CALCULATION OF THE CONSTANT OF WALL TURBULENCE

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CALCULATION OF THE CONSTANT OF WALL TURBULENCE

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1. Turbulent flows possess certain conservative properties with respect to the averaged characteristics of the flow. This circumstance makes it possible to construct a phenomenological theory of the averaged flow without considering its detailed internal structure.

In a wall-bounded turbulent boundary layer one can distinguish three characteristic regions: the viscous sublayer, in which molecular friction is substantially greater than turbulent friction; the region of the quadratic law of friction, in which the Prandtl–Taylor law holds with a high degree of accuracy ([1,2])

$$\tau_{\mathrm{T}} = \rho(\varkappa y\, du/dy)^2, \tag{1}$$

where $\varkappa$ is the Prandtl-Karman constant ([3]), determined experimentally in semiempirical theories; and the outer region of the boundary layer.

The region of the Prandtl–Taylor law is of greatest importance for the formation of the velocity profile in the wall-bounded turbulent boundary layer; this is due to three important circumstances:

First, as the Reynolds number increases, the relative thickness of the viscous sublayer tends to zero, while in the outer region of the boundary layer the flow velocity tends to the velocity of the undisturbed stream or to the velocity on the axis of the channel.

Second, the region of the Prandtl–Taylor law is highly conservative with respect to external influences (longitudinal pressure gradient, degree of turbulence of the external stream, etc.) ([4,5]).

Third, the viscous sublayer exhibits stability with respect to any external disturbances, which also determines its thickness ([6,7]).

2. As a generalization of the above-mentioned properties of the turbulent boundary layer, one may put forward the principle of maximum stability of the averaged turbulent flow, which was formulated mathematically in

([8]). In the latter work, as a measure of the stability of the flow, the functional

$$\Pi = \sup_a Y, \tag{2}$$

was introduced, where Y is the logarithmic decrement of damping and a is the wave number.

It was assumed here that, in the first approximation, the tensor of turbulent stresses does not vary when small disturbances are imposed on the flow, and the quantity Y can be calculated on the basis of the solution of the usual Orr–Sommerfeld equation ([9]). The basis for this is that, for the maximally stable velocity profile, the function $Y(a)$ has two equal maxima lying near the boundaries of the spectrum. The presence of two "dangerous" disturbances corresponds to two natural length scales of the turbulent flow—the size of the boundary layer or channel and the thickness of the viscous sublayer. It is then evident that very long waves cannot interact substantially with turbulent pulsations, the maximum intensity of which lies in the medium-wave part of the spectrum. As for short-wave disturbances, with increasing

filling of the velocity profile are localized in the viscous sublayer, where the level of turbulent friction is small. This is clearly seen in Fig. 1, where the dependence of the position of the critical layer on the parameter χ, chosen here as a measure of the filling of the velocity profile, is plotted. As is known ([9]), in the critical layer the local flow velocity coincides with the propagation velocity of the disturbance under consideration, and its position is the principal factor characterizing the stability of the flow. As turbulence increases, the critical layer of short-wave disturbances sinks into the viscous sublayer, while the critical layer of long-wave disturbances remains in the region of intense turbulence.

Fig. 1. Position of the critical point for short-wave (1) and long-wave (2) disturbances

As was shown in ([8]), the dependence of the functional (2) on the flow parameters is not smooth, which creates considerable difficulties for computations. It therefore seems reasonable to pass to a certain integral functional. The latter can be constructed as follows. Suppose that at the initial instant of time, in some cross-section of the channel, a velocity disturbance in the form of a δ-function is imposed. The kinetic energy of the disturbed motion is

$$E \sim \int_{-\infty}^{\infty} v^2 dx. \tag{3}$$

If the disturbances decay, then the integral exists

$$I_1 = \int_0^\infty dt \int_0^\infty v^2 dx, \tag{4}$$

having the dimension of action. For a given disturbance, the energy of an individual Fourier harmonic is proportional to $\exp(2\alpha Y t)$, and the desired functional can be given the form

$$I = -\int_0^\infty \frac{d\alpha}{\alpha Y}. \tag{5}$$

It is essential that the main contribution to this integral is made by short- and long-wave disturbances. As a consequence, as the calculations showed, the maximally stable profile found from (2) also realizes the minimum of the functional (5). For unstable profiles, when $Y(\alpha)$ changes sign, the functional I has no meaning.

