THE CONSTANT OF STEPWISE IONIZATION OF ATOMS

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Abstract

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THE CONSTANT OF STEPWISE IONIZATION OF ATOMS

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1. Stepwise ionization of atoms in a discharge tube occurs when the time for an excited atom to escape to the walls of the tube is much longer than the time for its destruction under the action of electron impact, i.e., stepwise ionization is realized for sufficiently large transverse dimensions of the discharge tube and high electron densities. In the process of stepwise ionization, an excited atom passes through a whole sequence of excited states, with transitions between these states caused by collisions of the atom with electrons. Under the conditions considered (large dimensions of the discharge tube), the ionization constant does not depend on the dimensions of the tube, and since the velocity distribution function of the free electrons is Maxwellian (high electron density), this quantity depends only on the electron temperature, the electron density, and the properties of the atoms being ionized. The aim of the present work is to find these dependences.

2. Let us establish the relation between the constants of stepwise ionization in two different atomic gases. In doing so we shall assume that the electron temperature is much smaller than the excitation energy of the atoms. The density of free electrons produced as a result of stepwise ionization per unit time is equal to

$$\frac{dN_e}{dt} = N_e \sum_n k_n N_n,$$

where N_e is the electron density; N_n is the density of atoms in state n; k_n is the constant of direct ionization of these atoms. The stepwise-ionization constant k_{ion} is introduced on the basis of the relation $dN_e/dt = k_{\text{ion}} N_{aN} e$, where N_a is the density of atoms in the ground state. Hence

$$k_{\text{ion}} = \sum_n k_n \frac{N_n}{N_a}. \tag{1}$$

In deriving the relation between the constants of stepwise ionization in different gases, we use the fact that the properties of a strongly excited atom do not depend on the kind of atom, since they are determined by the Coulomb interaction of the electron with the charge of the atomic core. Therefore the constants of direct ionization of an excited atom k_n are the same for atoms of any kind. In addition, the ratio of the densities of atoms $N_n^{(1)}/N_n^{(2)}$ in identical strongly excited states in two different gases, at identical electron densities and temperatures, does not depend on the number of this state. Denoting $N_n^{(1)}/N_n^{(2)} = s$ and taking into account that at low electron temperatures the quantity k_{ion} is determined by the destruction of strongly excited atoms, we obtain for the ratio of the constants of stepwise ionization of atoms of different kinds, at identical electron density and temperature:

$$k_{\text{ion}}^{(1)}/k_{\text{ion}}^{(2)} = sN_a^{(2)}/N_a^{(1)}.$$

Here the indices 1, 2 refer to atoms of the corresponding kind.

To determine the proportionality coefficient s, which does not depend on the electron density, let us consider the case of high electron density, when the collision frequencies of an electron with an excited atom significantly exceed the radiation frequencies of the excited atom. In this case the density of excited atoms is determined by the Boltzmann law

$$N_n = \frac{g_n}{g_a} N_a \exp\left(\frac{I_n - I}{T}\right),$$

where T is the electron temperature; $g_n = g_i g_e n^2$; g_i is the statistical weight of the ion; $g_e = 2$; g_a is the statistical weight of the atom in the ground state; I, I_n are the ionization potentials of atoms in the ground and excited states. Using this, we find the proportionality coefficient, which leads to the following relation between the stepwise-ionization constants for atoms of different kinds at the same temperature and electron density:

$$k_{\text{ion}}^{(1)} \frac{g_{1a}}{g_{1i}} \exp\left(\frac{I_1}{T}\right) = k_{\text{ion}}^{(2)} \frac{g_{2a}}{g_{2i}} \exp\left(\frac{I_2}{T}\right), \tag{2}$$

Here g_{1a}, g_{2a} and I_1, I_2 are the statistical weights and ionization potentials of the corresponding atom in the ground state; g_{1i}, g_{2i} are the statistical weights of the ion.

