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Abstract

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MECHANICS

V. S. LODSHININ

ABSOLUTELY CONTINUABLE LIMITING MODES OF MOTION OF MACHINE UNITS WITH PIECEWISE-MONOTONE CHARACTERISTICS

(Presented by Academician I. I. Artobolevskii on 25 V 1970)

1. For fairly broad classes of machine units with monotone and piecewise-monotone characteristics, the principal tendencies in the behavior of the parameters describing their dynamics are revealed in absolutely continuable limiting modes of motion.

In the present article we consider the question of the conditions for the occurrence and the topological structure of such modes. For definiteness and brevity of exposition, throughout we shall speak of modes with respect to the kinetic energy T of a machine unit.

Accordingly, the equation of motion of the driving member ([1]) is written in the form

$$dT/d\varphi = M(\varphi, T) = M\,(\varphi, T) - M\,(\varphi, T). \tag{1}$$

It is assumed that

1°. The reduced moment of inertia $I = I(\varphi)$ depends on the angle of rotation φ of the driving member, and the reduced moment $M(\varphi, T)$ of all forces acting on the unit is a function that is piecewise-monotone with respect to the kinetic energy T, defined and continuous in the strip

$$-\infty < \varphi < +\infty, \quad 0 \leq T \leq \widetilde{T}, \tag{2}$$

where $\widetilde{T}$ is the maximum possible value of the kinetic energy T of the unit that can be imparted to it by the acting forces.

The steepness $M'_T(\varphi, T)$ of the reduced moment of all acting forces is assumed to be discontinuous in the strip (2).

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2. As is known ([2]), the behavior of the kinetic energy T of a machine unit is closely connected with the structure of its inertial curve.

Theorem 1. *Suppose that the reduced moment $M(\varphi, T)$ of all acting forces satisfies the conditions listed in 1° and, in addition, there exists a pair of functions*

$$T = \gamma_1(\varphi), \quad T = \gamma_2(\varphi), \quad 0 \le \gamma_1(\varphi) \le \gamma_2(\varphi) \le \widetilde{T}, \tag{3}$$

$$-\infty < \varphi < +\infty,$$

such that

$$M[\varphi, \gamma_1(\varphi)] \cdot M[\varphi, \gamma_2(\varphi)] \le 0, \quad -\infty < \varphi < +\infty. \tag{4}$$

Then there exists an inertial curve $\mathcal{T}$ of the motion of the machine unit, defined for every value of the angle of rotation φ of the driving member.

In the general case, under the conditions of Theorem 1, for each value of the angle of rotation φ there will exist one or several values of the kinetic energy

$$T = \tau_i(\varphi), \quad \tau_i(\varphi) < \tau_{i+1}(\varphi), \quad -\infty < \varphi < +\infty, \quad i = 1, 2, \dots, n, \tag{5}$$

for which the reduced moments of the driving forces and of the resistance forces mutually balance:

$$M[\varphi, \tau_i(\varphi)] \equiv 0, \quad -\infty < \varphi < +\infty.$$

The functions $T = \tau_i(\varphi)$ and the graphs corresponding to them shall be called single-valued branches of the inertial curve $\mathcal{T}$ of the motion of the machine unit.

The number of single-valued branches of the inertial curve is quite definite for each individual machine unit over a rather broad range of variation of the loading law of the working machine.

Machine units whose inertial curves are represented by one single-valued branch are widespread. These include, in particular, units with negative load, $M'_T(\varphi, T) < 0$, of the reduced moment of all acting forces, for example, units in which the reduced moments $M_d(\varphi, T)$ of the driving forces decrease with increasing kinetic energy T, while the moments $M_c(\varphi, T)$ of the resistance forces increase with the same parameter.

A fairly significant class of machine units consists of units whose inertial curves are made up of two or three single-valued branches. Such, for example, are the inertial curves of machine units with asynchronous three-phase-current motors

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under certain laws of their loading, and the inertial curve of ship motion in the planing regime ([3]).

A branch $T = \tau_i(\varphi)$ of the inertial curve $\mathcal{T}$ of the motion of a machine unit will be called stable (unstable) if, upon passing through it from below upward, the reduced moment $M(\varphi, T)$ of all acting forces changes its sign from plus (minus) to minus (plus). In practice, in the successive transition from the first (lower) branch $T = \tau_1(\varphi)$ to the next, the character of their stability simply alternates.

