UNIVERSAL LAW OF TURBULENT HEAT AND MASS TRANSFER FROM A WALL AT LARGE REYNOLDS AND PECLET NUMBERS

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Abstract

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UNIVERSAL LAW OF TURBULENT HEAT AND MASS TRANSFER FROM A WALL AT LARGE REYNOLDS AND PECLET NUMBERS

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1. The principal dimensionless characteristic of turbulent heat and mass transfer from a plane wall $y = 0$ to a plane-parallel flow of fluid moving along the Ox axis is the Stanton number

$$\mathrm{St} = j_w / c_p \rho U (\vartheta_w - \Theta)$$

or

$$\mathrm{St} = j_w / \rho U (\vartheta_w - \Theta)$$

(the first formula refers to heat transfer, and the second to mass transfer). Here j_w is the flux of admixture or heat in the Oy direction at $y = 0$, and ϑ_w is the wall temperature or admixture concentration at the wall (j_w and ϑ_w are assumed constant), ρ is the density of the fluid, c_p its heat capacity, U a characteristic velocity of the fluid, and Θ a characteristic temperature (or admixture concentration). The fluid is assumed incompressible, and the admixture (or heat) is assumed not to affect the flow (i.e., to be dynamically passive). As the quantities U and Θ, it is theoretically most convenient to take the values $u_m = \bar{u}(L)$ and $\vartheta_m = \bar{\vartheta}(L)$ of the mean velocity $\bar{u} = \bar{u}(y)$ and the mean temperature (or concentration) $\bar{\vartheta} = \bar{\vartheta}(y)$ at the center of a pipe or channel (for flows in pipes and plane channels) or at the outer boundary of the boundary layer (for the case of flow past a plate); in practice, however, in the study of heat and mass transfer in pipes and channels, as U and Θ one usually uses the more simply measured mean velocity u_{av} and mean temperature ϑ_{av} of the entire mass of flowing fluid (the corresponding value of St we shall denote by St_{av}). The numbers St and St_{av}, or the Nusselt numbers associated with them,

$$\mathrm{Nu} = \mathrm{St}\,\mathrm{Re}\,\mathrm{Pr}$$

and

$$\mathrm{Nu}_{av} = \mathrm{St}_{av}\mathrm{Re}_{av}\mathrm{Pr},$$

are functions of the Reynolds number $\mathrm{Re} = UL/\nu$, or $\mathrm{Re}_{av} = 2LU_{av}/\nu = DU_{av}/\nu$, and the Prandtl number (thermal or diffusional) $\mathrm{Pr} = \nu/\chi$, where ν is the kinematic viscosity coefficient and χ is the coefficient of thermal diffusivity or diffusion; an extensive literature is devoted to the semiempirical (and purely empirical) study of their dependence on Re and Pr (see, for example, ([1–4])). It turns out, however, that at sufficiently large Reynolds and Peclet numbers $\mathrm{Pe} = \mathrm{Re}\,\mathrm{Pr}$ the general form of the dependence of St on Re and Pr can be obtained from similarity considerations alone.

2. For definiteness, we shall speak of heat transfer (bearing in mind that everything said is also valid for mass transfer). Near a smooth wall the profile $\bar{\vartheta}(y)$ depends only on the parameters ν, χ, $u_* = \sqrt{\tau_w/\rho}$ (where τ_w is the shear stress at the wall), and $j_w/c_p\rho$. Consequently,

$$\vartheta_w - \bar{\vartheta}(y) = \theta_* \varphi(y_+, \mathrm{Pr}), \quad \text{where } \theta_* = \frac{j_w}{c_p \rho u_*}, \quad y_+ = \frac{y u_*}{\nu} \tag{1}$$

(see, for example, ([1])). On the other hand, at a sufficiently large distance from the wall, at large Re and Pe, the turbulent viscosity and thermal diffusivity dominate over the molecular ones, and the heat-transfer regime does not depend on ν or χ (cf. the statement of the general similarity principle with respect to the Reynolds number in ([5])). Therefore, for large values of y, Re, and Pe,

$$\bar{\vartheta}(y) - \vartheta_m = \theta_* \varphi_1(\xi), \quad \text{where } \xi = y/L. \tag{2}$$