3. Let us consider in more detail the case of large electron density, when radiation of excited atoms plays no role and the density of excited atoms obeys the Boltzmann law: $N_n = N_a g_n/g_a \exp(-\varepsilon_n/T)$, where ε_n is the excitation energy of the corresponding level, $g_n = g_e g_i n^2$ is its statistical weight. In this case the ionization constant is

$$k_{\text{ion}} = \frac{1}{N_a} \sum_n k_n N_n = \frac{2g_i}{g_a} \int n^2 \, dn \, k_n \exp\left(-\frac{\varepsilon_n}{T}\right).$$

Since in inelastic collisions with an excited atom mainly energies of the order of the electron temperature are transferred, the main contribution to this integral is made by excited states with ionization energies of the order of the electron temperature, $me^4/\hbar^2 n^2 \sim T$. Taking this into account, we obtain

$$k_{\text{ion}} \sim \frac{g_i}{g_a} n^3 \langle v\sigma \rangle \exp\left(-\frac{I}{T}\right) \sim \frac{g_i}{g_a} \frac{me^{10}}{\hbar^3 T^3} \exp\left(-\frac{I}{T}\right),$$

where the characteristic velocity of electron collisions with an atom is $v \sim \sqrt{T/m}$, and the ionization cross section $\sigma \sim e^4/T^2$ of an excited atom with binding energy $me^4/\hbar^2 n^2 \sim T$ coincides, in order of magnitude, with the cross section for exchange of two free electrons with kinetic energies of the order of the electron temperature, by an energy of the order of the temperature T. Thus, in the present case, for the constant of stepwise ionization of an atom by electron impact we obtain

$$k_{\text{ion}} = A \, \frac{g_i}{g_a} \frac{me^{10}}{\hbar^3 T^3} \exp\left(-\frac{I}{T}\right), \tag{3}$$

where the coefficient A does not depend on the electron temperature or on the kind of atom.

Table 1

Plasma electron temperature, 10^3 K	Hydrogen	Hydrogen	Hydrogen	Hydrogen	Alkali metal	Alkali metal	Alkali metal	Alkali metal	Hydrogen-like	Hydrogen-like	Hydrogen-like	Hydrogen-like
Plasma electron temperature, 10^3 K	4	8	16	32	1	2	4	8	4	8	16	32
A	0.035	0.011	0.009	0.025	0.051	0.036	0.022	0.031	0.16	0.045	0.031	0.066

Table 1 gives the values of the coefficient A obtained by processing the calculations of Bates, Kingston, and McWhirter ([1]) for the stepwise-ionization constant. In doing so, the ionization constant calculated in that work in the limit of high electron densities was substituted into the right-hand side of formula (3), and from the resulting relation the coefficient A was found. According to the result obtained, the coefficient A should be the same in all cases. The dependence obtained is valid in the limit of low temperatures $T/I \ll 1$, when free electrons are formed as a result of ionization of highly excited atoms. The fact that processing the calculations does not lead to identical values of A should be attributed to a shortcoming of the calculations of Bates, Kingston, and McWhirter ([1]). Indeed, in that work, for the cross sections of inelastic transitions between atomic states under the action of electron impact, semiempirical Gryzinski formulas were used, whose accuracy is very limited. An error in the cross section accumulates in calculating the ionization constant, so that the error in the ionization constant may be larger than the error in the cross section of an inelastic transition—roughly speaking, by as many times as the number of transitions made by the valence electron before ionization.

4. Let us use the information that we can extract from the process of three-body recombination of electrons and ions in the limit when the electron density is high, so that radiation plays no role. The process under consideration proceeds according to the scheme $2e + X^+ \to e + X$ and is diametrically opposite to the process of stepwise ionization that interests us. According to studies of the mechanism of three-particle recombination ([2–6]), the recombining electron spends the main part of the recombination

time on levels with ionization energy of the order of the temperature. This agrees with our assertion, according to which stepwise ionization is mainly determined by the destruction of excited atoms with ionization potential of the order of the temperature.

