ON THE POSSIBILITY OF THE PRODUCTION OF ELECTRON-POSITRON PAIRS IN VACUUM UPON FOCUSING LASER RADIATION

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Abstract

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PHYSICS

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ON THE POSSIBILITY OF THE PRODUCTION OF ELECTRON-POSITRON PAIRS IN VACUUM UPON FOCUSING LASER RADIATION

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1. The possibility of "pulling out," by means of an external field, real pairs of particles–electrons and positrons–from the vacuum, predicted by quantum electrodynamics, has not yet been experimentally confirmed. At the same time, this theoretical result is of such fundamental significance from both the physical and the philosophical points of view that the constant striving for its experimental verification is entirely understandable. The difficulties of the experiment here, as is known, are connected with the fact that it is necessary to have electric fields with intensities E approaching the critical value $E_c = m^2c^3/e\hbar = 1.32 \cdot 10^{16}$ V/cm. There is apparently no basis for expecting static fields with such high intensities to be obtained in the near future. In this connection it seems of interest to clarify the possibilities of a laser experiment. In our opinion, the transition to the picosecond range of pulse durations of laser radiation has made these possibilities more realistic, although still difficult for experimental realization. Below we consider the problem of the production of electron-positron pairs in the electromagnetic field arising upon focusing coherent laser radiation into vacuum by means of an ideal lens.*

2. We shall proceed from the expression for the nonlinear additional term L_1 to the classical density of the Lagrangian function of the electromagnetic field $L_c = {}^1/_2(E^2 - H^2)$ (see, for example, ([2])):

$$L_1 = \frac{e^2}{8\pi^2\hbar c}\int_0^\infty \frac{e^{-\eta}}{\eta^3}\left[i\eta^2 b\,\frac{\cos\left(\frac{\eta}{E_c}\sqrt{a+2ib}+\text{c. c}\right)}{\cos\left(\frac{\eta}{E_c}\sqrt{a+2ib}-\text{c. c}\right)} + E_c^2 - \frac{\eta^2}{3}a\right]d\eta, \qquad (1)$$

where $a = E^2 - H^2$, $b = \mathbf{E}\cdot\mathbf{H}$ are the invariants of the radiation field. Here we are dealing with slowly varying fields $\mathbf{F}(\mathbf{E}, \mathbf{H})$, for which $(\hbar/mc)|\operatorname{grad}\mathbf{F}| \ll |\mathbf{F}|$ and $(\hbar/mc^2)|\partial\mathbf{F}/\partial t| \ll |\mathbf{F}|$, which is always fulfilled for laser (optical) radiation. In this case the Lagrangian function (1) depends on the values of the field invariants at the given instant of time and coincides with the Lagrangian function of a constant field with instantaneous values of the intensities. In a special coordinate system in which the intensity vectors of the electric and magnetic fields $\mathbf{E}'$ and $\mathbf{H}'$ are parallel, expression (1) can be transformed to the form

$$L_1 = -\frac{m^4c^5}{8\pi^2\hbar^3}\int_0^\infty \frac{e^{-\eta}}{\eta^3}\left[\frac{E'}{E_c}\eta\operatorname{ctg}\left(\frac{E'}{E_c}\eta\right)\eta\frac{H'}{E_c}\operatorname{cth}\left(\frac{H'}{E_c}\eta\right) - 1 + \frac{\eta^2}{3E_c^2}({E'}^2 - {H'}^2)\right]d\eta, \tag{2}$$

where

$$E' = \left\{{}^1/_2\left(a + \sqrt{a^2 + 4b^2}\right)\right\}^{1/2}, \qquad H' = b/E'. \tag{3}$$

The integral (2), representing the Lagrangian function of a slowly varying field, contains singularities at $(E'/E_c)\eta = \pi n$ $(n = 1, 2, ...)$,

* Qualitatively this question was discussed in ([1]).

if only the conditions $a < 0,\ b = 0$, requiring the presence of a single magnetic field in some special coordinate system, are not satisfied. The existence of singularities in (2) is an analytic expression of the fact that, in a system initially in the vacuum state, pairs of particles are produced. J. Schwinger([3]) showed that the probability that the system will remain in the vacuum state, i.e., that real pair creation will not occur, is equal to $\left|\exp\frac{i}{\hbar}W\right|^2 = \exp\left(-2\ \mathrm{Im}\,\frac{W}{\hbar}\right)$, where $W = \int L_1 d^4x$ is the action integral, with the integration path in (1) lying above the real axis. Therefore the number of pairs produced in a volume element ΔV during a time Δt is determined by twice the imaginary part of the Lagrangian function, $2\hbar^{-1}\,\mathrm{Im}(L_1\Delta V\Delta t)$. Hence, on the basis of (2), for the number n of electron-positron pairs produced in an electromagnetic field per unit volume per unit time, we obtain the expression*

$$n = \frac{mc^2}{4\pi^3\hbar\lambda_c^3}\left(\frac{E'}{E_c}\right)^2\sum_{n=1}^\infty\frac{1}{n^2}\left(n\pi\frac{H'}{E'}\right)\operatorname{cth}\left(n\pi\frac{H'}{E'}\right)\exp\left(-\frac{n\pi E_c}{E'}\right), \tag{4}$$

