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Abstract

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AN ITERATIVE PROCESS FOR COMPUTING THE CHARACTERISTIC CRITERION OF A PERIODIC LIMITING REGIME OF MOTION OF A MACHINE UNIT

1. The question of the actual determination of the characteristic criterion $\chi_\xi(\varphi)$ of the periodic limiting regime $T = T_\xi(\varphi)$ of motion of a machine unit is closely connected with the dynamic calculation of the machine and therefore has great theoretical and applied significance ([1–3]).

In the general case, in order to find the characteristic criterion $\chi_\xi(\varphi)$, and to take into account the influence on the machine links of the inertial forces of the initial motion in the sense of N. E. Zhukovskii ([4]), it is necessary to know the periodic limiting regime $T = T_\xi(\varphi)$ of motion of the machine unit. But this latter problem is solvable in quadratures only in rare cases, and therefore the criterion $\chi_\xi(\varphi)$, generally speaking, is not computed in closed form.

In the present note a uniformly convergent iterative process is constructed, allowing one to compute the characteristic criterion $\chi_\xi(\varphi)$ with any degree of accuracy; a convenient relation is given that makes it possible at each step of the iterative process to estimate the error with which the approximation $\chi_k(\varphi)$ reproduces the criterion $\chi_\xi(\varphi)$.

It is assumed that the equation of motion of the machine unit has been reduced to the form

$$dT/d\varphi = M(\varphi, T), \tag{1}$$

where:

1°. The reduced moment $M(\varphi, T)$ of all acting forces is a function defined and continuous in the strip

$$0 \le T \le \hat{T}, \qquad -\infty < \varphi < +\infty, \tag{2}$$

where $\hat{T}$ is the maximum possible value of the kinetic energy of the motion of the machine unit that the acting forces can impart to it.

2°. $M(\varphi, 0) > 0, \quad M(\varphi, \hat{T}) < 0.$

3°. The slope of the reduced moment of all acting forces is continuous and negative in the strip (2), $M_T'(\varphi, T) < 0$.

4°. The reduced moment $M(\varphi, T)$ has period ξ with respect to the angle of rotation φ: $M(\varphi + \xi, T) = M(\varphi, T)$.

The reduced moment of inertia of the masses of all links is regarded as a function of the angle of rotation of the reduction link, $I = I(\varphi)$.

Under the conditions considered, the slope of the moment of all acting forces is bounded below and above by certain negative constants

$$-\lambda_2 \leq M_T'(\varphi, T) \leq -\lambda_1 \qquad (0 < \lambda_1 \leq \lambda_2). \tag{3}$$

Introduce the notation

$$\tau_* = \inf_{0 \leq \varphi < \xi} \tau(\varphi), \qquad \tau^* = \sup_{0 \leq \varphi < \xi} \tau(\varphi),$$

where $T = \tau(\varphi)$ is the inertial curve of the motion of the machine unit ([5]).

Starting from an arbitrarily chosen ξ-periodic function $T_1(\varphi)$, defined, continuous, and satisfying in the interval $-\infty < \varphi < +\infty$ the inequality

$$\tau_* \leq T_1(\varphi) \leq \tau^*, \tag{4}$$

construct a functional sequence $T_k(\varphi)$ $(k = 1, 2, ...)$, defined by the recurrence law

$$T_{k+1}(\varphi) = \frac{e^{-\lambda_2 \varphi}}{e^{\lambda_2 \xi} - 1} \int_{\varphi}^{\varphi + \xi} e^{\lambda_2 t} \{M[t, T_k(t)] + \lambda_2 T_k(t)\}\, dt. \tag{5}$$

The latter converges uniformly on the entire number line to the periodic limiting regime

$$T_k(\varphi) \to T_\xi(\varphi), \qquad -\infty < \varphi < +\infty,$$

as $k \to \infty$ in (5).

