OBSERVABLE EFFECTS IN RELATIVISTIC TENSOR THEORIES OF GRAVITATION IN FLAT SPACE

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Abstract

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OBSERVABLE EFFECTS IN RELATIVISTIC TENSOR THEORIES OF GRAVITATION IN FLAT SPACE

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1. The purpose of the present note is to show that theories of gravitation in which the equivalence principle is violated in one way or another, and above all relativistic tensor theories of gravitation in flat space (r.t.t.f.s.)*, can be reconciled with the already available experimental data only if, in these theories, instead of the Newtonian gravitational potential one uses a gravitational potential depending on distance as $r^{-1}\exp(rR^{-1})$**, with a range R not exceeding 10^9 light years, which amounts to 1/10 of the radius of the visible part of the Universe***.

Since we are interested above all in estimate-type conclusions concerning the difference between the gravitational potential of r.t.t.f.s. and the Newtonian one, it is sufficient to carry out the calculations in any one of the r.t.t.f.s., for example, in the linear tensor theory of gravitation (l.t.t.).

2. In l.t.t. we shall consider the action functional of a system of N material points interacting through gravitational and electromagnetic fields in the form:

$$\begin{gathered} S=(32\pi G)^{-1}c^3(1+3a)(1+4a)^{-1}\int\left(\Psi_{ij,k}\Psi^{ij,k}+a\Psi_{,k}\Psi^{,k}\right)d^4x+ \\ +\sum_{\nu=1}^{N}\left[-m_\nu c\int\left(1+\frac{1}{2}\Psi_{ij}u_\nu^i u_\nu^j\right)ds_\nu+\sum_{\nu=1}^{N}\left(-e_\nu c^{-1}\int A_i\,dx_\nu^i\right)-\right. \qquad (1) \\ \left.-(16\pi c)^{-1}\int F_{ik}F_{lm}\eta^{il}\left[\eta^{km}\left(1+\frac{1}{2}\Psi\right)-2\Psi^{km}\right]d^4x,\right. \end{gathered}$$

where $\Psi_{ij}(x^0,x^1,x^2,x^3)$ denotes the gravitational tensor potential (Newtonian in the case of a static spherically symmetric field, since in the first term of expression (1) we have for the time being omitted the terms ensuring a Neumann form of the potential); A_i is the electromagnetic potential; $F_{ik}=A_{k,i}-A_{i,k}$;

Latin indices take the values $0, 1, 2, 3$; lowering and raising of indices is carried out with the aid of the Minkowski tensor: $\eta_{ij} = 2\delta_{0i}\delta_{0j} - \delta_{ij}$; $\Psi = \eta^{ij}\Psi_{ij}$; $ds = \sqrt{\eta_{ij}dx^i dx^j}$. The coefficient a is an undetermined parameter of the theory, and for its value one must also find restrictions following from experimental data; the factor $(1+3a)(1+4a)^{-1}$ in (1) has been chosen so that the constant G has the meaning of the Newtonian gravitational constant in regions sufficiently far from gravitating sources (with the exception of the N material points under consideration); in these regions the quantities c, e_ν have, obviously, the meaning of the velocity of light and of charges. Expression (1) differs from the expression proposed by Capella ([6]) only in notation and by the addition of the third term.

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* By relativistic tensor theories of gravitation in flat space we mean the Birkhoff-Moshinsky theory ([1],[2]) and all its subsequent linear and nonlinear modifications ([3–6]).

** A gravitational potential of this form was first proposed by Neumann ([7]) in connection with an attempt to construct cosmology within the framework of Newtonian mechanics.

*** This circumstance justifies the neglect of the gravitational field of the Universe that was allowed in work ([8]). Therefore the formulas given there describe not only qualitatively but also quantitatively an effect accessible to experimental verification, following from r.t.t.f.s. (an anomalous red shift).

As is known [6], the first two terms in (1) lead, for the field of masses at rest, to a gravitational potential of the form $\Psi_{0\alpha} = 0$, $\Psi_{\alpha\beta} = b\Psi_{00}\delta_{\alpha\beta}$, where $b = a(1+3a)^{-1}$; $\alpha, \beta = 1, 2, 3$; Ψ_{00} is some function of the coordinates. If, in addition to the gravitational field of the N material points under consideration, there is also the gravitational field of some other mass M at rest, then the potential Ψ_{ij} will be the sum of the potential $\hat{\Psi}_{ij}$ of this mass and the potential Ψ'_{ij} produced by the N material points under consideration.

