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Abstract

Full Text

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*Cybernetics
and Control Theory*

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ON THE QUESTION OF THE EFFICIENCY OF LARGE SYSTEMS

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Let us consider a macromodel of an information-control system C, consisting of a controlled object O and a controlling system Y, connected with one another by a flow of control information i (Fig. 1). We shall regard O as belonging to the class of large systems in the sense of the classification in ([1,2]). The belonging of Y to the same class will be considered a consequence of the necessary similarity of Y and O.

Let us define: (I) The goal for O is to obtain a positive effect ε^+ of some nature (the release of a hidden resource of the object or the achievement of the required behavior) by consuming the flow i. (II) The goal for Y is the processing of the flow i through the consumption of the cost ε^- (negative effect) for its creation and functioning. (III) The goal for C (the goal of control) is to obtain the positive resultant effect

Fig. 1. Macromodel of an information-control system

$$\varepsilon = \varepsilon^+ - \varepsilon^- > 0 \tag{1}$$

by applying Y to O. As a measure of efficiency for C, one may take the quantity ε or the ratio $\varepsilon^+/\varepsilon^{-*}$.

Let us investigate the general character of the influence of constraints in O and Y on the achievable values of the efficiency of C ([2]). For this purpose we construct static characteristics of the controlled object X_O, the controlling system X_Y, and the control system as a whole X_C in generalized "macrocoordinates" i and

ε. Since O "consumes" i and "releases" ε^+, we construct its characteristic as $X_O = \varepsilon^+(i)$. Conversely, Y "consumes" ε^- and "releases," processes i, and therefore $X_Y = i(\varepsilon^-)$.

In the chosen coordinates O can be characterized by two basic macroparameters: E—the potential effect contained in O; I—the informational complexity of O (the required magnitude of the flow i for releasing E). Therefore the idealized X_O may be represented by a step function (Fig. 2A), where segment a depicts the zone of initial "insensitivity"(the flow is insufficient and O is not yet controllable), and c the zone of "saturation" (the effect does not increase, since the resources of O have already been exhausted). We shall say that the macroparameters E and I constrain the region of admissible behavior of the object

$$D_0\{\varepsilon^+ \leq E,\ i \geq I\}. \tag{2}$$

The macroparameters of Y are: P—the potential capacity or productivity of Y (the maximum i that it is capable of processing); S—the costs for Y, characterizing its complexity. The idealized X_Y will likewise be represented by a step function (Fig. 2B), where segment d depicts

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* Some assumptions: 1) We suppose that ε^+ and ε^- are scalars of the same nature (or that a transformation exists making them comparable). 2) We do not distinguish direct and feedback flows from the general circulating flow i. 3) We do not consider the behavior of C in time and the strategy for achieving $\varepsilon > 0$. 4) If O already contains controlling subsystems and functions with some ε_0, we refer them to the initial reasoning and to the supplementary level of control for achieving an additional effect.

the still inactive (unfinished) Y, and f—the "overload" zone, when the capabilities of a given Y, under the chosen principles of its construction, have already been exhausted and its productivity no longer increases, regardless of any expenditures. The macroparameters S and P delimit the region of admissible control of the controlling system

$$D_Y\{\varepsilon^- \geq S,\ i \leq P\}. \tag{3}$$

C is the result of coupling Y with O, and as its static efficiency characteristic we shall take the resulting effect ε as a function of i: $X_C = \varepsilon(i)$, which can be represented as the difference between X_O and the inverse X_Y: $X_C = X_O - \overline{X}_Y = \varepsilon^+(i) - \varepsilon^-(i)$. From (2), (3), and (1) it follows that the possibility of attaining the control goal exists only for such a combination of the parameters O and Y for which $E > S$ and $P > I$, i.e., when the intersection of the regions D_O and D_Y gives a nonempty region D_C of purposeful behavior C. Figs. 2B, 2 show that the control goal specified for O can be achieved only by applying Y with characteristic $\overline{X}_{Y3}$.

Fig. 2. Idealized (stepwise) static characteristics

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We transform the idealized X_O and X_Y into more realistic X'_O and X'_Y (Fig. 3). The slope of segment b in X'_O indicates the presence of a controllability range of the object, when an increase in i leads to an increase in ε^+. Segment d in X'_Y represents the initial expenditures for creating the minimal core of the controlling system capable of functioning; the slope of segment e is associated with the expenditures for expanding the core, leading to an increase in i, and with operating expenditures. The principal macroparameters for O and Y are transformed and detailed accordingly, and new ones may be introduced. For example, for O: the controllability threshold I_n and the saturation threshold I_v, as well as derived parameters: the mean flow $I_\text{sr} = (I_\text{v} + I_\text{n})/2$, the controllability range $2j = I_\text{v} - I_\text{n}$, the maximum relative controllability $\alpha = E/I_\text{v}$, and the differential controllability $\beta = E/(I_\text{v} - I_\text{n}) = E/2j$. For Y: initial expenditures S_n, saturating expenditures S_v, mean expenditures $S_\text{sr} = (S_\text{v} + S_\text{n})/2$, development range $2c = S_\text{v} - S_\text{n}$, maximum relative capacity $\mu = P/S_\text{v}$, and differential capacity $\nu = P/(S_\text{v} - S_\text{n}) = P/2c$.