(this law of the temperature defect is less well known than the analogous Karman law of velocity defect, but it is not new—cf., for example, ([6,7])). Let now Re and Pe be so large that there exists an "overlap interval" of values of y, within which both laws (1) and (2) are simultaneously valid. In this case, literally repeating the simple argument first indicated by Izakson ([8]) as applied to the velocity field (see also ([1,5])), one can verify that within this interval both functions φ and φ_1 must be logarithmic:

$$\varphi(y_+, \mathrm{Pr}) = A \ln y_+ + B, \qquad \varphi_1(\xi) = -A \ln \xi + B_1. \tag{3}$$

Here the coefficients in the expression for φ, generally speaking, may depend on Pr, but do not depend on the character of the flow in the "core of the stream," while the coefficients of the function φ_1 do not depend on the molecular constants, but may be different in the case of a circular pipe, a plane channel, and a boundary layer on a plate; consequently A is a universal constant (namely $A = P/\varkappa$, where $\varkappa \approx 0.4$ is the Karman constant, and P is the turbulent Prandtl number). Substituting (3) into (1) and (2) and adding the results, we obtain

$$\mathrm{St} = \sqrt{c_f/2}\,[A \ln(\mathrm{Re}\sqrt{c_f/2}) + A_1(\mathrm{Pr})]^{-1}, \qquad A_1(\mathrm{Pr}) = B(\mathrm{Pr}) + B_1, \tag{4}$$

where $c_f = \lambda/4 = 2(u_*/U)$ is the friction-resistance coefficient, whose dependence on Re has been studied in detail (see, for example, (1,9,10)). The general law of heat transfer (4) is obviously related to the analogous classical Prandtl–Nikuradze resistance law derived similarly, which has the form $\sqrt{2/c_f} = A_2 \ln(\mathrm{Re}\sqrt{c_f/2}) + B_2$; at fixed Pr, like the latter, it contains only two undetermined coefficients.

3. The coefficient A, apparently, is most accurately determined from data of careful measurements of the temperature profile in a slightly heated circular pipe at Re of the order of $10^4 \div 10^5$, carried out by Johnk and Hanratty (6) and by (11). These authors independently obtained results according to which $P \approx 0.87 \div 0.9$, i.e. $A \approx 2.2$. The coefficient B_1 may be different for pipes, channels, and boundary layers; here we restrict ourselves only to flows in circular pipes. If it is assumed that the profile $\vartheta(y)$ can be regarded as logarithmic all the way to the pipe axis, then $B_1 = 0$; however, the data of (6) show that a more accurate estimate is $B_1 \approx 0.6$. Finally, according to the data of works (6,11), at $\mathrm{Pr} \approx 0.7$ (for heat transfer in air) $B(0.7) \approx 3.4$. Thus, at $\mathrm{Pr} \approx 0.7$ both coefficients of formula (4) for a circular pipe may be considered known.

When comparing formula (4) for $\mathrm{Pr} = 0.7$ with empirical data on heat transfer, it should be taken into account that the latter data always refer to $\mathrm{St}_{av} = \mathrm{St}/(1-\Delta)$, where $\Delta = (\vartheta_{av} - \vartheta_m)/(\vartheta_w - \vartheta_m)$. For an approximate estimate of Δ at $\mathrm{Pr} \geq 0.7$, one may neglect the sublayer of molecular viscosity and thermal conductivity, and consider the defect laws logarithmic all the way to the axis. Then, analogously to the derivation of formula (5.47) in book (1), we obtain: $\Delta \approx 3.4\mathrm{St}/(c_f/2)^{1/2}$, whence it follows that Δ is of the order of $0.2 \div 0.1$ at $\mathrm{Pr} \approx 0.7$ and can be easily calculated using formula (4).

