{2/3} \quad (^4)^*.$$

2. In the range of parameters ($N_e \gtrsim 5 \cdot 10^{13}$ cm$^{-3}$, $T_e \lesssim 50$ eV), the electric fields of ion-acoustic and still lower-frequency plasma oscillations are quasistatic for excited hydrogen atoms ($\gamma_a > \omega_{pi}$). The profiles of hydrogen lines that do not have a central Stark component ($Ly_\beta$, $H_\beta$, etc.) then turn out, over most of their extent, to be proportional to the distribution functions of the low-frequency electric field arising as the sum of statistically independent contributions from individual ions located near the emitter and from collective electrostatic oscillations with frequencies $\omega \lesssim \omega_{pi}$:

$$W(E,\beta) = \iint W_{\text{H}}(E')\, W_{\text{R}}(E'')\, \delta(E - E' - E'')\, dE'\, dE''. \tag{1}$$

The distribution of electric fields from individual ions $W_{\text{H}}(E)$ is described by the Holtsmark function ([8]), while the distribution of fields of oscillations with statistically independent phases $W_{\text{R}}(E)$ is described by the Rayleigh function ([10]). In formula (1) the parameter $\beta = E_{\text{H}}/E_{\text{R}}$ is the ratio of the field scales in the Holtsmark and Rayleigh distributions.

At a sufficiently high level of turbulence

$$\xi_i = \int E_k^2\, dk/8\pi N T_e = \langle E^2\rangle/8\pi N T_e \gg e^2 N^{1/3}/T_e,$$

the maximum of the distribution function $W(E,\beta)$ is shifted toward stronger fields ($E_{\max} \approx E_{\text{R}} > E_{\text{H}}$). The half-widths of lines without a central component will then be substantially larger than their values in an equilibrium plasma of the same concentration, and the characteristic dips in the central regions of the lines will be expressed much more clearly, since the nonadiabatic broadening of individual Stark components remains the same. In this case the mean strength of the nonequilibrium electric fields of low-frequency plasma oscillations can be determined from the Stark splitting $(\Delta\lambda)_{\text{exp}}$ of spectral lines,

$$\widetilde{E}_{\text{cp}} = \frac{4\pi\hbar c\,(\Delta\lambda)_{\text{exp}}}{3\,[n(n_1 - n_2) - n'(n_1' - n_2')]_{\text{cp}}\, e a_0 \lambda_0^2}, \tag{2}$$

where $(\Delta\lambda)_{\text{exp}}$ is the measured half-width, $\lambda_0$ is the wavelength of the line, and

$$\frac{3}{2} e a_0 [n(n_1 - n_2) - n'(n_1' - n_2')]_{\text{cp}}$$

is the component-averaged value of the difference of the projections of the dipole moment in the upper and lower states.

When only one ion-acoustic branch of electrostatic oscillations is pumped, the half-widths of the $H_\alpha$ and $H_\gamma$ lines should practically not increase in comparison with their values in an equilibrium plasma. The half-width of $H_\alpha$ will still be determined entirely by electron impact broadening of the unshifted component, while the half-width of $H_\gamma$ may even decrease because of the more substantial "smearing" of the side components. However, the wings of the lines will be more intense than in an equilibrium plasma, and the side components may "split off."

3. The frequency of electron Langmuir oscillations is $\sqrt{M_i/m_e}$ times higher than the ion-sound frequency and satisfies the nonadiabaticity criterion $\omega_{pe} > \omega_e$. Therefore the lifetime of the atom at a given Stark sublevel $\tau_a$ must depend on the level of turbulence at high frequencies. In order to calculate the corresponding transition frequency

$$\gamma_a^{\text{noise}} = 1/\tau_a^{\text{noise}},$$

it is necessary, as usual, to find the mean effect of these oscillations on the amplitude of the atomic state ([3], [4]). The averaging is carried out over a time interval sufficiently large in comparison with the period of the plasma oscillations, but sufficiently small in comparison with the time of growth—