Theorem 2. *Suppose that the reduced moment $M(\varphi, T)$ of all acting forces satisfies conditions 1° and, in addition:*

2°. *There exist differentiable functions*

$$T = \theta_1(\varphi), \quad T = \theta_2(\varphi), \quad 0 \le \theta_1(\varphi) \le \theta_2(\varphi) \le \widetilde{T},$$

$$-\infty < \varphi < +\infty, \tag{6}$$

for which the relation holds

$$\{M[\varphi, \theta_1(\varphi)] - \theta_1'(\varphi)\}\,\{M[\varphi, \theta_2(\varphi)] - \theta_2'(\varphi)\} \le 0, \quad -\infty < \varphi < +\infty. \tag{7}$$

Then there exists at least one absolutely continuable integral of the kinetic energy of the unit, wholly contained in the strip

$$\theta_1(\varphi) \le T \le \theta_2(\varphi), \quad -\infty < \varphi < +\infty. \tag{8}$$

Consider, for definiteness, the case when

$$M[\varphi, \theta_1(\varphi)] - \theta_1'(\varphi) \ge 0, \quad M[\varphi, \theta_2(\varphi)] - \theta_2'(\varphi) \le 0, \quad -\infty < \varphi < +\infty. \tag{9}$$

When these inequalities are satisfied, the curves $T = \theta_1(\varphi)$, $T = \theta_2(\varphi)$ are curves of one-sided conductance for the solutions of equation (1) of the motion of the machine unit. The strip (8), obviously, is a strip of stability: the integral curves $T = T(\varphi)$ of equation (1) that enter this strip as they proceed to the right, with further increase of the angle of rotation φ of the reduction link, cannot leave it. It is also easy to see that every solution $T = T(\varphi)$ of equation (1) that has points in common with the strip (8) is unboundedly continuable to the right.

Let

$$T = T[\varphi, \bar{\varphi}, \theta_2(\bar{\varphi})]$$

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be a solution of equation (1), determined by the initial conditions $T(\bar{\varphi}) = \theta_2(\bar{\varphi})$, where $\bar{\varphi}$ is any real number.

Taking $\bar{\varphi}$ as a parameter, consider the family of solutions

$$T = T[\varphi, \bar{\varphi}, \theta_2(\bar{\varphi})], \quad \bar{\varphi} \leq \varphi < +\infty. \tag{10}$$

By virtue of inequalities (9) and the uniqueness property it will be monotone

with respect to the parameter $\bar{\varphi}$ and bounded. Therefore, for any value of the angle of rotation φ there will exist a finite limit

$$\lim_{\bar{\varphi} \to -\infty} T[\varphi, \bar{\varphi}, \theta_2(\bar{\varphi})] = T^*(\varphi), \qquad -\infty < \varphi < +\infty. \tag{11}$$

The function $T^*(\varphi)$ is defined on the entire number line and, as the limit of solutions, is itself in turn a solution of equation (1) of the motion of the machine aggregate.

It is clear from the course of the reasoning that there can be no absolutely continuable integrals of the kinetic energy lying in the strip (8) above the solution $T = T^*(\varphi)$, and in this sense $T = T^*(\varphi)$ is the uppermost of them.

Quite analogously, considering the family of solutions

$$T = T[\varphi, \bar{\varphi}, \theta_1(\bar{\varphi})], \qquad \bar{\varphi} \leq \varphi < +\infty, \tag{12}$$

which is monotone with respect to the parameter $\bar{\varphi}$, one can verify the existence, for every φ, of the finite limit

$$\lim_{\bar{\varphi} \to -\infty} T[\varphi, \bar{\varphi}, \theta_1(\bar{\varphi})] = T_*(\varphi), \qquad -\infty < \varphi < +\infty, \tag{13}$$

which is the lowermost absolutely continuable integral of the kinetic energy of the motion of the machine aggregate in the strip (8).

