THEORY OF ELEMENTARY PARTICLES AND A NONINVARIANT VACUUM

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THEORY OF ELEMENTARY PARTICLES AND A NONINVARIANT VACUUM

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Recently Coleman (^{1}) proved a theorem stating that invariance of the vacuum is invariance of the world. The latter means that if the vacuum $|0\rangle$ (which by definition is a translationally invariant state) has the property

$$Q|0\rangle = \int j_0(\mathbf{x}, t)\, d^3x\, |0\rangle = 0, \tag{1}$$

then there exists a current-conservation law leading to conservation of the charge Q in time, i.e., the charge commutes with the Hamiltonian. As the charges Q one may take the generators of any internal symmetry group in the theory of elementary particles (for example, the groups $SU(2)$ or $SU(3)$). Important consequences follow from Coleman' s theorem concerning the properties of the vacuum when various interactions are considered in quantum field theory. Thus, for example, a theory with an interaction noninvariant with respect to some group of internal transformations and with an invariant vacuum is impossible. Consequently, in the theory of weak interactions the vacuum cannot be a state with zero strangeness (invariant with respect to gauge transformations leading to conservation of strangeness); in the theory of electromagnetic and weak interactions the vacuum cannot be an isotopically invariant state. Coleman' s theorem has been proved only for internal transformations whose generators are certain charges; for discrete transformations (C, P, CP, T, CPT) the theorem, generally speaking, does not apply.

The question arises: what does noninvariance of the vacuum mean? In fact, from the point of view of the usual formalism of quantum field theory, which uses the apparatus of Fock quantization, writing the charges in normal form automatically leads to invariance of the vacuum with respect to any internal transformations, and the assertion of its noninvariance is somewhat absurd (in Segal' s terminology (^{2}), the Fock vacuum is a "universally invariant" state).

In the present work a mathematical construction is proposed for a field theory with a vacuum noninvariant with respect to groups of internal transformations. The problem of different definitions of the vacuum in quantum field theory is essentially connected with the existence of representations of the canonical commutation relations for systems with an infinite number of degrees of freedom that are unitarily inequivalent to the Fock one (^{3}). We shall write the commutation relations for Bose fields in Weyl form, i.e., for $w(z)$ (where $z \in \Omega$ is a certain complex test function) one has

$$\begin{aligned} &w(z)w(z') = \exp \tfrac{1}{2} \operatorname{Im}(z, z')\, w(z+z'), \\ &w(f) = u(f) = \exp i\varphi(f), \qquad f \in \Omega_R, \\ &w(ig) = v(g) = \exp i\pi(g), \qquad g \in \Omega_R. \end{aligned} \tag{2}$$

Here $\Omega = \Omega_R \oplus i\Omega_R$, and

$$\varphi(f) = \int \varphi_s^i(\mathbf{x}, t) f_s^i(\mathbf{x})\, d\mathbf{x}, \tag{3}$$

where i, s are isotopic and spin indices.

Further, following Segal (^{2}), let us define the generating functional of the representation $E(w(z)) = E_w(z)$ as a certain complex-valued function E_w on Ω, possessing the properties

$$E(w^*(z)w(z)) \geqslant 0, \qquad E_w(0) = 1. \tag{4}$$

By the well-known theorem on representations of normed rings (^{4}), the functional $E(w(z))$ determines, up to unitary equivalence, a representation of the C^*-algebra W (the so-called Weyl algebra) associated with the Weyl system, i.e.,

$$E_w(z) = \langle w_E(z), v_E \rangle, \qquad w_E(z) \in W, \tag{5}$$

where v_E is the cyclic vector of the representation, and $w_E(z)$ is the operator realizing the representation.

Knowing the concrete form of $E_w(z)$, one can always construct, by means of the well-known Gelfand–Naimark–Segal (G.N.S.) construction (^{4}), a representation of the Weyl algebra W.

On the other hand, considering the functional $E_w(z)$ only on Ω_R and assuming Ω_R to be endowed with the topology of a nuclear space, it is not difficult to notice (^{11}) that $E_w(f)$ is the Fourier transform of a measure μ_E of sets in the rigged Hilbert space $\Omega_{RH} \subset \Omega_R \subset \Omega^*_{RH}$. We shall be interested in representations with a vacuum. In the present theory, the vacuum is defined as a translationally invariant vector. Such a vector exists in the Hilbert space of the representation of relations (2) if and only if the measure μ_E is equivalent to some Euclidean-invariant measure (^{5}). The vacuum is unique if the measure μ_E is ergodic with respect to Euclidean transformations (5). Finally, the measure μ_E is of essential importance for establishing the equivalence of representations of commutation relations: representations of relations (4) invariant with respect to time inversion are unitarily equivalent if and only if the corresponding measures μ_{E_1} and μ_{E_2} are equivalent (5,6).

Let there be some group of automorphisms of the Weyl algebra W. We shall call a representation W invariant with respect to this group of automorphisms if there exists a representation of the group by unitary operators in the Hilbert space of the representation W. There is a theorem (7) stating that a representation W with a vacuum invariant with respect to transformations of some group of automorphisms is invariant with respect to it. Therefore it is clear that if we construct examples of noninvariant representations with a vacuum, then these will also be examples of a field theory with a noninvariant vacuum.

Let us first formulate a theorem giving a condition necessary and sufficient for the invariance of a representation.

