ON THE CONDITION AT THE END OF A CRACK

Corresponding Member of the Academy of Sciences of the USSR L. A. GALIN, Ya. B. FRIDMAN,

1969

SovietRxiv

Abstract

Full Text

UDC 539.371

THEORY OF ELASTICITY

Corresponding Member of the Academy of Sciences of the USSR L. A. GALIN,
Ya. B. FRIDMAN,
G. P. CHEREPANOV, E. M. MOROZOV, V. Z. PARTON

ON THE CONDITION AT THE END OF A CRACK

The development of a crack in a solid is determined by a certain additional condition, specified at its end and not following from the equations of motion and deformation of the solid.

Here, in order to find this additional condition, general considerations are invoked that make it possible to determine its form for any model of a continuous medium. The formulation of this condition to some extent resembles the general formulation of rheological laws (^{1}). To determine this phenomenologically interpreted condition, direct experimental methods can be and are applied (^{2}).

In the case of elastic bodies, the general considerations prove sufficient to formulate exhaustively the limiting condition at the end of a crack, coinciding with the concept of brittle or quasi-brittle fracture (Irwin' s condition). It becomes clear why the various physical theories proposed at different times by Griffith (^{3}), Neuber (^{4}), Irwin (^{5}), Orowan (^{6}), Williams (^{7}), and others lead to one and the same limiting condition.

1. General form of the condition. Consider a continuous medium having surfaces of displacement discontinuity—cracks. Let us single out a neighborhood of some arbitrarily chosen point O of a smooth crack contour, small in comparison with the characteristic linear dimension of the crack. Let xyz be a system of rectangular Cartesian coordinates with origin at the point O, the y-axis perpendicular to the crack surface, and the z-axis directed along the contour. We shall represent the small neighborhood under consideration in the xy-plane by an infinite region having an unloaded cut along the negative semiaxis x. The parameters describing the state of the medium in the small neighborhood under consideration do not depend on z. Suppose that there is local symmetry of the phenomena with respect to the x-axis. We restrict ourselves to the consideration of quasi-static cracks and small deformations.

Let the singular solution of the mathematical problem posed for the chosen rheological model of the medium be determined up to a constant, i.e., a quantity

a independent of x and y (the free parameter of the singular solution).

From energetic considerations it follows (^{15}) that a continuous solution of the stated problem for any correctly formulated model of the medium must necessarily be singular (unbounded at the point O) either in stresses, or in deformations, or in both together. Of course, the singular solution must satisfy the natural physical condition of bounded energy expenditure in an arbitrarily small neighborhood of the point O of the medium.

Let the parameter a in the general case represent some functional characteristic of the loading process, crack development, boundary conditions, the shape of the body including the crack, etc.; this characteristic, for a fixed system of cracks, can in principle be found from the solution of the mathematical problem as a whole for any rheological model of the medium. It naturally includes this external parameter (for example, the crack length). Moreover, up to this point the calculation is carried out

for a body with cuts, and the transition to cracks is carried out after the introduction of some proposition on the mechanism of fracture in the neighborhood of the boundary of the cut. The ultimate aim of the calculation is to find the dependence of the crack length (or of some other external parameter) on time and on the loading process.

Owing to the local character of fracture (it is assumed that fracture always occurs at the end of the crack), the physical law determining the fracture of the material at the end of the crack can be written in the form of some functional relation between the following parameters: the parameter a, the crack length l, and the time t. Other variable quantities cannot enter this additional condition at the end of the crack because of the local character of fracture. It also naturally includes certain material constants characterizing its physical properties. Time can enter the additional condition explicitly only in nonstationary problems; in this case it characterizes the influence of the initial conditions. Physical constants, generally speaking, may vary with time (aging of the material near the end of the crack). Let us note that the additional condition may also include parameters that have no singularity at the end of the crack (for example, in some cases the temperature or the concentration of components of a solid solution).

In what follows, as the parameter a we take the stress intensity factor N, by which is meant the quantity

$$N = \lim_{y=0,\ x\to 0} \left(\sigma_y \sqrt{x}\right).$$

The quantity N has meaning only for materials close, in the sense indicated below, to linearly hereditary (in particular, elastic) bodies.

For a theoretical determination of the additional condition it is most natural to invoke physical notions of surface energy and fracture energy, analogous to the

Griffith–Irwin–Orowan concepts, as well as the modified Neuber concept; this approach is developed in works ([15,16]).

2. Elastic-plastic body. Near the end of a crack in an elastic-plastic body there will exist a region D of plastic deformations, while outside the region D the deformations will be purely elastic. In accordance with Saint-Venant' s principle, one may say that at distances large in comparison with the characteristic linear dimension of the region D, but small in comparison with the crack length, the elastic distribution of stresses and displacements is determined by the stress intensity factor N. Specifying this parameter determines, in particular, the size and shape of the plastic region, and the distribution of strains and stresses in it. Cracks whose size is comparable with the size of the plastic region will not be considered here. For an elastic-plastic body all parameters of the medium do not depend explicitly on time (but only through N). Therefore, in the present case, in addition to the general assumption on the local character of fracture, it is natural to suppose that time does not enter into the additional condition. This assumption is unconditionally adopted in all works on quasi-brittle fracture.

From this there follows, for example, the important conclusion that if, during the loading process, the quantity N did not decrease (this most commonly occurring case is analogous to the simple loading path of a plastic body), then after the crack has traversed a distance large in comparison with the size of the plastic region, so that the influence of the initial conditions becomes negligibly small, the stress intensity factor will be equal to a certain material constant (the Irwin –Orowan concept of quasi-brittle fracture ([5,6]); see also ([15,16,9,10])). Thus, this concept corresponds to reaching a stationary regime in a nonstationary problem.