Obviously,

$$\theta_1(\varphi) \leq T_*(\varphi) \leq T^*(\varphi) \leq \theta_2(\varphi), \qquad -\infty < \varphi < +\infty, \tag{14}$$

and all absolutely continuable integrals of the kinetic energy $T = T(\varphi)$ contained in the strip (8) fill completely the domain

$$-\infty < \varphi < +\infty, \qquad T_*(\varphi) \leq T \leq T^*(\varphi). \tag{15}$$

The case of inequalities in the opposite sense,

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$$M[\varphi,\theta_1(\varphi)] - \dot\theta_1(\varphi) \le 0, \qquad M[\varphi,\theta_2(\varphi)] - \dot\theta_2(\varphi) \ge 0, \quad -\infty < \varphi < +\infty, \tag{16}$$

is exhausted analogously. We note only that in the present case the strip (8) is a strip of instability: any integral curve $T = T(\varphi)$ of equation (1) of the motion of the machine aggregate that has left the strip (8) at some value of the angle of rotation φ of the drive member will never again enter it in its course to the right.

Corollary. *If the reduced moment $M(\varphi, T)$ of all acting forces satisfies conditions 1° and, in addition, there exists a pair of real numbers α and β, $0 < \alpha \le \beta \le \hat T$, such that for every value of the angle of rotation φ of the drive member the relation*

$$M(\varphi,\alpha)M(\varphi,\beta) \le 0, \qquad -\infty < \varphi < +\infty, \tag{17}$$

is fulfilled, then there exists at least one absolutely continuable integral $T = T(\varphi)$ of the kinetic energy of the motion of the machine aggregate satisfying the inequality

$$\alpha \le T(\varphi) \le \beta, \qquad -\infty < \varphi < +\infty. \tag{18}$$

Hence, as a special case, there follows the known result ([2]) on the existence of an absolutely continuable integral of the kinetic energy of an aggregate when the reduced moment of all acting forces has negative steepness, $M'_T(\varphi, T) < 0$, and $\alpha = 0,\ \beta = \hat T$.

3. The practical use of Theorem 2 and of the corollary following from it is connected with the question of identifying strips of stability and instability for the integral curves of equation (1) of the motion of a machine aggregate. Here we shall indicate one case of practical interest in which the identification of such strips does not cause difficulties.

Suppose that the single-valued branches (5) of the inertial curve $\mathcal{T}$ of the motion of the machine unit are known to us. Introducing the notation

$$\tau_*^{(i)} = \inf_{|\varphi|<\infty} \tau_i(\varphi), \qquad \tau_{(i)}^* = \sup_{|\varphi|<\infty} \tau_i(\varphi), \tag{19}$$

we consider the strips

$$-\infty < \varphi < +\infty, \qquad \tau_*^{(i)} \le T \le \tau_{(i)}^*, \quad i = 1, 2, \dots, n. \tag{20}$$

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Practically important operating regimes of motion of machine units are realized on the decreasing portions of the working characteristics of engines. It is therefore natural to assume that

$$\tau^*_{(i)} < \tau_*^{(i+1)}, \qquad i = 1, 2, \ldots, n-1. \tag{21}$$

The strips (20) are then separated. In the case of a stable (unstable) branch $T = \tau_i(\varphi)$ of the inertial curve, we shall have

$$M(\varphi, \tau_*^{(i)}) \geq 0 \ (\leq 0), \qquad M(\varphi, \tau^*_{(i)}) \leq 0 \ (\geq 0), \qquad -\infty < \varphi < +\infty, \tag{22}$$

and the strip (20) with number i will be a strip of stability (instability). By virtue of the corollary of Theorem 2, in each of them there exist absolutely continuable integrals of the kinetic energy.

Theorem 3. *Suppose that the reduced moment $M(\varphi, T)$ of all the acting forces satisfies conditions 1°, and that its slope in the strip of stability (instability) is bounded below and above by negative (positive) constants*

$$-\lambda_2^{(i)} \leq M'_T(\varphi, T) \leq -\lambda_1^{(i)} < 0 \qquad (0 < \mu_1^{(i)} \leq M'_T(\varphi, T) \leq \mu_2^{(i)}). \tag{23}$$

Then in it there exists, moreover uniquely, an absolutely continuable integral $T = T^{(i)}(\varphi)$ of the kinetic energy; the latter is an asymptotically stable (unstable) limiting regime of motion of the machine unit.

The most important for practice are the asymptotically stable absolutely continuable limiting regimes. As for unstable regimes, they express bifurcation states of the dynamic equilibrium of mechanical systems: the slightest deviation from any one of them leads either to the transition of the unit to one of the stable limiting regimes of motion, or to the complete damping of its kinetic energy and stopping.

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Note: Figure translations are in progress. See original paper for figures.

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