Let us make in (1) the substitution: $c' = (\varepsilon\mu)^{-1/2}c$, $G' = K^{-2}G$, $e'_\nu = K^{-1}\varepsilon^{-1/2}e_\nu$, $\Psi'_{ij} = K^{-2}\varepsilon\mu\Psi_{ij}$, $A'_0 = K^{-1}\varepsilon^{1/2}A_0$, $A' = K^{-1}\mu^{-1/2}A$, $x'^0 = (\varepsilon\mu)^{-1/2}x^0$, $x'^\alpha = x^\alpha$, $S' = K^{-2}S$, where $\varepsilon = 1 - \frac{1}{2}\hat{\Psi}_{00} - \frac{1}{2}\hat{\Psi}_{11}$, $\mu = (1 + \frac{1}{2}\hat{\Psi}_{00} + \frac{1}{2}\hat{\Psi}_{11})^{-1}$, $K = (1 - \frac{1}{2}\hat{\Psi}_{00} - \hat{\Psi}_{11})^{1/2}$.

It is easy to verify that, in this case, S' is expressed in terms of x'^0, x'^α, c', G', e'_ν, Ψ'_{ij}, A'_i in the same way as S is expressed in terms of x^0, x^α, $c, \ldots, A_i$, i.e., by formula (1), with the only difference that in the round brackets in the second term there appear additional terms which, in the case where $\frac{v^2}{c^2}\hat{\Psi}_{00} \ll 1$, have the order of smallness $\frac{v^4}{c^4}\hat{\Psi}_{00}^2$ and $\frac{v^2}{c^2}\Psi'_{00}\hat{\Psi}_{00}$; moreover, in the first and last terms there appear additional terms (which we shall not write out) giving a contribution (of order $\hat{\Psi}_{00}$) to quantities connected with the dependence of the gravitational field on time, and also to quantities connected with the interaction of the gravitational and electromagnetic fields:

$$S' = (32\pi G')^{-1}{c'}^3(1+3a)(1+4a)^{-1}\int\left(\Psi'_{ij,k'}\Psi'^{ij,k'} + a\Psi'_{,k'}\Psi'^{,k'} + ...\right)d^4x'$$
$$+\sum_{\nu=1}^{N}\int dt\left\{-\frac{1}{2}m_\nu {c'}^2\Psi'_{00} + \frac{1}{2}m_\nu v_\nu^2 + \frac{1}{8}m_\nu v_\nu^4 {c'}^{-2}(1+\varphi_1) - \frac{1}{4}m_\nu v_\nu^2(\Psi'_{00}\right.$$
$$\left. +2\Psi'_{11})(1+\varphi_2) + ...\right\} + \sum_{\nu=1}^{N}\left(-e'_\nu {c'}^{-1}\int A'_i\,dx^i_\nu\right)$$
$$-(16\pi c')^{-1}\int F'_{ik}F'_{lm}\left\{\eta^{il}\left[\eta^{km}(1+\frac{1}{2}\Psi') - 2\Psi'^{km}\right] + ...\right\}d^4x', \quad (2)$$

where the notation is $t = x^0/c = x'^0/c'$, $F'_{ik} = A'_{k,i'} - A'_{i,k'}$; $\varphi_1 = (\varepsilon\mu)^{-1}K^{-2}(1-\frac{3}{2}\hat{\Psi}_{00} - 2\hat{\Psi}_{11}) - 1 = -(1+b)(1+\frac{3}{2}b)\hat{\Psi}_{00}^2 + ...$; $\varphi_2 = (\varepsilon\mu)^{-1} - 1 = (1+b)\hat{\Psi}_{00} + ...$

The ellipses in the braces of the second term of expression (2) denote quantities of higher orders of smallness in v^2/c^2 or in $\hat{\Psi}_{00}$ than $m\frac{v^4}{c^2}\hat{\Psi}_{00}^2$ and $mv^2\Psi'_{00}\hat{\Psi}_{00}$.

It follows from equality (2) that if an observer carries out measurements on the indicated N material points at nonrelativistic velocities and with an accuracy lower than v^2/c^2, then he will not detect any deviations from Newtonian mechanics and Maxwellian electrodynamics. The observer will not notice the presence of $\hat{\Psi}_{ij}$ different from zero, since he will not know that the quantities c', G', e'_ν, which he will take to be the speed of light, the gravitational constant, and the charges, differ from the constants c, G, e_ν appearing in (1). But it turns out that in measurements in the relativistic domain (we also include among these measurements the shifts of planetary perihelia, the deflection of light rays by the Sun, and the radiation of gravitational waves) the observer will immediately be convinced of the presence of the quantities Ψ_{ij}. For example, for the ratio of the maximum possible velocity V' of a material point to the speed of light c', the formula will hold:

