ON THE FORMULATION OF THE PROBLEM OF THE INTERACTION WITH THE GROUND OF A ROLLING WHEEL WITH AN ELASTIC TIRE DURING ITS OSCILLATIONS

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ON THE FORMULATION OF THE PROBLEM OF THE INTERACTION WITH THE GROUND OF A ROLLING WHEEL WITH AN ELASTIC TIRE DURING ITS OSCILLATIONS

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The equations relating the oscillations of a rolling wheel with an elastic tire and the reaction on the tire from the ground caused by these oscillations are usually drawn up under the following assumptions (^{1}):

1. The tire is considered as an elastic body having a finite number of degrees of freedom.

2. The points of the tire surface in the zone of its contact with the ground during rolling do not slip relative to the ground.

3. Between the components of the tire deformation there exists an additional relation, the form of which is prescribed by different authors in different ways, on the basis of observations of the process of tire rolling.

We shall show that the last assumption is superfluous, since the relation between the components of the tire deformation which is lacking for the construction of a closed system of rolling equations follows from the first two assumptions.

Let us explain this by the example of setting up the equations of rolling of a tire under transverse oscillations of the wheel rim, analogous to the equations obtained by M. V. Keldysh (^{2}). Let the unperturbed motion of the wheel be rectilinear rolling of the latter with constant velocity V under a constant vertical load. Choose a rectangular coordinate system $oxyz$, moving with the same velocity, whose ox axis lies on the ground and is directed opposite to the motion, and whose oy axis is directed normal to the ground (Fig. 1). Place the axis $o'x'$ of the rectangular coordinate system $o'x'y'z$ on the line of intersection of the diametral plane of the undeformable wheel rim with the plane of the ground, and draw the axis $o'y'$ through the center of the wheel. We shall express the perturbed motion of the rim by the displacement $\delta(t)$ of the point

Fig. 1

Figure 1: Fig. 1

o' along the axis oz, where t is time, by the angle $\theta(t)$ between the axes ox and $o'x'$, and by the angle $\psi(t)$ between the axes oy and $o'y'$, taking the quantities δ, θ, and ψ to be small. We shall call the line formed by the points of the tire surface which, in the unperturbed motion, lie in the diametral plane of the rim the middle line. We shall characterize the points of the middle line in the unperturbed motion by the distance s from the point o' along the arc of the middle line, taking positive values of s to be those measured in the direction of the axis $o'x'$. Let the points of the middle li—

Fig. 1

s_1 and s_2 lie, respectively, at the beginning and at the end of the zone of contact of the tire with the ground.

Considering the tire to be an ideally elastic weightless body, let us assume that the reaction on the tire from the ground, caused by the perturbed motion of the rim, is determined by the angle ψ and the displacement $\Lambda(s,t)$ of the tire centerline from the plane $o'x'y'$. We shall represent the tire as a beam connected to the wheel rim through an elastic foundation, the axis of the beam coinciding with the tire centerline. Suppose that the axis of the beam can leave the plane $o'x'y'$ only through shear deformation in the material of the beam and tensile or compressive deformation in the elastic foundation. Suppose, moreover, that rotation of the wheel rim through an angle ψ, in the absence of adhesion of the tire points to the ground surface, produces a displacement $\psi f(s)$ of the centerline from the plane $o'x'y'$. Then the equilibrium equation for the tire centerline can be written in the following form:

$$G\,\partial^2\Lambda/\partial s^2 - k\Lambda = (G\,d^2 f/ds^2 - kf)\psi + q, \qquad (1)$$

where G is the shear stiffness of the beam; k is the stiffness of the foundation connecting the beam with the wheel rim; $q(s,t)$ is the intensity of the lateral load applied to the centerline on the segment within the zone of contact of the tire with the ground.

Represent the function Λ on the segment of the centerline in the zone of contact of the tire with the ground by the sum

$$\Lambda(s,t) = \lambda(t) + s\varphi(t) + s^2\varepsilon(t); \qquad s_1 < s < s_2, \qquad (2)$$

where $\lambda(t)$, $\varphi(t)$, and $\varepsilon(t)$ are the unknown functions. The assumption that, during rolling of the tire, points of the centerline in the contact zone do not slip relative to the ground leads to the following equation, which the function Λ must satisfy:

$$V\,\partial\Lambda/\partial s + \partial\Lambda/\partial t + d\delta/dt + s\,d\theta/dt + V\theta = 0; \qquad s_1 < s_2 < s_2. \tag{3}$$

Let the function (2) satisfy equation (3) in a neighborhood of the point o' to within small quantities of order $O(s^2)$. This imposes the following relations on the functions $\lambda(t)$, $\varphi(t)$, and $\varepsilon(t)$:

$$V\varphi + d\lambda/dt + d\delta/dt + V\theta = 0; \qquad V\varepsilon + d\varphi/dt + d\theta/dt = 0. \tag{4}$$

The lateral force and the moment of the forces about the vertical axis, acting on the tire from the ground, are expressed in terms of the functions λ, φ, ε, and the value of the angle ψ by means of equation (1).

To find an equation supplementing relations (4) between the functions $\lambda(t)$, $\varphi(t)$, and $\varepsilon(t)$, which characterize the deformation of the tire, let us turn to the features of the behavior of the function Λ in neighborhoods of the points s_1 and s_2. From the homogeneity of the adhesion conditions of the tire points with the ground it follows that the function Λ must be continuous at the points s_1 and s_2 and must have, at these points, continuous derivatives with respect to s up to one and the same order. At the same time, the disturbance of the equilibrium position of the tire caused by a change in the load q at the points s_1 and s_2 instantaneously encompasses the entire portion of the centerline free from contact, whereas within the contact zone the disturbance can propagate only from the point s_1 to the point s_2 with velocity V. For this reason, at the point s_1 there is preserved a dependence between the values of the function Λ and its derivatives with respect to s that follows from the equilibrium conditions of the segment of the tire centerline outside the zone of contact with the ground. In our case, from equation (1) it follows that at each instant of time the value of the derivative $\partial\Lambda/\partial s$ to the left of the point $\hat{S}_1$ (Fig. 1) depends linearly on the angle ψ and on the values of Λ to the left of the point $\hat{S}_1$ and to the right of the point $\hat{S}_2$:

$$\partial\Lambda(\hat{S}_1-,t)/\partial s = L\Lambda(\hat{S}_1-,t) + M\Lambda(\hat{S}_2+,t) + N\psi(t), \tag{5}$$

where the constants L, M, and N are determined by the coefficients G and k, as well as

function f. Consequently, at the point $\hat{S}_1$ the function Λ and its first derivative with respect to s remain continuous, while at the point $\hat{S}_2$, where there is no direct connection between the equilibrium conditions of the midline on both sides of the front of the zone of contact of the tire with the ground, only the function Λ remains continuous. Thus, relations (2), (4), and (5) give a closed system of equations for the rolling of the tire.

In order to obtain more accurate equations for the rolling of the tire during transverse oscillations of the wheel rim, the equilibrium conditions of the tire midline can be expressed by a differential equation of higher order, and the condition of adhesion of the midline to the ground can also be imposed more accurately by increasing the number of terms on the right-hand side of expression (2). The equations of tire rolling during longitudinal oscillations of the wheel can be formulated in the same way, if the deformation of the tire is characterized by the displacement of the points of its midline in the direction tangent to the midline of the unperturbed motion from the position occupied by these points in the unperturbed motion.

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5 XI 1968

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Note: Figure translations are in progress. See original paper for figures.