ON THE INVERSE PROBLEM OF MAGNETIC PROSPECTING

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Abstract

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ON THE INVERSE PROBLEM OF MAGNETIC PROSPECTING

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In proving uniqueness theorems and in actually finding solutions of the inverse problem of the Newtonian potential, it is assumed that the density of the disturbing body is given ([1–3]). Since in practice the magnitude of the density is unknown and is chosen on the basis of some extraneous considerations, the correspondence between the solution found and the real body depends on the correctness of the density hypothesis, for different bodies with different densities can create identical external potentials ([4]).

For the case of magnetic masses, the inverse problem can be solved with unknown magnetization if the method of artificial magnetization is applied. In this method one measures the external magnetic field induced by the disturbing body in the presence, outside the body, of a loop of insulated wire with a constant electric current ([5]). What is essential here is that the induced magnetization depends functionally on the relative position of the body and the loop; by measuring the induced field for several positions of the loop and knowing the propagation law of the inducing field, we obtain information on the magnetic properties and the form of the body.

The first proof of the uniqueness theorem for bodies of arbitrary shape and magnetization, for a large number of loop positions, was given by M. M. Lavrent' ev in a report at the First All-Union Conference on Methods for Interpreting Gravitational and Magnetic Anomalies in May 1964, and was later generalized in ([6]). Below a uniqueness theorem is given for a two-dimensional star-shaped body when it is magnetized by the field of an infinitely long ungrounded cable with constant current. It is assumed that the body is filled with a substance of constant magnetic permeability μ; the demagnetization effect is taken into account.

In this case we shall solve the direct problem by the method set forth in ([7]), i.e., the induced potential V at a point $P(x, z)$ outside the body is computed as the potential of a simple layer with unknown density σ, and σ is determined from a linear Fredholm integral equation of the second kind. For convenience, instead of the potential V we shall investigate the analytic function $U = \partial V/\partial x + i\,\partial V/\partial z$.

Let us write the expressions for U and σ in polar coordinates, assuming that the contour of the body Γ is determined uniquely by the radius s and the angle φ:

$$U(l,\alpha,q,\gamma) = \int_0^{2\pi} K(l,\alpha,q,\gamma,s,\varphi)\sigma(\varphi)\,d\varphi,$$

$$\sigma(\varphi) = \lambda \int_0^{2\pi} Q(s,\varphi,\rho,\beta)\sigma(\beta)\,d\beta + \lambda f(q,\gamma,s,\varphi). \quad (1)$$

Here $P(l,\alpha)$ is the observation point; (q,γ) is the trace of the cable in the plane of the drawing; (s,φ) and (ρ,β) are the fixed and current points on the contour of the body; f is the normal component of the intensity of the magnetizing field at the point (s,φ); $\lambda = \dfrac{1}{\pi}\dfrac{\mu-1}{\mu+1}$.

Let the zeroth approximation s_0 and λ_0 be known, i.e.

$$U_0 = \int_0^{2\pi} K_0\sigma_0\,d\varphi, \qquad \sigma_0 = \lambda_0 \int_0^{2\pi} Q_0\sigma_0\,d\beta + \lambda_0 f_0. \quad (2)$$

When s_0 and λ_0 are changed to s_1 and λ_1, with $s_1/s_0 < 1$ and $\lambda_1/\lambda_0 < 1$, U_0 and σ_0 receive the corresponding increments U_1 and σ_1. Expanding the functions K, Q, and f in Taylor series in powers of s_1 and s_1', and neglecting terms of second order of smallness with respect to s_1 and s_1', from (1) and (2) we obtain

$$U_1 = \int_0^{2\pi} \left(\frac{\partial K_0}{\partial s_0}s_1 + \frac{\partial K_0}{\partial s_0'}s_1'\right)\sigma_1\,d\varphi + \int_0^{2\pi} K_0\sigma_1\,d\varphi, \quad (3)$$

$$\sigma_1 = \lambda_1 \int_0^{2\pi} Q_0\sigma_0\,d\beta + \lambda_0 \int_0^{2\pi} \left(\frac{\partial Q_0}{\partial s_0}s_1 + \frac{\partial Q_0}{\partial s_0'}s_1'\right)\sigma_0\,d\beta + \lambda_1 f_0 + \lambda_0\left(\frac{\partial f_0}{\partial s_0}s_1 + \frac{\partial f_0}{\partial s_0'}s_1'\right).$$

Consider the case of a body close to a circular cylinder of radius R. Placing the origin at the center of the circle and setting $s_0 = R = \text{const}$, we find from (3)

$$\sigma_1 = \lambda_1 f_0 + \lambda_0\frac{\partial f_0}{\partial s_0}s, \qquad U_1 = \lambda_1 \int_0^{2\pi} K_0 f_0\,d\varphi + \lambda_0 \int_0^{2\pi} \frac{\partial}{\partial R}(K_0 f_0)s_1\,d\varphi. \quad (4)$$

Under the assumptions made, the following holds.