Theorem. *In order that an irreducible representation of the Weyl algebra be invariant with respect to some group of automorphisms, it is sufficient that the null-space of the generating functional be mapped into itself under these automorphisms. This requirement is also necessary if the vacuum $|\Phi_0\rangle$ has the property $U(g)|\Phi_0\rangle = \exp i\omega(g)|\Phi_0\rangle$, where $U(g)$ is a representation of the group, which is always true for groups commuting with translations when the vacuum is unique.*

The proof of sufficiency is based on the well-known remark (8) on the connection between the invariance of the null-space and the similarity operation, and on the boundedness of the operator $U(g)$, which follows from certain theorems in the theory of representations of C^*-algebras cited in (7).

Necessity is obvious.

To construct noninvariant representations it is sufficient to take a generating functional such that, under transformations of the group, it changes its form, while the corresponding measures μ_E and μ_{Fg} must be inequivalent.

Let us give physical examples of such noninvariant representations.

1. **Violation of isotopic invariance.** $E_w(z)$, where z are multicomponent functions realizing a representation of $SU(2)$, has the form

$$E_w(z) = \exp -\|^{1/4}(1+\tau_3)z\|^2, \tag{6}$$

$$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

for isospinors, and

$$\tau_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

for isovectors. The representation fixed by (6) is inequivalent to the Fock representation, when

$$E_w^F(z) = \exp -^{1/4}\|z\|^2,$$

by the well-known theorem on the equivalence of Gaussian measures in function space (^{3}). Indeed, under transformations of the group of isotopic rotations $SU(2)$,

$$E_w(z) \to E_w\big(\exp {}^{1/2} i\tau_2 \alpha z\big)$$

or

$$E_w(z) \to E_w\big(\exp {}^{1/2} i\tau_1 \alpha z\big)\,,$$

and the inequivalence of the corresponding measures proves the inequivalence of the representations. The vacuum of the theory (6) is constructed as the equivalence class of unity with respect to the measure corresponding to (6), and this state is no longer isotopically invariant.

2. **Violation of $SU(3)$.**

$$E_w(z) = \exp -^{1/4}\|\bar{z}^\alpha_\beta z^\beta_\alpha + a\,[\bar{z}^\alpha_\beta z^\beta_\gamma (\lambda_8)^\gamma_\alpha + z^\beta_\gamma z^\alpha_\beta (\lambda_8)^\gamma_\alpha]\|, \tag{7}$$

where z are multicomponent functions transformed according to the octet representation of $SU(3)$. The noninvariance of the vacuum is proved in the same way as in case (1). The representation (7) resembles in form the representations studied by Araki and Woods (^{11}) in the theory of a Bose gas. These representations are reducible.

In view of the insufficient clarity of the physical meaning of the reducibility of representations of the canonical commutation relations, let us consider another representation, also possessing the property that $SU(3)$ is broken in accordance with the Gell-Mann–Okubo formula:

$$\begin{aligned} E_w(z) = \exp -^{1/4}\|\bar{z}^\alpha_\beta z^\beta_\alpha &+ a\,[\bar{z}^\alpha_\beta z^\beta_\gamma (\lambda_8)^\gamma_\alpha + z^\beta_\gamma z^\alpha_\beta (\lambda_8)^\gamma_\alpha] \\ &+ a\{[\operatorname{Im}\bar{z}^\alpha_\beta \operatorname{Im} z^\beta_\gamma (\lambda_8)^\gamma_\alpha + \operatorname{Im}\bar{z}^\beta_\gamma \operatorname{Im} z^\alpha_\beta (\lambda_8)^\gamma_\alpha] \\ &- [\operatorname{Im}\bar{z}^\alpha_\beta \operatorname{Im} z^\beta_\gamma (\lambda_3^{-1})^\gamma_\alpha + \operatorname{Im}\bar{z}^\beta_\gamma \operatorname{Im} z^\alpha_\beta (\lambda_8^{-1})^\gamma_\alpha]\}\|. \end{aligned} \tag{8}$$

In this case, annihilation and creation operators of particles may be introduced in the representation space by means of the well-known Bogoliubov transformation:

$$\alpha(f) = \frac{(1+a\lambda_8)^2+1}{2(1+a\lambda_8)}\, a(f) + \frac{(1+a\lambda_8)^2-1}{2(1+a\lambda_8)}\, a^+(f). \tag{9}$$

3. **Violation of strangeness.** As the corresponding representation one may take a representation characterized by a Gaussian measure with a nonzero first moment $m(f) = \langle \Phi_0 | \varphi(f) | \Phi_0 \rangle$, or take

$$E^s_w(z) = \exp -^{1/4} \| \bar{z} z + ia(\bar{z}\bar{z} - zz) \|.$$

These representations are noninvariant with respect to the transformations $z \to z \exp i\alpha$. The representation $E^s_w(z)$ is also noninvariant with respect to a CP-transformation, which corresponds to the conception of a CP-noninvariant vacuum expressed by us earlier (^{10}).

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References

[1] S. Coleman, J. Math. Phys., **7**, 787 (1966).
[2] I. Segal, *Mathematical Problems of Relativistic Physics*, Moscow, 1968.
[3] R. Haag, Lectures in Theor. Phys., **3**, 1960, p. 353.
[4] M. A. Naimark, *Normed Rings*, Moscow, 1956.
[5] H. Fukutome, Progr. Theor. Phys., **23**, 989 (1960).
[6] H. Araki, J. Math. Phys., **1**, 492 (1960).
[7] I. Montvay, Nuovo Cim., **40**, 121 (1965).
[8] A. N. Vasil' ev, ZhETF, **49**, issue 10 (1965).
[9] E. Shilov, Fandyk Thin, *Integral, Measure and Derivative in Linear Spaces*, "Nauka," 1967.
[10] A. A. Grib, Vestn. Leningrad. Univ., **22**, 50 (1967).
[11] H. Araki, E. J. Woods, J. Math. Phys., **4**, 637 (1963).