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CALCULATION OF IONIZATION INSTABILITY

IN A LOW-TEMPERATURE MAGNETIZED PLASMA

1°. Ionization instability ([1]), discovered comparatively recently in a low-temperature magnetized plasma, occurs in many magnetohydrodynamic devices. This type of instability is apparently one of the basic and simplest cases of the development of turbulence in a low-temperature plasma. Its study is therefore of interest from both the applied and the general-physical points of view.

Experimental and theoretical investigations of this phenomenon are contained in ([2,3]), as well as in a number of other works (see the review ([4])). However, the study of ionization instability cannot be regarded as complete. Experimental results are few, and the analytical theory of the origin and development of magnetic strata has been constructed under the restrictions of linearity or one-dimensionality.

In the present work the study of ionization instability has been carried out on a specific physical model of a plasma by means of the numerical solution of a correspondingly formulated mathematical problem. Modern computational methods make it possible to carry out such a treatment under sufficiently general assumptions. However, there is no need to take many different effects into account at once. It is more expedient first to analyze the simplest physical model in which the mechanism of ionization instability, responsible for the formation of strata, is incorporated. Subsequent refinement of the model would make it possible to estimate the qualitative and quantitative influence of various factors. Analysis of the results of the numerical calculations given below has made it possible

Fig. 1.

Figure 1: Fig. 1.

to draw certain conclusions about the mechanism and the basic properties of the nonlinear, nonstationary process of formation and behavior of strata in a magnetic field.

2°. We shall consider the following formulation of a two-dimensional problem, borrowed from experiment ([2]). Let a plasma with a constant concentration of free electrons $n = n_0$ be located in the rectangular region $ABCD$ (Fig. 1). The external magnetic field is constant and directed along the z axis, perpendicularly upward from the plane of the figure. In the plasma, between the ideally sectioned electrodes AB and CD, an electric current flows $\mathbf{j}(j_x, j_y, 0)$, $j_x = 0$, $j_y = -j_0$, $j_0 > 0$. At the electrodes $j_y = -j_0$ is maintained constant. The walls AD and BC are made of an ideal dielectric, on which $j_x = 0$.

Suppose that at the time $t = 0$ a perturbation of the spatial distribution of the electron concentration has occurred. We shall be interested in the subsequent behavior of the concentration of free electrons and of the electric-current-density vector. Concerning the physical properties of the plasma we shall make the following assumptions: 1. The electron temperature T_e is constant. 2. T_e considerably exceeds the ion temperature T_i. 3. The frequency of collisions of electrons with ions is much greater than the frequency of their collisions with neutral atoms, so that the collision time of electrons is inversely proportional to the concentration, $\tau = \tau_0 n_0/n$, where τ_0 and n_0 are chosen scales, and the plasma conductivity is constant, $\sigma = \sigma_0$. 4. Diffusion processes-

···processes (radiation, electron thermal conductivity, thermodiffusion of electrons, etc.) are not taken into account. Under these assumptions, the energy-balance equation, which determines the rate of ionization and recombination, can be written in the form (3)

$$I\,\partial n/\partial t = (j_x^2 + j_y^2)/\sigma - \chi n T_e/\tau,$$

where I is the ionization potential, and χ is the fraction of the electron energy lost in one collision with an ion. However, the electron concentration does not exceed the value n^* attained upon complete ionization of the plasma (as is usually the case, the complete ionization of the readily ionized component is meant). The spatial distribution of the electric current is described by the equations $\operatorname{div}\mathbf{j} = 0$, $\operatorname{rot}\mathbf{E} = 0$. Ohm' s law is taken in the form $\mathbf{j} + \mathbf{j} \times \vec{\Omega} = \sigma\mathbf{E}$, $\vec{\Omega} = ec\tau\mathbf{H}/m_e$, where e and m_e are the charge and mass of the electron, and c is the speed of light in vacuum.

Fig. 1.

Fig. 2.

Fig. 2.

Figure 2: Fig. 2.

Fig. 3.

Figure 3: Fig. 3.

Fig. 3.

Introduce the vector potential of the electric current $\Psi(0, 0, \psi)$ so that $\mathbf{j} = \operatorname{rot} \Psi$, and write the initial equations in dimensionless form. For this purpose, as the fundamental scales of measurement we choose the plasma conductivity σ_0, the initial values of the electric-current density j_0 and electron concentration n. For the potential ψ we obtain the equation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} - \frac{\partial \Omega}{\partial y}\frac{\partial \psi}{\partial x} + \frac{\partial \Omega}{\partial x}\frac{\partial \psi}{\partial y} = 0, \qquad \Omega = \frac{H}{n}. \tag{1}$$

The equation for the ionization rate takes the form

$$\partial n/\partial t = (\partial \psi/\partial x)^2 + (\partial \psi/\partial y)^2 - n^2. \tag{2}$$

On AB and CD ($AB = CD = L$), from the condition $j_y = -1$ it follows that $\psi = x$, and from the condition $j_x = 0$ on AD and BC we obtain $\psi = 0$ on AD and $\psi = L$ on BC.

3°. Numerical integration of the nonlinear system (1)–(2) is constructed as follows. First, the solution of the stationary equation (1) at the new time instant is found with coefficients computed from the values of n in the preceding time layer. Then, in accordance with the difference analogue of equation (2), the values of n are recalculated for the new time instant. The solution of the boundary-value problem for equation (1) was carried out with the aid of

the locally one-dimensional method (5) with symmetrization of directions (6). The differential operators in each direction are approximated by homogeneous through-counting difference schemes (7). Because the coefficients at the first derivatives in equation (1) vary strongly in space, the difference schemes are monotonized (8); these schemes are solved by the method of sweep with fluxes (9). A square difference grid with step h is used. Its elements are cells over which all the physical quantities under consideration are averaged. The values of these quantities are assigned to the centers of the cells.