Theorem. *Let a star-shaped body, close in form to a circular cylinder and filled with a substance of constant magnetic permeability μ, be magnetized by the field* *of a steady electric current in an ungrounded cable located outside the body. Then, from the induced external magnetic potentials V, specified for two positions of the cable, the contour of the body and the permeability μ are determined uniquely.*

Proof. Since the density σ is "carried" by the linearization onto the circle of radius R, U_1 has no singularities outside this circle. Continuing U_1 analytically up to the boundary of the circle Γ, i.e. letting l tend to R, we obtain a singular integral equation with a singularity at $\varphi = \alpha$

$$\overline{U}_1 = \lim_{l\to R} U_1 = \lambda_1 \int_0^{2\pi} \overline{K}_0 f_0 \, d\alpha + \lambda_0 \int_0^{2\pi} \frac{\partial}{\partial R} (\overline{K}_0 f_0) s_1 \, d\alpha, \tag{5}$$

where

$$\overline{K}_0 = \lim_{l\to R} K_1 = \frac{e^{-i(\varphi+\alpha)/2}}{2\sin(\varphi-\alpha)/2}.$$

Here both integrals exist in the sense of the Cauchy principal value. We transform (5) by setting $\tau = Re^{i\varphi}$ and $t = Re^{i\alpha}$:

$$U_1(t) = \int_\Gamma \frac{w(\tau)}{\tau - t} \, d\tau, \tag{6}$$

where

$$w(\tau) = \left(\lambda_1 f_0 + \lambda_0 \frac{\partial f_0}{\partial R} s_1\right) e^{-i\varphi}. \tag{7}$$

Inverting the singular integral (6) according to ([8]) and taking (7) into account, we obtain

$$\lambda_1 f_0(\psi) + \lambda_0 \frac{\partial f_0(\psi)}{\partial R} s_1 = -\frac{1}{\pi^2} \int_0^{2\pi} \frac{e^{i(\psi-\alpha)/2}}{2\sin(\psi-\alpha)/2} \overline{U}_1(\alpha) \, d\alpha = B(\psi), \tag{8}$$

where $B(\psi)$ is a known function at the point (R, ψ) on the circle Γ.

To determine the unknowns λ_1 and s_1, we compose a second equation of type (8) for another position of the source of the magnetizing field and solve

system

$$\begin{aligned} \lambda_1 f_{0\mathrm{I}} + \lambda_0 \frac{\partial f_{0\mathrm{I}}}{\partial R} s_1 &= B_{\mathrm{I}}, \\ \lambda_1 f_{0\mathrm{II}} + \lambda_0 \frac{\partial f_{0\mathrm{II}}}{\partial R} s_1 &= B_{\mathrm{II}}, \end{aligned} \tag{9}$$

where the indices I and II correspond to the first and second positions of the cable, whose coordinates are $(x_{0\mathrm{I}}, h)$ and $(x_{0\mathrm{II}}, h)$.

The determinant D of system (9) for the case of a body close to a cylinder is not identically zero. Indeed,

$$D = (h^2 - R^2 - x_{0\mathrm{I}} x_{0\mathrm{II}}) \sin\psi - (x_{0\mathrm{I}} + x_{0\mathrm{II}})(h\cos\psi - R).$$

Without loss of generality, put $x_{0\mathrm{I}} = -x_{0\mathrm{II}}$. Since $h > R$ (the cable is located outside the body), we have

$$D = (h^2 - R^2)\sin\psi \neq 0, \qquad \text{if } \psi \neq 0 \text{ and } \psi \neq \pi.$$

Thus, from system (9) we uniquely determine the unknown constant λ_1 and the unknown function s_1. The zero approximation for bodies close to cylinders may be chosen by the formula $R = \sqrt{x_0^2 + h^2 - \Delta^2}$, where Δ is the distance between the cable and the nearest point at which $\partial v/\partial x = 0$. To determine the coordinates of the cable trace (x_0, h), it is sufficient to find the center of the cylinder (the origin of coordinates), which is computed uniquely from the external potential. This completes the proof of the theorem.

Remark 1. The method for solving the direct problem of the artificial magnetization method may be applied to the computation of inductive magnetic fields from homogeneous bodies magnetized by a constant magnetic field or by quasistationary daily magnetic variations of the Earth.

Remark 2. The numerical solution of integral equation (1) in the direct problem of the artificial magnetization method should be carried out by the P-step method of steepest descent ([9]), since the norm of the operator with kernel λQ may be close to 1, or even greater than 1, for the case of nonconvex bodies.

Remark 3. The numerical solution of the inverse problem is equivalent to solving an integral equation of the first kind, which can be carried out with the aid of regularization methods developed by A. N. Tikhonov ([10]).

In conclusion, I take this opportunity to express my gratitude to M. M. Lavrentiev for his constant attention to the work.

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Note: Figure translations are in progress. See original paper for figures.