3. In flow past an impermeable wall with a sufficiently small pressure gradient, the shear stresses in the region where law (1) holds are close to the friction at the wall, as a result of which a linear law of turbulent viscosity holds

$$\mu \ll \mu_{\mathrm{T}} = \chi \rho u_* y. \tag{6}$$

In the region of the viscous sublayer ([10]), as $y \to 0$,

$$\mu \gg \mu_{\mathrm{T}} \sim y^4. \tag{7}$$

Thus, the effective viscosity forming the turbulent velocity profile,

$$\mu_* = \mu + \mu_{\mathrm{t}}, \tag{8}$$

is a complex function of the distance from the wall, and its form is known only in the region where the local laws (1) and (7) are valid.

However, owing to the degeneration of the viscous sublayer and the maximum filling of the velocity profile in the core of the flow as $\mathrm{Re} \to \infty$, the profile calculated from the effective viscosity

$$\mu_* = \mu + \varkappa \rho v^* y, \tag{9}$$

approaches, with increasing Reynolds number, the true distribution of velocities. At the same time, such a profile has only one variable parameter—the Prandtl-Kármán constant, about which it is known that at sufficiently large Reynolds numbers it is indeed self-similar.

Fig. 2. Dependence of the integral functional on profile fullness

Figure 1: Fig. 2. Dependence of the integral functional on profile fullness

Fig. 2. Dependence of the integral functional on profile fullness

Consequently, computation of the quantity $\varkappa$ from the condition of a minimum of the functional I when using the law of friction (9), being formally correct as $\mathrm{Re} \to \infty$, should in practice give the true result throughout the entire range of developed turbulent flows ($\mathrm{Re} \gg \mathrm{Re}_{\mathrm{cr}}$).

Figure 2 shows the dependence $I(\varkappa)$ for the effective viscosity (9), calculated deliberately at not very large Reynolds numbers. As can be seen, the functional I is smooth and has a distinct minimum at $\varkappa \approx 0.4$, which agrees well with all the experimental material presently available ([11]).

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REFERENCES

1. G. J. Taylor, Proc. Roy. Soc., **A 135**, 685 (1932).
2. L. Prandtl, Neuere Ergebnisse der Turbulenzforschung, VDI, Zs., **77**, No. 5 (1933).
3. Th. Karman, Proc. Intern. Congr. Appl. Mech., Cambridge, 1934.
4. F. Clauser, J. Aeronaut. Sci., **21**, No. 2 (1954).
5. M. Pichal, Turbulentni mezni vrstva na rovinne desce v Proudeni s vysokonintenzitou turbulence, Strojnicky Časopis, **16**, No. 2 (1965).
6. S. S. Kutateladze, ZhTF, **5**, issue 10 (1935).
7. S. S. Kutateladze, *Fundamentals of the Theory of Heat Transfer*, Moscow-Leningrad, 1962.
8. M. A. Goldshtik, DAN, **182**, No. 5 (1968).
9. Lin Tszyan-tszyao, *Theory of Hydrodynamic Stability*, IL, 1958.

10. L. D. Landau, E. M. Lifshitz, *Mechanics of Continuous Media*, 1944.

11. M. P. Escudier, The Distribution of the Mixing Length in Turbulent Flows Near Walls. Mach. Eng. Dep., Imperial College of Science and Technology, London, 1965.

Note: Figure translations are in progress. See original paper for figures.