CONSTRUCTION OF UNIAXIAL REGULAR GEAR MECHANISMS BY MEANS OF COMBINATORIAL DIAGRAMS

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CONSTRUCTION OF UNIAXIAL REGULAR GEAR MECHANISMS BY MEANS OF COMBINATORIAL DIAGRAMS

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In the synthesis of a uniaxial regular gear mechanism (u.r.g.m.) ([1–6]) according to the system of connections that it must reproduce, one can determine all sets of three-link differential mechanisms corresponding to this system ([2]). To construct, from each such set, the desired u.r.g.m., all links having the same designation in the different three-link mechanisms of the set must be combined into a single link. However, other links of the mechanism may obstruct such a combination. In many cases the problem of determining all essentially different possibilities for arranging the mechanisms of the set in space, if it is solved by attempting to draw the mechanism diagrams directly, is very laborious. The solution of this problem is considerably simplified by introducing a special symbolic representation of mechanisms ([1,2]), but even when this representation is used one often has to deal with the necessity of investigating and drawing a very large number of different variants.

The present note sets forth a method by which the determination of all possible variants of arranging a u.r.g.m. in space is reduced to the consideration of specially constructed combinatorial diagrams. In this case the problem is completely formalized, which makes it possible to use an electronic digital computer for its solution. The formalization also guarantees that not one of the possible arrangements of the u.r.g.m. in space will be missed.

We shall call the **output** pq of a central ([2]) link p of a u.r.g.m. to its other central link q: a) if link p is meshed with the satellites of link q or if the satellites of link p are meshed with link q, the set of points on the surfaces of the teeth of link p or of its satellites which come into contact with the teeth of link q or of its satellites; b) if link p is equipped with a clutch whose engagement connects it with link q, the set of points of link p that are in contact with link q when the clutch is engaged.

The set of points of a working ([2]) link p of a u.r.g.m. to which, by the operating conditions of the mechanism, access from outside must be provided—for example, the place of contact of this link with a link not belonging to the mechanism

Fig. 1

Figure 1: Fig. 1

and supplying to link p a torque from outside—will, in accordance with (^{2}), be assigned to the outputs $p\gamma$ of link p to the fixed link γ. The symbol γp has an analogous meaning in this case.

To solve the question of the possibility of arranging the mechanism in space, the list of outputs of all central links to one another must be known.

Suppose, for example, that it is required to construct a gearbox that is a u.r.g.m. with three degrees of freedom (^{1,2,4}), having the outputs shown in Fig. 1. The gearbox contains 7 central links—the fixed link γ and the moving links $0, \infty, 1, 2, 3,$ and 4. The driving link 0 and the driven link ∞ are connected with moving links that supply torques to the mechanism from outside (Figs. 1a and 1b). The gearbox contains three differential three-link mechanisms (Figs. 1d, 1e, and 1zh) and four clutches (Figs. 1v, 1g, 1z, 1i), two of which (Figs. 1v and 1g) are brakes, i.e., when engaged they connect the moving central links with the fixed link—

Then the link γ here has outputs $\gamma 0$ (Fig. 1a), $\gamma\infty$ (Fig. 1b), $\gamma 1$ (Fig. 1c), and $\gamma 2$ (Fig. 1e); link 0 has outputs 0γ (Fig. 1a), 0∞ (Fig. 1d), and 03 (Fig. 1h); link 1 has outputs 1γ (Fig. 1c), $(14)_1$ (Fig. 1g), 13, and $(14)_2$ (Fig. 1e). In the same way the outputs of the remaining central links may also be recorded.

Let link p have outputs to links $q_1, q_2, \dots, q_k$—k outputs in all. We shall call a certain **sequence of outputs** of link p the sequence

$$pq_1\ pq_2\ pq_3 \dots pq_k,$$

as well as any other sequence obtained from it by a cyclic permutation. It is clear that the maximum possible number of distinct sequences of outputs of link p is equal to $(k-1)!$.

Fig. 1

For each central link of the single-axis regular gear mechanism being constructed, including the fixed link γ, we shall record one of its sequences of outputs. Such a list of sequences of outputs of the central links will be called a **combinatorial scheme** (K-scheme).