$\lambda_c = \hbar/mc$. In what follows we shall confine ourselves to consideration of the first term of the sum (4). We note that in the case of a plane wave $(a = b = 0)$, for arbitrary intensity and spectral composition, production of particle pairs is impossible.

3. The strengths of the electric and magnetic fields $\mathbf{E}(P,t) = \mathrm{Re}[\mathbf{e}(P)e^{-i\omega t}]$, $\mathbf{H}(P,t) = \mathrm{Re}[\mathbf{h}(P)e^{-i\omega t}]$ at a point P located near the focus of an ideal lens are determined by the relations([8])

$$e_x(P) = -iA(I_0 + I_2 \cos 2\varphi), \quad e_y(P) = -iAI_2 \sin 2\varphi,$$
$$e_z(P) = -2AI_1 \cos\varphi,$$
$$h_x(P) = -iAI_2 \sin 2\varphi, \qquad h_y(P) = -iA(I_0 - I_2 \cos 2\varphi),$$
$$h_z(P) = -2AI_1 \sin\varphi,$$

where $A = kfE_0$; E_0 is the amplitude of the electric-field strength in the incident plane wave; f is the focal length of the lens; $k = \omega/c$;

$$I_\mu(k\rho, kz, \theta_0) = 4\int_0^{\theta_0} \cos^{1/2}\theta \, \sin^{\mu+1}\left(\frac{\theta}{2}\right) \cos^{3-\mu}\left(\frac{\theta}{2}\right) J_\mu(k\rho \sin\theta) e^{ikz\cos\theta}\, d\theta; \tag{5}$$

J_μ are Bessel functions ($\mu = 0, 1, 2$); $\theta_0 = \arctan(d/2f)$; d is the diameter of the lens; (ρ, z, φ) are the coordinates of the point in a cylindrical coordinate system with its center at the focus and with polar axis coinciding with the optical axis of the lens. We have

$$a = A^2 \cos 2\varphi \left[(I_1^2 - I_0 I_2)e^{-2i\omega t} + \text{c.c.} + 2|I_1|^2 + 2\,\mathrm{Re}(I_0 I_2^*)\right] = A^2 \delta \cos 2\varphi, \tag{6}$$

$$b = \frac{1}{2} A^2 \sin 2\varphi \left[(I_1^2 - I_0 I_2)e^{-2i\omega t} + \text{c.c.} + 2|I_1|^2 + 2\,\mathrm{Re}(I_0 I_2^*)\right] = \frac{1}{2} A^2 \delta \sin 2\varphi,$$

where

$$\delta = \alpha \cos(2\omega t + \gamma) + \beta, \qquad \alpha = 2|I_1^2 - I_0 I_2|, \qquad \beta = 2|I_1|^2 + 2\,\mathrm{Re}(I_0 I_2^*). \tag{7}$$

* It can be shown([7]) that in the case of a constant homogeneous electric field E, acting over a length $l \gg 2mc^2/eE$, direct calculation of the flux of Dirac vacuum electrons from the lower to the upper energy band leads to a formula coinciding with the first term of the sum (4) at $H' = 0$. This result is closely connected with the so-called paradox of Klein([4,5]). We also note that a problem formally equivalent to the problem of pair creation in a constant homogeneous electric field was considered by A. G. Aronov and G. E. Pikus in studying the tunneling current in semiconductors in a transverse magnetic field([6]); in doing so they used approximate wave functions describing quasistationary states of an electron in the valence band and in the conduction band. The result obtained in([6]) actually coincides (to within replacement of the factor 1/36 appearing in([6]) by $1/4\pi^2$) with the first term of the infinite sum (4) at $H' = 0$.