*According to calculations ([9]), the ratio of the half-widths of the Balmer lines $H_\alpha$, $H_\beta$, $H_\gamma$ and $H_\delta$ is $1 : 20 :$

and $1 : 100 : 5 : 400$ when the calculated data ([9]) are extrapolated to the region $N_e \lesssim 10^{14}$ cm$^{-3}$ and $T_e \approx 50$ e

where usually the Doppler broadening of the $H_\alpha$ and $H_\gamma$ lines exceeds the Stark broadening.

vibrations and over the lifetime of the atom in the given state. In addition, one must average over the chaotic phases of the plasma oscillations. The corresponding calculations lead to the following expression, which is valid under the condition that the Stark splitting between neighboring sublevels is small in comparison with the electron plasma frequency:

$$\gamma_\alpha^{\text{noise}} = 6\pi N v_{T_e} \cdot n^2 \left[n^2 - (n_1 - n_2)^2 - m^2 + 1\right] \frac{a_0 v_{T_e}}{\omega_{pe}} \left\{ \int E_k^2 \, dk \, / \, 8\pi N T_e \right\}. \tag{3}$$

In a thermodynamically equilibrium plasma the noise level $\xi_e = \xi_{T_e} =$

$$= \int \frac{E_k^2 \, dk}{8\pi N T_e} \approx \left( N \frac{v_{T_e}^3}{\omega_{pe}^3} \right)^{-1} = (N r_{\text{De}}^3)^{-1},$$

and the contribution of Langmuir oscillations to the total transition frequency proves to be small compared with the contribution of individual electron collisions:

$$(\gamma_\alpha^{\text{coll}} / \gamma_\alpha^{\text{noise}}) \approx {}^3/_{4\pi} \ln(T_e / \hbar n^2 \omega_{pe}) \sim 10.$$

However, in a turbulent plasma, where $\xi_e \gg \xi_{T_e} = (N r_D^3)^{-1}$, the transition frequency $\gamma_\alpha^{\text{noise}}$ caused by Langmuir oscillations may significantly exceed the electron collision frequency. This will primarily affect the shapes of the $Ly_\alpha$, $H_\alpha$, $P_\alpha$ lines, whose profiles are determined essentially only by the broadening of the central Stark component, which carries the main fraction of the total line intensity.

The turbulence level $\xi_e$ of Langmuir oscillations can be measured from the broadening of these lines,

$$\xi_e \approx (\Delta\lambda_{1/2})_{\text{exp}} \omega_{pe} m_{ec} / 3\lambda_0^2 n^4 a_0 N T_e \tag{4}$$

provided, of course, that other broadening mechanisms are unimportant here, and that $N$ and $T_e$ are known.

The development of turbulence at Langmuir frequencies will not, however, affect the half-widths of the $Ly_\beta$, $Ly_\delta$, $H_\beta$, and $H_\delta$ lines as long as $\gamma_\alpha^{\text{noise}}$ remains smaller than the mean Stark splitting between neighboring levels $\Delta\omega \sim 3n\hbar / e m_{eE} 0$, i.e. as long as