Figure 1 shows the experimental dependence $N = N(\Delta l)$ (Δl is the increment of crack length in the subcritical state), plotted from the results of work ([11]). A sheet specimen of D16T-1 with a central crack was subjected to a single tensile loading. The curve was obtained for two initial crack lengths–20 and 40 mm. Here, as in the case of viscous bodies, the intensity factors tend toward a certain asymptotic value.

3. Time effects. The presence of time in the additional condition may be connected either with the nonstationarity of the process (the influence of the initial conditions) or with aging of the material. When the action of nonlinear deformation laws is confined to a small region D near the crack tip (so that the concept of the stress-intensity factor N is meaningful), aging of the material in the general case of a stationary crack is expressed by the phenomenological dependence $N =$

[Fig. 1 and Fig. 2]

Fig. 1 **Fig. 2**

$N_0(t)$, where t is the time from fracture (the beginning of crack growth) counted from the moment of load application, corresponding to the given N. Determination of the function N_0 must constitute the subject of theoretical or direct experimental investigation.

The presence of the crack velocity $\dot{l}$ in the additional condition (entering through the fracture constants) indicates the influence of viscous deformations or creep deformations on crack growth. In the simplest case of a stationary process, when the characteristic increment of crack length is large compared with the characteristic linear size of the region D, the additional condition at the crack tip has the form $N = N_1(\dot{l})$; determination of the function N_1 must constitute the subject of theoretical or direct experimental investigation.

As an example, Fig. 2 shows the dependence $N = N_1(\dot{l})$, obtained by recalculating the experimental data of work ([8]). A strip of cellulose acetate containing an edge crack was subjected to long-term static loading at constant stress. The curve shown in Fig. 2 corresponds to stress values 7.4, 7.6, 7.8 kg/mm^2. In work ([16]), on the basis of a modified Neuber concept, a theoretical dependence $N = N_1(\dot{l})$ was obtained, agreeing with experiments on long-term strength.

4. Elastic body. For ideally brittle bodies, when the size of the region D is zero, the generally accepted assumption of the local character of fracture yields the limiting Irwin condition $N = K_c$, where K_c is a material constant (the Irwin constant ([9])). This condition constitutes the basic concept of brittle fracture ([9]). Let us briefly recall how various authors arrived at this condition. According to the most natural and general physical ideas of Griffith-Irwin-Orowan, to form a unit area of crack surface it is necessary to expend a certain energy G_c, which is a constant characteristic of the material. The mathematical formulation of these views, which

Irwin gave in (9), for the plane-stress state, reduces to the limiting condition $N^2 = EG_c$, where E is Young' s modulus. Williams related crack growth to the limiting radius of curvature at the crack end (7). Neuber put forward the concept of a "plastic" particle: near the end of an elastic crack there is a plastic region, the size d of which is a structural constant of the material (4). Leonov and Panasyuk assumed that, over a certain segment on the continuation of the crack, the stress σ_y is equal to the theoretical strength, while the displacement v of the opposite faces at the point O is equal to a material constant (12). Barenblatt believed (13) that, over a segment of length d on the continuation of the crack, the stress σ_y, as well as the quantity d, are material constants*. McClintock supposed that at a distance d ahead of the crack, the shear strain in the plastic region reaches a limiting value for each material (14). All the indicated concepts, as $d \to 0$, naturally lead to Irwin' s limiting condition.

The main task of fracture mechanics at the present time consists in developing theoretical and experimental methods for determining the functional dependence among the indicated invariant characteristics, with the aim of creating effective methods for calculating the strength of parts with cracks. A series of experimental studies on materials with known rheological properties should also be devoted to this, and the law governing the transfer of test results from specimens to full-scale parts should be established.

Received
25 VII 1968

CITED LITERATURE

1. L. I. Sedov, *Introduction to the Mechanics of a Continuous Medium*, Moscow, 1962.
2. B. A. Drozdovskii, V. M. Markochev, Ya. B. Fridman, DAN, **174**, No. 4 (1967).
3. A. A. Griffith, Phil. Trans. Roy. Soc., A, **221** (1920).
4. H. Neuber, *Kerbspannungslehre*, Moscow, 1947.
5. G. R. Irwin, In: *Fracturing of Metals*, Cleveland, 1948.
6. B. O. Orowan, In: *Fatigue and Fracture of Metals*, N. Y., 1950.
7. M. L. Williams, Proc. Crack Propagation Symposium, Cranfield, 1962.
8. S. P. Zhurkov, E. E. Tomashevskii, ZhTF, **27**, No. 6 (1957).
9. G. R. Irwin, In: *Handbuch der Physik*, Vol. 6, Berlin, 1958.
10. G. P. Cherepanov, PMM, **32**, No. 6 (1968).
11. V. M. Markochev, Author' s abstract of candidate dissertation, Moscow, 1966.
12. M. Ya. Leonov, V. V. Panasyuk, Prikl. Mekh., Vol. **5** (1959).
13. G. I. Barenblatt, PMM, **23**, No. 3-5 (1959).
14. F. A. McClintock, J. Appl. Mech., **25**, No. 4 (1958).
15. G. P. Cherepanov, Intern. J. Solids and Structures, **4**, 811 (1968).
16. G. P. Cherepanov, Mekh. Tverd. Tela, No. 1 (1969).
17. D. D. Ivlev, Zhurn. Prikl. Mekh. i Tekh. Fiz., No. 6 (1967); Mekh. Tverd. Tela, No. 6 (1967).
18. E. M. Morozov, V. Z. Parton, Mekh. Tverd. Tela, No. 6 (1968).

* The works of G. I. Barenblatt in the part concerning the modeling of phe-

nomena at the crack end, as shown by a number of authors (17, 18), contain errors.

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.