γ-phase. Hence, according to (3), we obtain

$$E' = A\sqrt{\delta}\,|\cos\varphi| \quad \text{for } \delta > 0; \qquad E' = A\sqrt{-\delta}\,|\sin\varphi| \quad \text{for } \delta < 0. \tag{8}$$

The number of pairs, averaged over the field period, produced per unit time in the volume V of the focus has the form*

$$N = \frac{mc^2\omega}{8\pi^3\hbar\lambda_c^3}\left(\frac{A}{E_c}\right)^2 \int_V \alpha\sin 2\varphi\ \mathrm{cth}(\pi\,\mathrm{tg}\,\varphi) \int_0^{\pi/\omega} \left|\cos 2\omega t + \frac{\beta}{\alpha}\right| \times$$

$$\times \exp\left(-\lambda \Bigg/ \sqrt{\left|\cos 2\omega t + \frac{\beta}{\alpha}\right|}\right)\, dt\,\rho\,d\rho\,dz\,d\varphi,$$

where $\lambda = \pi E_c/(A\sqrt{\alpha}\,|\cos\varphi|)$, $\alpha = \alpha(\rho, z)$, $\beta = \beta(\rho, z)$. Successive asymptotic estimates of the time integral for $\lambda \gg 1$ and of the integral over φ for $\pi E_c/A\sqrt{\alpha \pm |\beta|} \gg 1$ lead to the formula

$$N = \frac{mc^2}{\sqrt{2}\,\pi^4\hbar\lambda_c^3}\left(\frac{A}{E_c}\right)^3 \int_{-\infty}^{\infty}\int_0^{\infty} \alpha^{-1/2}\left[(\alpha + |\beta|)^2 \exp\left(-\frac{\pi E_c}{A\sqrt{\alpha + |\beta|}}\right) + \right.$$

$$\left. +(\alpha - |\beta|)^2 \exp\left(-\frac{\pi E_c}{A\sqrt{\alpha - |\beta|}}\right)\right]\rho\,d\rho\,dz. \tag{9}$$

For $\beta \neq 0$ the main contribution to the integral is made by the first term in the integrand (for $|\beta| \geq \alpha$ the second term should be set equal to zero).

To calculate the last integral it is necessary to know the explicit form of the functions $\alpha = \alpha(\rho, z)$ and $\beta = \beta(\rho, z)$. The integrals (5) in the particular case $z = 0$ were considered in works (8–11) in the limit of small angles θ_0 and for $\theta_0 \to 90^\circ$.

In (12) the results of numerical integration are presented for the case $\theta_0 = 45^\circ$. The value of the integral (9) is determined, evidently, by the maxima of the function $\alpha + |\beta|$. Let us consider the integrals (5) for small z (obviously, the plane $z = 0$ is the plane of extremal values of $\alpha + |\beta|$). Passing in (5) to a new integration variable $u = \sin\theta$ ($u_0 = \sin\theta_0$),

$$I_\mu(k\rho, kz, u_0) = 2\int_0^{u_0} f_\mu(u^2) J_\mu(k\rho u) \exp\left(ikz\sqrt{1-u^2}\right) u^{\mu+1}\,du \quad (\mu = 0, 1, 2), \tag{10}$$

where

$$f_{00}(\xi) = \frac{1}{2}\left[(1-\xi)^{-1/4} + (1-\xi)^{1/4}\right], \qquad f_{10}(\xi) = \frac{1}{2}(1-\xi)^{-1/4},$$

$$f_{20}(\xi) = (2\xi)^{-1}\left[(1-\xi)^{-1/4} - (1-\xi)^{1/4}\right].$$

Expanding the exponential in a series,

$$\exp\left(ikz\sqrt{1-u^2}\right) = 1 + ik(1-u^2)^{1/2}z - \frac{1}{2}k^2(1-u^2)z^2 + \cdots,$$

we obtain

$$I_\mu(k\rho, kz) = I_{\mu0}(k\rho) + iI_{\mu1}(k\rho)z - I_{\mu2}(k\rho)z^2 + \cdots \tag{11}$$

$$I_{\mu\nu}(k\rho) = 2\left(\frac{k}{2}\right)^\nu \int_0^{u_0} f_{\mu\nu}(u^2)J_\mu(k\rho u)u^{\mu+1}\,du \quad (\mu = 0,1,2;\ \nu = 1,2), \tag{12}$$

where $f_{\mu\nu}(u^2) = 2(1-u^2)^{\nu/2}f_{\mu0}(u^2)$. Representing further the functions $f_{\mu\nu}(u^2)$ by Taylor series in powers of $(u_0^2 - u^2)$,

$$f_{\mu\nu}(\xi) = \sum_{n=0}^{\infty} \frac{(-1)^n f_{\mu\nu}^{(n)}(\xi_0)(\xi_0-\xi)^n}{n!},$$

where $\xi = u^2$, $f_{\mu\nu}^{(r)}(\xi) = d^n f_{\mu\nu}(\xi)/d\xi^n$, $\xi_0^2 = u_0^2$, we obtain

$$I_{\mu\nu}(k\rho) = \left(\frac{k}{2}\right)^\nu \left(\frac{k\rho}{2}\right)^\mu \sum_{n=0}^{\infty}(-1)^n \frac{\Lambda_{n+\mu+1}(k\rho u_0)}{2^{\mu+\nu+1}(n+\mu+1)!} f_{\mu\nu}^{(n)}(u_0^2) \quad (\mu,\nu = 0,1,2), \tag{13}$$