Theorem 1. *If the reduced moment $M(\varphi, T)$ of all acting forces satisfies conditions 1^0–4^0, then the functional sequence*

$$\chi_k(\varphi) = \frac{M[\varphi, T_k(\varphi)]}{T_k(\varphi)} - \frac{\dot{I}(\varphi)}{I(\varphi)}, \qquad -\infty < \varphi < +\infty, \quad k = 1, 2, \ldots, \tag{6}$$

converges uniformly on the entire number line to the characteristic criterion $\chi_\xi(\varphi)$ *of the periodic limiting regime* $T = T_\xi(\varphi)$ *of motion of the machine aggregate:*

$$\chi_k(\varphi) \to \chi_\xi(\varphi), \qquad -\infty < \varphi < +\infty,$$

as $k \to \infty$.

Indeed, carrying out the obvious identical transformations and using Lagrange' s theorem, we find

$$\chi_k(\varphi) - \chi_\xi(\varphi) = \frac{1}{T_k(\varphi)}\left\{M'_T(\varphi, c_k) - \frac{M[\varphi, T_\xi(\varphi)]}{T_\xi(\varphi)}\right\}[T_\xi(\varphi) - T_k(\varphi)]. \tag{7}$$

where $c_k \in (T_k(\varphi), T_\xi(\varphi))$.

It is easy to show that, for all k, the inequalities

$$\tau_* \le T_k(\varphi) \le \tau^*, \qquad -\infty < \varphi < +\infty, \quad k = 1, 2, ... \tag{8}$$

will hold.

On the basis of Lemmas 1 and 2 of [5], replacing in the latter T by $\tau^* - \tau_*$, we have

$$|M'_T(\varphi, c_k)| \le \lambda_2, \qquad |M[\varphi, T_\xi(\varphi)]| \le \lambda_2(\tau^* - \tau_*). \tag{9}$$

Therefore, from (7), (8), and (9) we obtain the estimate

$$|\chi_k(\varphi) - \chi_\xi(\varphi)| \le \frac{1}{\tau_*}\left\{\lambda_2 + \frac{\lambda_2(\tau^* - \tau_*)}{\tau_*}\right\}|T_k(\varphi) - T_\xi(\varphi)| = \frac{\lambda_2 \tau^*}{\tau_*^2}|T_k(\varphi) - T_\xi(\varphi)|. \tag{10}$$

By virtue of the uniform convergence of the sequence (5), for every $\varepsilon > 0$, with $\varepsilon_1 = \varepsilon\tau_*^2/\lambda_2\tau^*$, there is a number K, depending only on ε_1 (and, consequently, on ε), such that the inequality

$$|T_k(\varphi) - T_\xi(\varphi)| < \varepsilon_1$$

is satisfied for all numbers $k > K(\varepsilon)$, and moreover at once on the entire number line.

Taking estimate (10) into account, we see that

$$|\chi_k(\varphi) - \chi_\xi(\varphi)| < \varepsilon$$

for all $k > K(\varepsilon)$ and $\varphi \in (-\infty, +\infty)$.

Thus, the constructed uniformly convergent iterative process (6) makes it possible to compute the characteristic criterion $\chi_\xi(\varphi)$ of the periodic limiting regime $T = T_\xi(\varphi)$ of motion of the machine aggregate with any desired degree of accuracy.

Theorem 2. *Under the conditions considered, for the error r_k with which the approximation $\chi_k(\varphi)$ reproduces the characteristic criterion $\chi_\xi(\varphi)$ of the periodic limiting regime $T = T_\xi(\varphi)$, at each step*

of the iterative process (6) the estimate is valid

$$r_k = \sup_{0 \leqslant \varphi < \xi} \left|\chi_k(\varphi) - \chi_\xi(\varphi)\right| \leqslant \frac{\lambda_2 \tau_*}{\tau_*^2} \left(\frac{\lambda_2}{\lambda_1} - 1\right) \rho_k, \qquad k = 2, 3, \dots, \tag{11}$$

where

$$\rho_k = \sup_{0 \leqslant \varphi < \xi} \left|T_k(\varphi) - T_{k-1}(\varphi)\right|. \tag{12}$$

2. Table 1 gives the results of tabulating the successive approximations $\chi_k(\varphi)$ to the characteristic criterion $\chi_{2\pi}(\varphi)$ of a periodic–