The results of a numerical calculation of Nu_{av} by formula (4) with coefficients $A = 2.25$ and $A_1 = 4.0$ (taking into account the correction Δ) are presented in Fig. 1 together with empirical data (borrowed from 13 works, many of which are indicated in (1,4)) and the results of calculation by the approximate empirical formula $\mathrm{Nu}_{av} = 0.018\,\mathrm{Re}_{av}^{0.8}$, recommended in (2).

4. For formula (4) to be usable at other Pr, it is necessary to specify the values of the function $B(\mathrm{Pr})$, describing the temperature (or concentration) drop in the wall layer extending to the lower boundary of the logarithmic layer. The form of this function in the case $\mathrm{Pr} \ll 1$ (heat transfer in liquid metals) and in the case $\mathrm{Pr} \gg 1$ (heat transfer in liquid oils or mass transfer in droplet liquids) can be estimated with the aid of

using Levich' s "three-layer model" (12), according to which

$$A_1(\mathrm{Pr}) = B(\mathrm{Pr}) + B_1 = C \overset{(n-1)/n}{\mathrm{Pr}} + 5.25 \ln \mathrm{Pr} + C_1, \qquad (5)$$

where, in accordance with the data of recent studies (13–16), one should take

Figure 1: Comparison of heat-transfer measurement data in circular tubes (Pr ≈ 0.7) (points) with M. A. Mikheev' s empirical formula (dashed curve) and the authors' formula (solid curve).

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Figure 2

Figure 2: Figure 2

$n = 3$, $C = 12.5$, while C_1 can depend only weakly on Pr, so that, in the first approximation, $C_1 = \text{const}$. If $C_1 = -5.0$ (taking into account that $B(0.7) = 3.4$), then the corresponding formula

Fig. 1. Comparison of heat-transfer measurement data in circular tubes (at Pr ≈ 0.7) (points) with M. A. Mikheev' s empirical formula (dashed curve) and the authors' formula (solid curve)

$$\mathrm{N}_{av} = \frac{\sqrt{\lambda/8}\,\mathfrak{R}_{av}\,\mathrm{Pr}}{5.25\lg\left(\mathfrak{R}_{av}\,\mathrm{Pr}\,\sqrt{\lambda/8}\right) + 12.5\,\mathrm{Pr}^{2/3} - 5.0} \cdot \frac{1}{1-\Delta} \qquad (6)$$

(where $\lambda = 4c_f$, and the correction Δ will be negligibly small for all $\mathrm{Pr} \geqslant 10$) already turns out to correspond well to practically all available empirical data on heat and mass transfer in circular tubes for $\mathfrak{R}$ varying from $5 \cdot 10^3$ to $2 \cdot 10^5$ and Pr varying over 6 orders of magnitude—from 0.7 to 10^6 (see Fig. 2, the experimental data in which are borrowed from 35 works, most of which are cited in (4,16,17)).

The situation is somewhat more complicated with data on heat transfer in liquid metals, where $\mathrm{Pr} \ll 1$, which are all characterized by a large scatter of experimental points (see, for example, (3,12)) and refer to somewhat different conditions, when not θ_w is fixed but j_w. If here we use the same formulas (4) and (5), but only set $C_1 = 0$ (i.e., discard the free term -5.0 in the denominator of (6)) and calculate the correction Δ more accurately, taking into account the presence of a sublayer of molecular thermal conductivity with a linear profile of the mean temperature, then the calculated values of N_{av} will agree well with the available data on heat transfer

Fig. 2. Comparison of empirical data on heat and mass transfer at various values of Re and Pr (points) with the results of calculations by formula (6) (curves).
1 –Re = 5000; 2 –10,000; 3 –25,000; 4 –50,000; 5 –100,000; 6 –200,000

Fig. 3. Comparison of empirical data on heat transfer in liquid metals at Pr 0.025 (points) with the results of calculations by the modified formula (6)

Figure 3

Figure 3: Figure 3

(curve)

in liquid metals (see, for example, Fig. 3, which refers to data, characterized by a relatively small scatter, from 7 works on heat transfer in mercury and in a sodium-potassium alloy, for which Pr 0.025).

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Note: Figure translations are in progress. See original paper for figures.