In the sequence of outputs of some link, for example link p, select some output pq. Let ps be the output immediately preceding pq. Next consider the sequence of outputs of link q. Let in this sequence, before the output qp, there be immediately located the output qr (if qp is the first member of the sequence, then qr is the last member of this sequence). Compose the sequence $pq\ qr$. Now consider the sequence of outputs of link r. Let in it, before the output rq, there be located the output rt. By adding the member rt to the preceding sequence, we obtain $pq\ qr\ rt$. Continuing in the same way the selection of outputs from the K-scheme, we shall, as is easy to see, eventually arrive at the output sp, after which, if one follows the adopted method of composing the sequence, there follows the output pq, with which its composition began. The sequence $pq\ qr\ rt \dots sp$, as well as any other sequence obtained from it by cyclic permutation, will be called a **cycle**. The cycle $pq\ qr\ rt \dots sp$ will also be denoted by $pqrt \dots s$. Obviously, any two cycles that differ from one another do not contain a single common output.

Suppose that one cycle of the K-scheme has been composed. If it includes all outputs, then, according to what has been said, no other cycles can be composed for this scheme. But if there are outputs not included in the composed cycle, then, starting from one (any) of them, we construct one more cycle, and so on, until we exhaust all outputs of the K-scheme. In this way we obtain a complete list of all cycles of the given K-scheme.

Consider, for example, one of the possible K-schemes containing the outputs of Fig. 1.

1. $\gamma 0\ \gamma 1\ \gamma 2\ \gamma\infty$.
2. $0\gamma\ 0\infty\ 03$.
3. $\infty\gamma\ \infty 4\ \infty 3\ \infty 0$.
4. $1\gamma\ 13\ (14)_1\ (14)_2$.
5. $2\gamma\ 24$.
6. $30\ 3\infty\ 31$.
7. $4\infty\ 42\ (41)_2\ (41)_1$.

The total number of outputs of this K-scheme is 24. Let us begin composing the cycles of the scheme with the first member of sequence 1—$\gamma 0$. Following the method accepted above,

for composing cycles, we obtain the cycle: 1. $\gamma 0\ 03\ 31\ 1\gamma$. The exit 1γ closes the cycle, since in the row of exits of link γ the term $\gamma 1$ follows immediately after $\gamma 0$. The next term of the row of exits of link γ, which did not enter the preceding cycle, is the exit $\gamma 1$. Beginning the composition of the next cycle with it, we obtain: 2. $\gamma 1\ (14)_2\ 42\ 2\gamma$. Continuing the operation of composing cycles further, we arrive at the following cycles: 3. $\gamma 2\ 24\ 4\infty\ \infty\gamma$; 4. $\gamma\infty\ \infty 0\ 0\gamma$; 5. $0\infty\ \infty 3\ 30$; 6. $\infty 4\ (41)_1\ 13\ 3\infty$; 7. $(14)_1\ (41)_2$. The seven cycles listed contain all 24 exits of the K-scheme and therefore constitute a complete list of all distinct cycles for this scheme.

Let a cycle $pqr \dots s$ be given, consisting of k letters. Take an oriented topological k-gon $pqr \dots s$, whose vertices $p, q, r, \dots, s$ along the boundary of the polygon are arranged in the same order as the letters $p, q, r, \dots, s$ in the notation of the cycle.

The notation $pqr \dots s$ determines the orientation of the polygon. The sides of the polygon $pq, qr, \dots, sp$, bearing the same names as the corresponding exits of the cycle, will be called images of these exits; the vertices of the polygon, images of the links of the same name as them; and the entire oriented polygon will be called the image of the cycle.

Each cycle of a K-scheme has a corresponding image—a polygon, and each exit has a corresponding image—a side of one of the polygons. Let pq be one of the exits. Then in the K-scheme there is also the exit qp. We identify any two sides of the form pq and qp so that the ends of these sides with the same names are identified. Vertices of the polygons with the same names are also identified. As a result of all this, a two-dimensional polyhedral complex is obtained. In all ordinary single-axis gear mechanisms used in mechanical engineering, the set of exits leads to a connected complex. We shall consider only such connected complexes. The vertices of the complex will also be called images of links; the edges, images of exits, with each pair of exits pq and qp represented by one edge; and the two-dimensional elements of the complex—polygons—images of cycles.