$$\xi_e \lesssim 2\pi e^4 N^{2/3} / \hbar n^3 \omega_{pe} T_e. \tag{5}$$

The proportionality between the broadening of an individual Stark component and the energy density of Langmuir oscillations will hold as long as $\tau_\alpha \gg 2\pi\omega_{pe}^{-1}$. For $\tau_\alpha \lesssim 2\pi\omega_{pe}^{-1}$ ($\xi_e \gtrsim e^2 / 2n^4 a_0 T_e$), the conditions for applicability of the usual ([4,9]) method of calculating the nonadiabatic transition frequency are violated. The calculations can be performed in the case $\xi_e \gtrsim e^2 / 2n^2 a_0 T_e$, when the electric fields $E_{pe}$ of the Langmuir oscillations prove so large that, during a significant part of the period $\tau_{pe} = 2\pi/\omega_{pe}$, the atomic dipole moment adiabatically follows the vector $E_{pe}$, but in each period the adiabaticity of the precession is violated with probability equal to unity when, in the course of the oscillation, the field $E_{pe}$ reaches minimal values $|E_{pe}| \ll E_0$. Therefore, for all lines for which the energy of the upper level satisfies the condition $\varepsilon_n = e^2/2n^2a_0 < n^2\xi_{eT}e$, the half-widths will be identical and equal to $\Delta\omega_{1/2} \approx \omega_{pe}$. They will no longer increase with an increase in the noise level of high-frequency electrostatic oscillations. Now the minimum value $n_{\min}$ for which such proportionality still holds can serve as an indicator of the turbulence level:

$$\xi_e \approx e^2/2n_{\min}^2 a_0 T_e.$$

4. With simultaneous excitation in the plasma of high-frequency ($\omega \gtrsim \omega_{pe}$) electrostatic oscillations with level $(Nr_D^3)^{-1} \ll \xi_e < e^2/2n^4a_0T_e$ and low-frequency oscillations with level $e^2N^{1/3}/T_e < \xi_i < e^2/2n^2a_0T_e$, the half-widths of lines with a strong central Stark component ($Ly_\alpha, H_\alpha, H_\gamma,$ etc.) are determined by the energy density of Langmuir oscillations, whereas the half-widths of lines without a central component ($Ly_\beta, Ly_\delta, H_\beta,$ etc.) are determined by the mean field strength of low-

low-frequency oscillations. For $\xi_i \gtrsim \xi_i^{\text{cr}} \approx e^2/2n_i^2a_0T_e$, the precession frequency of the atomic dipole moment around the vector of the low-frequency electric field proves to be greater than $\omega_{pe}$, and the Langmuir oscillations prove to be adiabatic. In this case, for lines beginning from the level $n > n_i$, the quantity $\tau_a$ is determined only by electron collisions, and instead of the "smeared" profiles characteristic of nonadiabatic broadening by plasma oscillations, a rather distinct splitting should be observed, analogous to the case of excitation by low-frequency oscillations alone. For $\xi_e > e^2/2n_0^2a_0T_e$, the half-widths $\Delta\omega_{1/2}$ of lines beginning from levels $n > n_0$ must be identical and equal to $\omega_{pe}$. The presence of low-frequency turbulence will not affect those lines for which the high-frequency broadening exceeds the splitting in quasistatic fields,

$$\omega_{pe} \gtrsim \frac{3}{4}(n^2 - n'^2)\frac{ea_0}{\hbar}\widetilde{E}_{\text{cp}},$$

i.e., lines whose upper level satisfies the inequality $n_{\text{cr}} \lesssim e^2/a_0T_e\xi_i$. The level of low-frequency turbulence $\xi_i$ can therefore be determined from the maximum value $n_{\max} = n_{\text{cr}}$ at which the equality $\Delta\omega_{1/2} = \omega_{pe}$ is still satisfied, and above which $\Delta\omega_{1/2} > \omega_{pe}$, and the profiles of hydrogen lines without a central component exhibit the characteristic splitting.

In an experimental study of plasma turbulence it is extremely important to separate the broadening caused by collective oscillations from the broadening associated with an increase in the concentration of charged particles. This imposes rather stringent requirements on the rate of recording the spectral-line profiles and on the degree of ionization of the preliminary plasma, since in turbulent stochastic fields of electrostatic oscillations an avalanche ionization process of the high-frequency-breakdown type is possible. These requirements are best satisfied by using the method of high-speed electro-optical spectrochronography ([11]), and the first experimental results obtained by this method ([12,13]) are in good agreement with the theoretical concepts developed above.

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*Note: Figure translations are in progress. See original paper for figures.*