$$V'/c' = 1 - \frac{1}{2}(1+b)\hat{\Psi}_{00} + ... \quad (3)$$

The fact that the quantities t, ${x'}^\alpha$, v_ν, m_ν, e'_ν, c', G', Ψ'_{ij}, A'_i, F'_{ik} denote intervals of time, distances, velocities, masses, charges, etc., measured by an observer (situated near M) according to the usual rules for measuring quantities (not taking gravitation into account), follows from the similarity (within the limits mentioned above) of the action functionals S and S', and consequently from the similarity of the systems of equations of motion for material points and fields determined by them. In particular, the quantities m_ν and e'_ν appearing in (2) can be determined by an observer (near M) from the usual formulas after measuring the velocities and accelerations of the material points under consideration in their collisions; but formula (2) contains no indications as to precisely what the observer chooses as the units of length, time, and mass. However, this indefiniteness in the choice of scales does not introduce uncertainty into the physical meaning of the dimensionless quantities and ratios obtained from (2). Examples of such ratios are given by formula (3), as well as by the formulas presented in Sec. 3. In order to use formula (3) for experimental measurement (or estimation from above) of the quantity Ψ_{00}, one must make an ultrarelativistic particle and a light ray move along closed trajectories (respectively, in an accelerator and in a scheme with a Kerr cell or with a rotating mirror), and then, using the same standards of length and time, measure both trajectories and both periods of motion.*

The fact that no deviation of the quantity V'/c' from unity has been detected indicates that, within the framework of r.t.t., for the gravitational potential $\hat{\Psi}_{00}$ of the Universe near the Earth the strong inequality $|\hat{\Psi}_{00}| \ll 1$ must hold.** It follows from this, in particular, that in r.t.t. only the values $b = 3/4,\ a = -2/5$, indicated by Capella [(6)], can be reconciled with the data on the shift of Mercury' s perihelion and the deflection of rays by the Sun.

3. Let us write the exact expressions for the kinetic energy $\mathcal{E}'$ and momentum $\mathbf{p}'$ of a material point that follow from (1), (2):

$$\mathcal{E}' = K^{-2} m V'^2 \left[\left(1 + \hat{\Psi}_{00} + \tfrac{1}{2}\hat{\Psi}_{11}\right) \left(1 - \frac{v^2}{V'^2}\right)^{-1/2} - \tfrac{1}{2}\left(\hat{\Psi}_{00} + \hat{\Psi}_{11}\right)\left(1 - \frac{v^2}{V'^2}\right)^{-3/2} \right]; \tag{5}$$

$$\mathbf{p}' = K^{-2} m \mathbf{v} \left[\left(1 - \tfrac{1}{2}\hat{\Psi}_{11}\right) \left(1 - \frac{v^2}{V'^2}\right)^{-1/2} - \tfrac{1}{2}\left(\hat{\Psi}_{00} + \hat{\Psi}_{11}\right)\left(1 - \frac{v^2}{V'^2}\right)^{-3/2} \right] ***. \tag{6}$$

Formula (5) is conveniently rewritten in the form

$$lv/c' = \left[1 - \left(mc'^2/\mathcal{E}'\right)^2 f(y)\right]^{1/2}.$$

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* In practice, such a formulation of the question is made difficult by the fact that in r.t.t. the concept of an ultrarelativistic particle becomes more complicated in the presence of a gravitational field: not only the parameter $(1 - v^2/V^2)^{-1}$ is significant, but also the parameter $\hat{\Psi}_{00}(1 - v^2/V^2)^{-1}$; a quantitative treatment will be undertaken in Sec. 3.

** Let us note that, according to (1), for the field of static masses $\Psi_{00} < 0$ always. In the arguments given, substantiating inequality (4), for r.t.t.p. the expansion of the Universe was not taken into account, as a result of which its tensor gravitational potential $\hat{\Psi}_{ij}$ differs from the potential of any static mass. However, it is not difficult to show that taking this circumstance into account would only widen the range of observable effects to which r.t.t.p. would lead in the event of a violation of condition (4). For an analogous reason we exclude from consideration the variant $\hat{\Psi}_{00} \sim 1$, $|1 + b| \ll 1$, although it too would ensure fulfillment of the condition $|V'/c' - 1| \ll 1$.