Fig. 3. Detailed (trapezoidal and logistic) characteristics. A $-\operatorname{tg}\varphi = \alpha,\ \operatorname{tg}\psi = \beta$; B $-\operatorname{tg}\xi = \mu,\ \operatorname{tg}\theta = \nu$

For further detailing of the characteristics (the curves X''_o and X''_y in Fig. 3) we shall adopt the hypothesis that large systems behave in accordance with the "logistic" curve $(1, 3)$, whose equation in the coordinates (i, ε^+) has the form $\varepsilon^+(i) = \lambda/[1 + \gamma \exp(-\varkappa i)]$. Expressing its coefficients through the parameters E, I_cp, j, we obtain the logistic characteristic of the object

$$X_o = E/\{1 + \exp[2(I_\text{cp} - i)/j]\}.$$

Similarly, for y in the coordinates (ε^-, i), with parameters P, S_cp, c:

$$X_y = P/\{1 + \exp[2(S_\text{cp} - \varepsilon^-)/c]\}.$$

Hence the characteristic of system efficiency is:

$$X_c = X_o - \overline{X}_y = E/\{1 + \exp[2(I_\text{cp} - i)/j]\} - S_\text{cp} + (c/2)\ln(P/i - 1)\ \ast.$$

Let us turn to the interpretation of the results. In Fig. 4, from the shifted X_o and $\overline{X}_y$, X_c is constructed for the case of the correct combination of the parameters O and (the unsloped D_C). The resulting difference between the direct and inverse logistic characteristics demonstrates a certain general law of behavior of large systems.

The segments of X_c correspond to: A—the range of system underloading (the influence of initial costs and the threshold of controllability; preparation losses);

–the range of effective functioning of a developed C (the region of revenues, profitable management with the maximum effect at point M; consistency of the parameters O and); B–the range of overload and failure (the region of aging of C, exhaustion of the resources O and of the capacity ; losses from overstrain, the need for radical changes in the system).

Fig. 4. Logistic characteristic of the efficiency of a control system

To construct a dynamic macromodel of a large system, it is necessary to add the time coordinate. In the general case, the parameters considered for O and , and consequently also the form of X_c, become functions of time **. However, despite the dynamic transformation, the proposed X_c makes it possible to explain some cases of the behavior of large systems. For example:

I. Houm' s graph of profits and losses (5), depicting the economic efficiency of using computers to manage an economic object, contains successive (over time) phases of losses, profits, and again losses. The form of this graph can be explained as the dynamic generation of the static X_c, in connection with the growth over time of i and the passage through the ranges A, , and B (Fig. 4).

II. It appears possible to give additional justification for, or to refine, some conclusions from (4), based on an exponential model–

* Let us note that the left infinite branch of the logistic curve leads to an inaccurate representation of X_o, and especially X_y and X_c, in the region of small i. Usually the behavior of C in this region is not of particular interest. Otherwise, to represent X_y one may use a shifted logistic curve that intersects the ε^- axis at the point S_n.

** To construct them it is necessary to adopt certain methods for reducing the effect or costs to specified moments in time (4), and also to take into account the influence over time of the factors of "ontogenesis" (the development of the given specific system) and "phylogenesis" (the development of the given type of systems, for example the technical progress of an industry, which changes the criteria for evaluating the given system).

or O and a linear model Y: a) conclusion 1, concerning a decrease in additional efficiency and an increase in the payback period of the system as Y becomes more complicated and i increases, becomes valid only beyond the point M of the characteristic X_C; b) conclusion 2, concerning the need for accelerated execution of the first steps of automation, is supported by the need for rapidly traversing the range A; c) conclusion 3, concerning the economic advisability of leaving in O a share of disorder, is justified by the fact that eliminating residual disorder requires an increase in i, and this is unprofitable beginning with i_M.

III. Analysis of X_C makes it possible to indicate the direction of progress P for X_Y (Fig. 4), which is consistent with the second universal criterion of computer efficiency (6) (the minimum cost of effective speed, or the maximum ratio of productivity to processing cost). It becomes possible to derive this criterion (it coincides with $\mu = \max i/\varepsilon^-$) for various points of the characteristic X_Y, and also to bring the other parameters considered above to bear in assessing the quality of a computer and its suitability for controlling a given O.

For example: a) Fig. 2B shows the insufficiency of evaluating a control system (computer) by only the second universal criterion, whose value for systems with characteristics $\overline{X}_{Y1}$, $\overline{X}_{Y3}$, and $\overline{X}_{Y4}$ proves to be the same, but only the system with characteristic $\overline{X}_{Y3}$ is effective as applied to the given O. b) A qualitative assessment is possible of the transition to a more developed core of a computer operating system, which, despite an increase in initial costs (ΔS in Fig. 4) and a decrease in the limiting productivity ($-\Delta P$) spent on additional intrasystem needs, leads to an increase in capacities ($\Delta\nu$ and $\Delta\mu$) and ultimately to $\Delta\varepsilon_M$. c) It may be supposed that the empirical "Grosch' s law" known among computer developers (computer productivity grows in proportion to the square of its cost) is a consequence of the logistic character of the development ("phylogeny") of computers and acts temporarily, while development is represented by a segment of the logistic curve X_Y with rapid growth of $di/d\varepsilon^{-}$ under the existing structural and physical principles of computer construction. A revolution in these principles may be accompanied by a transition to a new logistic characteristic and by a resumption of the action of Grosch' s law.

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Note: Figure translations are in progress. See original paper for figures.