4°. Let us trace the development of the ionization instability in examples of specific calculations. In all the variants presented it was assumed that $\Omega = 10$, $AB = CD = 1$, $AD = BC = 2$. At the initial instant $t = 0$, the free-electron concentration was $n = 1$; $j_x = 0$; $j_y = -1$. The perturbation of the concentration n was introduced with the same magnitude Δn in a rectangle composed of several cells; the shape, position, as well as the amplitude and sign of the perturbation were varied.

At the first stage, over the characteristic time $t = t^* \sim 1$, the ionization instability manifests itself in the appearance of regular formations in which $n > 1$, separated by regions of lower concentration $n < 1$. In shape and position these formations can be compared with the striations observed in experiment. Figure 1 shows the striations (shaded in the figure) that occurred in the calculation with step $h = 0.1$ (number of nodes $N = 200$) at the time $t = 1.5$. The boundaries of the striations are the lines of equal concentration $n = 1$. A perturbation with amplitude 0.2 at $t = 0$ occupied 8 cells in the central part of the region (Fig. 1).

The induced electric currents flow along closed lines (Fig. 1). Combining with the external current, they force the resultant current to flow predominantly along the striations; the intense Joule heating inside them leads to strong ionization. The weakening of the resultant current and of the Joule heating in the "holes," on the other hand, promotes a further decrease in the number of free electrons in them owing to recombination.

At the second stage, the striations are destroyed, breaking up into a number of chaotic formations ("tongues," "loops," "bridges," etc.). The regular pattern of developed striations is replaced, for $t \gg t^*$, by an irregular pattern of "holes" and "bridges." Figure 2 shows the level lines $n = 1$ for the variant described above at $t = 7$; the regions with $n > 1$ are shaded.

Changing the shape and position of the perturbation at $t = 0$ showed that the initial conditions affect the shape and position of the striations at $t = t^*$. Thus, if $\Delta n > 0$, a striation arises at the site of the perturbation; if $\Delta n < 0$, a depression of the concentration n develops there.

The problem of ionization instability in an unbounded volume, as is clear from the original equations, has no characteristic size. However, in the numerical solution of the stated problem the characteristic size λ is the averaging scale, equal to the step of the difference grid, $\lambda = h$. In solving the problem with allowance for diffusion processes (radiation, thermal conductivity, etc.), the characteristic size of the problem would be the diffusion length for these processes. From this point of view, the step of the difference scheme may be interpreted in the same way as the diffusion length of certain physical processes. Therefore, reducing the step of the difference grid by a factor of two ($h = 0.05$; $N = 800$) leads, by the time $t = 1.5$, to a somewhat different arrangement of the striations (Fig. 3). At the site of the same perturbation (now occupying 32 cells), two striations arise, whose thickness is still equal to several averaging scales (several steps of the difference grid). Near the boundaries the striation pattern is disturbed only slightly, which is explained by the influence of the boundary conditions. The development of the process of striation formation in this case proceeds more intensely, and already by the time $t = 1.5$ signs are detected of the appearance

Fig. 4

Figure 4: Fig. 4

of an irregular pattern, which by the time $t = 7$ has little in common with the pattern in Fig. 2.

Let us consider the change with time of the mean quantities. Figure 4 shows $\langle n \rangle$, $\langle \tau \rangle = \langle 1/n \rangle$, $\sigma_{eff} = \langle j_y \rangle / \langle E_y \rangle$, $n_{\max}$, $n_{\min}$ for $N = 200$, $h = 0.1$ (solid lines) and for $N = 800$, $h = 0.05$ (dashed lines). At the first stage of striation formation ($t < t^*$) the sharpest change in these quantities occurs, in particular a decrease of σ_{eff}. This is explained by the fact that the preferential flow of the electric current takes place along the striations, which increases the effective resistance of the plasma, despite the constant conductivity $\sigma = 1$.

With decreasing h (increasing the number of nodes N) the thickness of the striations decreases and their number in the central part of the region increases, which leads to an increase of the current density in them. Therefore the decrease of σ_{eff} in this case occurs more intensively, as is seen from Fig. 4. In turn, the increase of the current density in the striations causes a more intensive release of Joule heat and, consequently, a more rapid increase of $n_{\max}$ (for $h = 0.05$, complete ionization $n_{\max} = n^* = 2.5$ is attained). When the striations are destroyed at the beginning of the second stage, the mean quantities change little. For $t \gg t^*$ the final establishment of $\langle n \rangle$, σ_{eff}, and $n_{\max}$ occurs; however, $n_{\min}$ continues to decrease, as a result of which $\langle \tau \rangle$ increases. The calculations showed that, for fixed h, the form and position of the initial perturbation have no substantial influence on the values of the mean quantities established for $t \gg t^*$.

Fig. 4

5°. On the basis of an analysis of the results of the numerical solution of the problem of the behavior of a local perturbation of the electron concentration in a magnetized plasma when an electric current flows through it, the following conclusions may be drawn:

1. The adopted physical model qualitatively reflects the most essential features in the behavior of the plasma known from experiment. The nonlinear mechanism of Joule heating and of energy transfer in collisions of electrons with ions gives rise, over the characteristic time t^*, to the manifestation of the ionization instability in the form of regular formations—striations.

2. At late stages of the process the striations break up, leading to an irregular spatial distribution of the electron concentration and of the components of the electric-current density.

3. Despite the essentially nonstationary character of the ionization instability, over several characteristic times the mean quantities become established.

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Note: Figure translations are in progress. See original paper for figures.