Table 1

	φ_0	φ_1	φ_2	φ_3	φ_4	φ_5
$\chi_1(\varphi)$	0,0057143	−0,048957	−0,095300	−0,12627	−0,13714	−0,12627
$\chi_2(\varphi)$	−0,050718	−0,084556	−0,10872	−0,11903	−0,11183	−0,085645
$\chi_3(\varphi)$	−0,055550	−0,089573	−0,11325	−0,12246	−0,11360	−0,085184
$\chi_4(\varphi)$	−0,055453	−0,089648	−0,11348	−0,12281	−0,11402	−0,085568
$\chi_5(\varphi)$	−0,055389	−0,089603	−0,11345	−0,12280	−0,11403	−0,085606
$\chi_6(\varphi)$	−0,055379	−0,089595	−0,11345	−0,12280	−0,11403	−0,085603

	φ_6	φ_7	φ_8	φ_9	φ_{10}
$\chi_1(\varphi)$	−0,095300	−0,048957	0,0057143	0,060387	0,10673
$\chi_2(\varphi)$	−0,043060	0,0084150	0,058095	0,095820	0,11527
$\chi_3(\varphi)$	−0,039812	0,014662	0,066733	0,10534	0,12381
$\chi_4(\varphi)$	−0,040013	0,014818	0,067375	0,10642	0,12513
$\chi_5(\varphi)$	−0,040065	0,014772	0,067365	0,10608	0,12524
$\chi_6(\varphi)$	−0,040065	0,014772	0,067366	0,10648	0,12523

Fig. 1

Figure 1: Fig. 1

	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}
$\chi_1(\varphi)$	0,13770	0,14857	0,13770	0,10673	0,060390
$\chi_2(\varphi)$	0,11505	0,097592	0,067382	0,029468	−0,011348
$\chi_3(\varphi)$	0,12147	0,10064	0,067479	0,027182	−0,015282
$\chi_4(\varphi)$	0,12247	0,10173	0,068293	0,027716	−0,014981
$\chi_5(\varphi)$	0,12260	0,10188	0,068424	0,027826	−0,074895
$\chi_6(\varphi)$	0,12261	0,10188	0,068434	0,027837	−0,014886

limit regime $T = T_{2\pi}(\varphi)$ of a rotor whose motion is described by the equation

$$\frac{dT}{d\varphi} = M_b(\varphi) - k\left(\sqrt{\frac{2T}{I}}\right)^n.$$

The case not integrable in quadratures is taken, when $n = 4$, with numerical data $M_b(\varphi) = 2 + \sin\varphi$ kgf, $I = 1$ kg $\cdot$ m^2, $k = 0{,}01$ kgf $\cdot$ sec^4, corresponding to a slow-speed rotor.

The successive approximations

$$\chi_k(\varphi) = \frac{M[\varphi, T_k(\varphi)]}{T_k(\varphi)} = \frac{2 + \sin\varphi - 0{,}04\,T_k^2(\varphi)}{T_k(\varphi)}, \qquad k = 1, 2, \dots, 6,$$

were computed at the points

$$\varphi_i = -\pi + \frac{\pi}{8} i, \quad i = 0, 1, 2, \dots, 16,$$

for $T_1(\varphi) \equiv 7$ J.

Using estimate (11), it is easy to verify that

$$r_6 = \sup_{|\varphi|<\pi} |\chi_6(\varphi) - \chi_{2\pi}(\varphi)| < 3 \cdot 10^{-5}.$$

Consequently, the 6th approximation $\chi_6(\varphi)$ reproduces the characteristic criterion $\chi_{2\pi}(\varphi)$ with at least 3 significant digits.

It is obvious that $|\chi_{2\pi}(\varphi)| < 0.126$; therefore, in any position of the rotor the inertial forces of the initial motion will amount to less than 6.3% of the inertial forces of the permanent motion.

Fig. 1

Figure 1 shows the graphs of the approximations $T_1(\varphi)$, $T_2(\varphi)$, $T_6(\varphi)$ to the periodic limiting regime $T = T_{2\pi}(\varphi)$ of the rotor motion, and the graphs of the corresponding approximations $\chi_1(\varphi)$, $\chi_2(\varphi)$, $\chi_6(\varphi)$ to the characteristic criterion $\chi_{2\pi}(\varphi)$.

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Note: Figure translations are in progress. See original paper for figures.