A NEW SOLUTION OF THE DIFFERENTIAL EQUATIONS OF MOTION OF A BODY HAVING A FIXED POINT, UNDER THE CONDITIONS OF S. V. KOVALEVSKAYA

MECHANICS

1969

SovietRxiv

Abstract

Full Text

UDC 531.38

MECHANICS

E. I. KHARLAMOVA, P. V. KHARLAMOV

A NEW SOLUTION OF THE DIFFERENTIAL EQUATIONS OF MOTION OF A BODY HAVING A FIXED POINT, UNDER THE CONDITIONS OF S. V. KOVALEVSKAYA

(Presented by Academician P. Ya. Kochina on 16 V 1969)

The problem of the motion of a body having a fixed point is reduced to the integration of the system of differential equations

$$\begin{aligned} A\,dp/dt &= (B-C)qr + \lambda_2 r - \lambda_3 q + \gamma'' e_2 - \gamma' e_3, \\ B\,dq/dt &= (C-A)rp + \lambda_3 p - \lambda_1 r + \gamma e_3 - \gamma'' e_1, \\ C\,dr/dt &= (A-B)pq + \lambda_1 q - \lambda_2 p + \gamma' e_1 - \gamma e_2, \end{aligned} \tag{1}$$

$$d\gamma/dt = r\gamma' - q\gamma'', \qquad d\gamma'/dt = p\gamma'' - r\gamma, \qquad d\gamma''/dt = q\gamma - p\gamma'. \tag{2}$$

Here p, q, r are the angular velocity of the body; $\gamma, \gamma', \gamma''$ is the vector indicating the direction of the force of gravity (its modulus Γ is equal to the product of the weight of the body by the distance of the center of mass of the body from the fixed point); $\lambda_1, \lambda_2, \lambda_3$ is the gyrostatic moment, e_1, e_2, e_3 is the unit vector of the ray going from the fixed point through the center of mass; A, B, C are the principal axial moments of inertia of the body.

In the absence of gyrostatic moment the problem has several particular solutions, some of which also extend to the case when the gyrostatic moment is different from zero. Thus, N. E. Zhukovskii (^{1}) and later V. Volterra (^{2}) obtained a solution generalizing the case of integrability belonging to L. Euler; L. N. Sretenskii (^{3}) obtained solutions, particular cases of which are the solutions of S. A. Chaplygin–D. N. Goryachev (^{4}) and V. Hess (^{5}). Generalizations of other solutions are given in works ($^{6-9}$) and others. But up to the present time no solutions have been found under the conditions of S. V. Kovalevskaya and with nonzero gyrostatic moment.

The solution of the system (1), (2) presented in this communication was obtained under the conditions

$$A = C = 2B, \qquad e_2 = e_3 = 0 \qquad (e_1 = 1).$$

The gyrostatic moment is orthogonal to the plane of the circular section of the ellipsoid of inertia

$$\lambda_1 = \lambda_3 = 0.$$

Under these assumptions equations (1) take the form

$$A\,dp/dt = -Bqr + \lambda_2 r, \qquad B\,dq/dt = -\gamma'', \qquad C\,dr/dt = Bpq - \lambda_2 p + \gamma', \quad (3)$$

and the system (2), (3) has the solution

$$Ap = c\cos\varepsilon + m\cos\sigma, \qquad Bq\,\frac{c}{2}\sin\varepsilon + n\sin\sigma,$$

$$\frac{n^2 - m^2}{m^2}\,C^2r^2 = -(n^2 - m^2)\cos^2\sigma + 2cm\cos\varepsilon\cos\sigma + 2cn\sin\varepsilon\sin\sigma + c^2,$$

$$\frac{n^2 - m^2}{n^2}\,A\gamma = -(n^2 - m^2)\cos^2\sigma + cm\cos\varepsilon\cos\sigma + cn\sin\varepsilon\sin\sigma,$$

$$2\,\frac{n^2 - m^2}{n^2}\,By' = cm\sin\varepsilon\cos\sigma - cn\cos\varepsilon\sin\sigma,$$

$$m\gamma'' = -n^2 r\cos\sigma.$$

The dependence of the auxiliary variable σ on time is established by inverting the integral

$$t = \frac{m}{n}\int_{\sigma_0}^{\sigma}\frac{d\sigma}{r(\sigma)}.$$

This solution contains 7 independent parameters $A, c, m, n, \varepsilon, \sigma_0, \alpha_0$ (the last appears in constructing the fixed hodograph of the angular velocity $^{(10)}$). Two of them can be expressed in terms of the quantities Γ and λ_2 from the relations

$$A\Gamma = \left|\frac{cn^3}{n^2 - m^2}\right|, \qquad \lambda_2 = \frac{c}{2}\sin\varepsilon.$$

Donetsk Computing Center
of the Academy of Sciences of the Ukrainian SSR

Received
11 V 1969

CITED LITERATURE

[1] N. E. Zhukovskii, *On the motion of a rigid body having cavities filled with a homogeneous drop liquid*, Works, 3, Moscow, 1936.
[2] V. Volterra, Acta Math., 22 (1899).
[3] L. N. Sretenskii, DAN, 149, No. 2 (1963).
[4] S. A. Chaplygin, *A new case of rotation of a heavy rigid body supported at one point*, Works, 1, Moscow, 1948.
[5] W. Hess, Math. Ann., 37, 2 (1890).
[6] P. V. Kharlamov, DAN, 154, No. 2 (1964).
[7] P. V. Kharlamov, PMM, 29, 2 (1965).
[8] E. I. Kharlamova, Collection *Mechanics of a Rigid Body*, issue 1, Kiev, 1969.
[9] E. I. Kharlamova, PMM, 30, 4 (1966).
[10] P. V. Kharlamov, PMM, 28, 3 (1964).

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.