* If $|\beta| > \alpha$ (it will be shown below that this condition is satisfied for the case of interest to us), then the electric-field strength E' is described by one of formulas (8). For $|\beta| < \alpha$, one part of the period E' is described by one of expressions (8), and another part by the other. However, in view of the averaging over the angle φ, for E' we may use one expression throughout the entire period.

where $\Lambda_n(z) = 2^n n! z^{-n} J_n(z)$. For $u_0 \le \sqrt{2}/2$ ($\theta_0 \le 45°$) the series (13) converge sufficiently rapidly. In the vicinity of the focal plane one may confine oneself to the first terms of the expansion of the function $\alpha + |\beta|$ in powers of z. From formulas (7) and (11) we obtain

$$\alpha + |\beta| = 4I_{10}^2 + 4\left(I_{01}I_{21} - 2I_{10}I_{12}\right)z^2 + \ldots \tag{14}$$

4. Let us consider the case $\theta_0 = 45°$. By calculating the coefficient of z^2 in (14) one can establish that the function $\alpha + |\beta|$ at $z = 0$ assumes maximum values. Using tables of the functions $\Lambda_n(z)$, from formula (13) we obtain $\max I_{10}(k\rho) = I_{10}(3.18) = 0.0698$. Thus, the main contribution to the integral (9) for $(\pi E_c/A) \gg 1$ is determined by the behavior of the function $\alpha + |\beta|$ in the vicinity of the maximum point ($k\rho_0 = 3.18;\ z = 0$). Applying the method of asymptotic estimates with successive integration of (9) with respect to ρ and z, we obtain

$$N = \frac{2mc^2\sqrt{2}}{\pi^4 \hbar \lambda_c^3} \left(\frac{A}{E_c}\right)^4 \frac{\rho_0}{\sqrt{\alpha}} (\alpha + |\beta|)^{7/2} \left[\frac{\partial^2(\alpha + |\beta|)}{\partial z^2} \frac{\partial^2(\alpha + |\beta|)}{\partial \rho^2}\right]^{-1/2} \times$$

$$\times \exp\left(-\frac{\pi E_c}{A\sqrt{\alpha + |\beta|}}\right), \tag{15}$$

where the values of the functions α, $\alpha + |\beta|$, and of the derivatives of $\alpha + |\beta|$ at the point ($k\rho_0 = 3.18;\ z = 0$) are understood. Using (14) and (13), we find $\partial^2(\alpha + |\beta|)/\partial z^2|_{(\rho_0,0)} = -0.0102k^2$; $\partial^2(\alpha + |\beta|)/\partial \rho^2|_{(\rho_0,0)} = 8I_{10} d^2 I_{10}/d\rho^2$. The derivative $d^2 I_{10}/d\rho^2$ is calculated by differentiating integral (12) with respect to the parameter, followed by expansion in a series, $d^2 I_{10}/d\rho^2|_{z=0} = -0.0156k^2$.

Substituting these values into formula (15), we obtain that, for the creation of one pair in vacuum when a laser-radiation pulse of duration 10^{-11} sec and wavelength $\lambda = 6900$ Å (ruby laser) passes through an ideal lens, the field strength in the incident plane wave must satisfy the relation $(E_c/kfE_0) = 2.9$, and the corresponding total laser power must be equal to

$$P = \pi f^2 \frac{cE_0^2}{8\pi} = \frac{cE_c^2}{8(2.9)^2 k^2} \simeq 10^{19} \text{ W}. \tag{16}$$

Such powers, of course, cannot yet be realized. However, in the growth of the powers of generation of coherent optical radiation by the methods of quantum electronics, “saturation” has not yet occurred. The transition from the nanosecond range of laser-pulse durations to the picosecond range has led to a jump in the possible generation powers by $4 \div 5$ orders of magnitude, and at present one can realistically hope to obtain, in the visible part of the spectrum, powers of 10^{14} W (for a duration of 10^{-12} sec). Further growth of the generation power seems possible along the path of transition to a shorter-wavelength range; in this connection, incidentally, according to (16), the pair-production threshold itself is also lowered ($P \sim \lambda^2$).

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Note: Figure translations are in progress. See original paper for figures.