ON THE QUESTION OF THE KINETIC DESCRIPTION OF RAPID PROCESSES IN A PLASMA

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Abstract

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ON THE QUESTION OF THE KINETIC DESCRIPTION OF RAPID PROCESSES IN A PLASMA

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The state of a plasma is usually described by the distribution function $f(t, \mathbf{r}, \mathbf{v})$, which is the probability density of the random coordinates $\mathbf{r}(t)$ and velocity $\mathbf{v}(t)$ of a particle. The distribution function depends on time t as on a parameter and satisfies a kinetic equation of the Fokker–Planck type

$$\frac{\partial f}{\partial t} = -v_i \frac{\partial f}{\partial x_i} - \frac{\partial}{\partial v_i}(A_i f) + \frac{1}{2}\frac{\partial^2}{\partial v_i \partial v_j}(D_{ij} f). \quad (1)$$

On the other hand, it is known from the theory of random processes [1] that an equation of type (1) exists only for Markov random processes $\{\mathbf{r}(t), \mathbf{v}(t)\}$, for which the rate of change of the distribution function $\partial f/\partial t$ at time t depends only on the value of f at this time and does not depend on the preceding history, i.e., on the values of f at preceding instants of time. In particular, the process $\{\mathbf{r}(t), \mathbf{v}(t)\}$ will be non-Markovian if $\{\mathbf{r}(t), \mathbf{v}(t)\}$ is the solution of a system of differential equations whose coefficients are random functions with a nonzero correlation time.

In the case of a plasma the state of a particle with mass m and charge e is determined by the equations

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}, \qquad \frac{d\mathbf{v}}{dt} = \frac{e}{m}\mathbf{E}(\mathbf{r}, t), \quad (2)$$

where the electric field $\mathbf{E}$ is a random process with a correlation function that may be approximated by the exponential

$$\langle E_i[\mathbf{r}(t), t] E_j[\mathbf{r}(t+\tau), t+\tau] \rangle = \delta_{ij}\sigma^2 e^{-\nu|\tau|}. \quad (3)$$

The correlation time $1/\nu$ usually has the order of the period of plasma oscillations.

To reveal the main features of this random process, we shall consider the simplest case of one-dimensional homogeneous (i.e., independent of **r**) motion. Then equations (2), (3) are simplified:

$$\dot{v} = \frac{e}{m} E, \tag{4}$$

$$\langle E(t)E(t+\tau)\rangle = \sigma^2 e^{-\nu|\tau|}. \tag{5}$$

The process $v(t)$ is non-Markovian, but it can be augmented to a Markovian one [1]. To do this, using the Wiener–Khinchin theorem, we transform relation (5) to the form

$$\dot{E} + \nu E = u(t), \tag{6}$$

where $u(t)$ is white noise having zero correlation time

$$\langle u(t)u(t+\tau)\rangle = 2\sigma^2 \nu \delta(\tau). \tag{7}$$

The two-dimensional random process $\{v(t), E(t)\}$, described by equations (4), (6), is Markovian, and its distribution function $f(v, E)$ satisfies the Fokker–Planck equation (1)

$$\frac{\partial f}{\partial t} = -\frac{e}{m} E \frac{\partial f}{\partial v} + \nu \frac{\partial (fE)}{\partial E} + \sigma^2 \nu \frac{\partial^2 f}{\partial E^2}. \tag{8}$$

If the electric field had no "memory," i.e., if it itself were white noise, then the distribution function would depend only on the velocity: $f(v)$. (The distribution function has the same form after the lapse of a time much longer than the correlation time. Such a consideration is used in the theory of stochastic acceleration of particles (2,3).) The simplest "memory" of the type (5) leads to the distribution function $f(v, E)$, or, by virtue of (4), $f(v, \dot{v})$. More complicated correlation functions of the electric field will lead to a dependence of the distribution functions on higher derivatives of the velocity:

$$f(v);\ f(v, \dot{v});\ f(v, \dot{v}, \ddot{v});\ f(v, \dot{v}, \ddot{v}, \dddot{v}), \ldots \tag{9}$$

Such a chain of one-particle distribution functions, depending on the velocity v and its derivatives (this possibility was first pointed out by Vlasov (4)), is analogous to Bogolyubov' s chain (5) $f(v_1), f(v_1, v_2), f(v_1, v_2, v_3), \ldots$ of many-particle distribution functions depending only on the velocity. The chain (9) may prove preferable in the case of strongly turbulent plasma, when the correlations of the distribution functions $g(v_1, v_2) = f(v_1, v_2) - f(v_1, v_2)$ are not small (6).

(At present, a quantitative theory of strongly turbulent plasma has not yet been created [7].)

It can be shown that in a quiescent plasma the velocity of particles is a Markov process, if one neglects terms that are small in comparison with the Coulomb logarithm.

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Note: Figure translations are in progress. See original paper for figures.