In the case of equilibrium between the processes of three-body recombination and stepwise ionization, the relation between the constants of stepwise ionization k_{ion} and three-body recombination α_{rec} has the form

$$k_{\text{ion}} N_{eN} a = N_{eN} i \alpha_{\text{rec}},$$

where N_a is the density of atoms in the ground state and N_i is the density of ions. In this case the dependence of the recombination coefficient on the electron density and temperature has the form ([2–6])

$$\alpha_{\text{rec}} = b N_{ee}^{10} / \sqrt{m} T^{9/2}, \tag{4}$$

where b is a constant of proportionality. Using the Saha formula

$$\frac{N_{eN} i}{N_a} = \frac{g_{eg} i}{g_a} \left(\frac{mT}{2\pi\hbar^2} \right)^{3/2} \exp\left(-\frac{I}{T} \right),$$

we again obtain for the ionization constant formula (3):

$$k_{\text{ion}} = A \frac{g_i}{g_a} \frac{me^{10}}{\hbar^3 T^3} \exp\left(-\frac{I}{T} \right), \qquad A = \frac{2b}{(2\pi)^{3/2}} \; (g_e = 2). \tag{5}$$

The values of the coefficient A obtained from processing the results of the calculations of Bates, Kingston, and McWhirter for the recombination constant are given in Table 2.

Table 3 gives the values of the coefficient in formula (4) that were found in works ([3–6]). In the work of Hinnov and Hirschberg ([3]), in deriving formula (4), Thomson' s theory was used for the cross section of excitation of an atom by electron impact; in the work of Makin and Keck ([4]), this formula was obtained on the basis of the variational method. In the work of Gurevich and Pitaev–

···[in the work] of Gurevich and Pitaevskii ([5]), the triple recombination of an electron and a multiply charged ion with large charge $Z \gg 1$ was investigated. In this case formula (4) contains one more factor Λ, which for $Z \gg 1$ is equal to $\Lambda = \ln Z$. For $Z \sim 1$

Table 2

Electron tem- pera- ture, 1000° K	0.25	0.5	1.0	2.0	2.0	4
b	4.1	2.6	2.4	19	0.97*	0.8**
A	0.52	0.33	0.3	0.24	0.12	0.1

* Hydrogen plasma. ** Alkali-metal plasma.

the quantity $\Lambda \sim 1$. Finally, in the work of D' Angelo ([6]), formula (4) was obtained on the basis of Thomson' s theory for triple capture. As can be seen, the values found in these works for the proportionality coefficient b agree well with one another, especially if one takes into account the crudeness of the models and approximations used in these works. The value of the coefficient A in Table 3 was found on the basis of formula (5). As can be seen, the results presented in Table 3 contradict the data of Bates, Kingston, and McWhirter ([1]) (Tables 1 and 2). This casts doubt on the reliability of the indicated calculations, which is connected with their use of Gryzinski' s formulas for transition cross sections.

Table 3

Lit. source	([3])	([4])	([5])	([6])
b	3.1	5.9	3.2Λ	4.2
A	0.38	0.73	0.39Λ	0.52

5. In conclusion, let us compare the constants of stepwise and direct ionization in the case of low temperatures and high electron densities. The ionization cross section of an atom by electron impact near the threshold is practically determined by a linear dependence on the difference between the electron energy and the ionization potential: $\sigma_{\text{ion}} = \sigma_0(\varepsilon/I - 1)$, where σ_0 is of atomic order of magnitude. From this we find, for the direct-ionization constant averaged over the Maxwellian electron distribution:

$$k_{\text{direct}} \approx \sqrt{8T/\pi m}\,\sigma_0 \exp(-I/T).$$

Comparing this with the stepwise-ionization constant (5), we find

$$k_{\text{direct}}/k_{\text{step}} \sim (\hbar^2 T/me^4)^{7/2} \ll 1.$$

Thus, at high electron densities and low temperatures, the stepwise mechanism of free-electron formation is realized. We note that the electron distribution function in the tail usually decreases much more steeply than the Maxwellian one, i.e., the direct-ionization constant is even smaller than we have obtained.

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Note: Figure translations are in progress. See original paper for figures.