Let us note that each edge of the complex occurs on the boundaries of polygons twice, and with opposite orientation. Any vertex p of each of the two-dimensional elements of the complex is incident to edges that are images of adjacent terms of the row of exits of link p. Conversely, any two adjacent terms of each row of exits have as their images two edges of some two-dimensional element of the complex, incident to one vertex of this element.

It follows from what has been said:

A polyhedron, of which the subdivision is a complex consisting of vertices—images of the links of some K-scheme, edges—images of the exits of these links, and two-dimensional elements—images of the cycles of the scheme, is an orientable closed two-dimensional manifold.

Consider the drawing S_1 introduced by M. A. Kreines ([2]) of a certain ordinary single-axis gear mechanism. For all pieces of the plane representing central links on S_1, choose one and the same direction of traversal; let us establish, for example, the direction of traversal along the boundary of a piece so that the piece being traversed lies on the left side of the traverser. Suppose link p has k exits $pq_1, pq_2, pq_3, \dots, pq_k$. Begin the traversal of the piece of the plane representing this link on S_1 with the part of the boundary of this piece that represents, for example, the exit pq_1, and suppose that, in the chosen direction of traversal, this exit on the boundary of the piece is followed by the exit pq_2, then by the exits $pq_3, \dots, pq_k$. Continuing the traversal along the boundary of piece p, after the exit pq_k we again arrive at the exit pq_1.* We shall call the list $pq_1\ pq_2 \dots pq_k$ the **order of exits** of link p of the ordinary single-axis gear mechanism under consideration.

We shall say that a K-scheme corresponds to a certain ordinary single-axis gear mechanism if

Fig. 2

Figure 2: Fig. 2

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* We assume that every link in the drawing S_1 is bounded by one simple closed contour; this is the case for all mechanisms used in practice.

only in the case when: a) the o.r.g.m. contains those and only those central links and terminals whose designations enter into the K-scheme; b) the order of the terminals of each central link of the o.r.g.m. coincides with the sequence of terminals of this link in the K-scheme. From what has been set forth, and also from the properties of symbolic representations ([2]), the following assertion follows:

In order that a K-scheme correspond to some o.r.g.m., it is necessary and sufficient that the polyhedral complex of this K-scheme be a subdivision of a topological sphere.

Let the number of vertices of the complex (the number of central links) be equal to n, the number of edges (the number of pairs of terminals of the type pq and qp) be equal to m, and the number of polygons (cycles) be equal to t. Then, since the Euler characteristic of a subdivision of the sphere is equal to two, the following corollary is valid:

Fig. 2

In order that a K-scheme correspond to some o.r.g.m., it is necessary and sufficient that the equality

$$n - m + t = 2 \tag{1}$$

hold.

For the K-scheme of the example given above we have $n = 7$, $m = {}^{24}/_{2} = 12$, $t = 7$; consequently, equality (1) is satisfied. Therefore, an o.r.g.m. can be constructed from this K-scheme. In Fig. 2a the complex corresponding to the K-scheme under consideration is shown, and in Fig. 2b the scheme of the desired o.r.g.m. is shown. The order of the terminals of any of the central links of the mechanism in Fig. 2b coincides with the sequence of terminals of this link in the K-scheme.

From what has been said there follows the following rule for finding all possible variants of the arrangement of a certain o.r.g.m. in space, if the list of its central links and the list of terminals of each of them are known: a) all possible, essentially distinct K-schemes satisfying the problem are written down; b) for each K-scheme all cycles corresponding to it are found; c) for each K-scheme the Euler characteristic of the complex representing the scheme is determined. Each K-scheme whose complex has Euler characteristic equal to two (equation

(1)) corresponds to an o.r.g.m. arranged in space. According to the remaining schemes, an o.r.g.m. cannot be constructed without introducing additional central links.

It can be shown that if the Euler characteristic is equal to zero (a subdivision of the torus), then in order to construct an o.r.g.m. it is necessary to introduce into it no fewer than one additional central link; if the Euler characteristic is equal to −2 (a subdivision of a sphere with two handles), then for placing the o.r.g.m. in space it is necessary to introduce no fewer than two additional central links.

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Note: Figure translations are in progress. See original paper for figures.