*** For the quantities $\mathcal{E}'$, $\mathbf{p}'$ the usual conservation laws hold, for example:

$$\sum_{\nu=1}^{N} \mathcal{E}'_\nu + (8\pi)^{-1} \int (\mathbf{E}'^2 + \mathbf{H}'^2)\, dr = \text{const},$$

where, as usual, $E'_\alpha = F'_{0\alpha}$, $H'_1 = F'_{23}, \ldots$. It also follows from (5) and (6) that the ratio $\mathcal{E}v/pc'^2 = 1$ holds in r.t.t. (to within corrections of order $\hat{\Psi}_{00}$) for all $v^2/{V'}^2$, which makes it possible to measure the energy from the momentum and velocity.

where $y = (mc'^2\mathcal{E}'^{-1})^2(-{}^1/_2\hat{\Psi}_{00} - {}^1/_2\hat{\Psi}_{11})^{-1}$, and before $\mathcal{E}'$ factors differing from 1 by quantities of order $\hat{\Psi}_{00}$ have everywhere been omitted; l denotes the distance traversed by the particle in the accelerator in one period; ν is the frequency of its revolution (coinciding with the frequency of the accelerating field);

$$f(y) = 2y^{-1}\{-1+{}^1/_4({}^1/_2 y)^{1/3}[(\sqrt{1+{}^4/_{27}y}+1)^{2/3}+(\sqrt{1+{}^4/_{27}y}-1)^{2/3}+{}^2/_3({}^1/_2 y)^{1/3}]^2\}.$$

The quantity l may be regarded as measured with an accuracy no worse than the ratio Δ of the half-width of the cross section of the vacuum chamber to the mean radius of the trajectory; this ratio is minimal for the Serpukhov accelerator (see the summary of data in ([9])) and is $\Delta = {}^1/_{2360}$. The frequency ν, obviously, is measured with much greater accuracy. Since no deviations from the usual formula $e\nu/c' = [1 - (mc'^2/\mathcal{E}')^2]^{1/2}$ have been observed, this indicates that in a r.t.t.f.s. the restriction $|f(y) - 1| < 2(\mathcal{E}'/mc'^2)\Delta$ must be satisfied; in this connection errors in the measurement of $\mathcal{E}'$ are immaterial, provided only that they are at least an order of magnitude smaller than $\mathcal{E}'$. The quantity $\mathcal{E}'$ may be obtained, for example, from the formula: $\mathcal{E}' = e'c'H'/2\pi\nu$, where H' is the mean magnetic-field strength along the trajectory; the applicability of this formula to a r.t.t.f.s. is easily verified from (1) and (2), taking (4) into account. Thus, on the basis of the data from the Serpukhov accelerator one may conclude: $|f(y)| < 3.1$, i.e. $y > 0.29$, hence:

$$|\hat{\Psi}_{00}| < {}^1/_{1600}; \tag{7}$$

here we take $b = {}^3/_4$, $\mathcal{E}'/mc'^2 = 70$.

It is known that the Newtonian gravitational potential of the visible part of the Universe is, in order of magnitude, close to unity; therefore inequality (7) immediately leads us to the conclusion formulated in Sec. 1 on the inevitability, in a r.t.t.f.s., of a Neumann gravitational potential with range R, at least by an order of magnitude smaller than the dimensions of the visible part of the Universe. This latter circumstance permits, in a quantitative estimate of R, neglect of the second derivatives in the equation for the Neumann potential:

$$\Delta\hat{\Psi}_{00} - R^{-2}\hat{\Psi}_{00} = K^2(\varepsilon\mu)^{-1} \cdot 8\pi G' {c'}^{-2} \rho*,$$

and consequently, taking (4) into account:

$$R = \sqrt{-\hat{\Psi}_{00}}\, c'(8\pi G'\rho)^{-1/2}.$$

Here ρ denotes the mean density of matter in the visible part of the Universe: $\rho = 3 \cdot 10^{-31}$ g/cm^3 (see ([10])); thus we obtain $R = \sqrt{-\hat{\Psi}_{00}} \cdot 0.47 \cdot 10^{24}$ km $= \sqrt{-\hat{\Psi}_{00}} \cdot 4.9 \cdot 10^{10}$ light years, or, taking (7) into account: $R < 1.2 \cdot 10^9$ light years.

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* This equation is obtained if the term $R^{-2}(\Psi_{ij}\Psi^{ij} + \alpha\Psi^2)$ is added to the integrand of the first term of equality (1); if one adds an expression bilinear

in Ψ_{ij} of general form, then this would lead to a potential of a point mass in the form of a superposition of two Neumann potentials with different R, which would not substantially affect the quantitative estimate for R.

Note: Figure translations are in progress. See original paper for figures.