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Abstract

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GEOPHYSICS

S. I. ZUBKOV

A GENERALIZED RHEOLOGICAL MODEL OF ROCKS

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The processes of deformation of rocks at high pressures and temperatures, and also under stresses exceeding the elastic limit, are to a considerable extent determined by irreversible deformations of viscous and quasiviscous flow, of diffusion and rupture character, respectively. Therefore, the mathematical description of such deformation processes cannot be confined to Hooke' s law and must take into account the rheological properties of rocks.

The first requirement for the theoretical rheology of rocks is that, in the model description of the rheological properties of rocks, their principal properties must be taken into account: elasticity, viscous and plastic flow, elastic aftereffect, and shear and tensile strengths. The second requirement for the theoretical rheology of rocks is the writing of the deformation equations of rheological models for the case of variable coefficients, since it is well known ([1]) that the rheological coefficients of rocks are variable quantities depending on time and on the physical parameters characterizing the state of the Earth' s material. Finally, the third requirement is the necessity of a tensor form for the deformation equations.

We shall call a rheological model of rocks satisfying the requirements listed above the generalized rheological model of rocks.

In Fig. 1 a generalized rheological model is presented, composed of serially connected Burgers and Shvedov–Bingham models. It reflects the principal rheological properties of rocks: elasticity G_2, viscous flow η_2, elastic aftereffect η_1, G_1, plastic flow σ^*_{ik}, η_3, shear strengths $\sigma^*_{xy}, \sigma^*_{yz}, \sigma^*_{zx}$, and tensile strengths $\sigma^*_{xx}, \sigma^*_{yy}, \sigma^*_{zz}$.

Fig. 1. Generalized rheological model of rocks for deformation under isotropic compression (extension)

The deformation equations of the Burgers–Shvedov–Bingham (B.–Sh.–B.) model with variable coefficients for shear were obtained by us by adding the deformations of the Hooke element G_2, the Newton element η_2, the Kelvin–Voigt element η_1, G_1, and the Shvedov–Bingham element σ^*_{ik}, η_3; in differentiating the deformations of the model with respect to time, the rheological coefficients were left under the derivative signs:

$$\tau_1 \frac{d^2\varepsilon_\tau}{dt^2} + \frac{d\varepsilon_\tau}{dt} + \frac{d\tau_1}{dt}\frac{d\varepsilon_\tau}{dt} = \tau_1 \frac{d^2(\sigma_\tau/G_2)}{dt^2} + \frac{d(\sigma_\tau/G_2)}{dt} + \frac{d(\sigma_\tau/G_1)}{dt} +$$

$$+\tau_1 \frac{d(\sigma_\tau/\eta_2)}{dt} + \tau_1 \frac{d(\sigma_\tau/\eta_3)}{dt} - \tau_1 \frac{d(\sigma^*_\tau/\eta_3)}{dt} + \frac{\sigma_\tau}{\eta_2} + \frac{\sigma_\tau}{\eta_3} - \frac{\sigma^*_\tau}{\eta_3} + \frac{d\tau_1}{dt}\left[\frac{d(\sigma_\tau/G_2)}{dt} + \frac{\sigma_\tau}{\eta_2} + \frac{\sigma_\tau}{\eta_3} - \frac{\sigma^*_\tau}{\eta_3}\right], \tag{1}$$

where $\tau = xy, yz, zx,\ \tau_1 = \eta_1/G_1$.

Equations (1) relate the tangential components of the stress tensors σ_{ik} and strain tensors ε_{ik}, $i, k = x, y, z$, in the rheological medium B.–Sh.–B.

Using the method of mathematical analogy with the transformations of elasticity theory, proposed in (2) and physically substantiated in (3), we obtain the relation between the normal components of the tensors σ_{ik} and ε_{ik} in the B.–Sh.–B. medium with variable coefficients:

$$2\left[\tau_1 \frac{d^2(e_n - \theta/3)}{dt^2} + \frac{d(e_n - \theta/3)}{dt} + \frac{d\tau_1}{dt}\frac{d(e_n - \theta/3)}{dt}\right] =$$

$$= \tau_1 \frac{d^2[(\sigma_n - P)/G_2]}{dt^2} + \frac{d[(\sigma_n - P)/G_2]}{dt} + \frac{d[(\sigma_n - P)/G_1]}{dt} + \tau_1 \frac{d[(\sigma_n - P)/\eta_2]}{dt} +$$

$$+\tau_1 \frac{d[(\sigma_n - P)/\eta_3]}{dt} - \tau_1 \frac{d[(\sigma^*_n - P^*)/\eta_3]}{dt} + \frac{\sigma_n - P}{\eta_2} + \frac{\sigma_n - P}{\eta_3} - \frac{\sigma^*_n - P^*}{\eta_3} +$$

$$+\frac{d\tau_1}{dt}\left\{\frac{d[(\sigma_n - P)/G_2]}{dt} + \frac{\sigma_n - P}{\eta_2} + \frac{\sigma_n - P}{\eta_3} - \frac{\sigma^*_n - P^*}{\eta_3}\right\}, \tag{2}$$

where $n = xx, yy, zz$, $P^* = (\sigma^*_{xx} + \sigma^*_{yy} + \sigma^*_{zz})/3$, $\theta = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$, $P = (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})/3$.

The equation of volumetric strain under all-round uniform (isotropic) compression (extension) of the B.–Sh.–B. medium is derived by a method entirely analogous to the method of deriving equations (1), and, in the case of variable coefficients, has the form

$$T_1 \frac{d^2\theta}{dt^2} + \frac{d\theta}{dt} + \frac{dT_1}{dt}\frac{d\theta}{dt} = T_1 \frac{d^2(P/K_2)}{dt^2} + \frac{d(P/K_2)}{dt} + \frac{d(P/K_1)}{dt} +$$

$$+ T_1 \frac{d(P/\zeta_2)}{dt} + T_1 \frac{d(P/\zeta_3)}{dt} - T_1 \frac{d(V^*/\zeta_3)}{dt} + \frac{P}{\zeta_2} + \frac{P}{\zeta_3} - \frac{V^*}{\zeta_3} +$$

$$+ \frac{dT_1}{dt}\left[\frac{d(P/K_2)}{dt} + \frac{P}{\zeta_2} + \frac{P}{\zeta_3} - \frac{V^*}{\zeta_3}\right], \quad (3)$$

where $T_1 = \zeta_1/K_1$, θ is the volumetric strain; P is the isotropic stress of compression or extension; V^* is the volumetric yield limit; K_1 and K_2 are the moduli of elastic aftereffect and elastic deformation under isotropic compression (extension), respectively; ζ_1, ζ_2, and ζ_3 are the volumetric viscosity of aftereffect, the relaxation volumetric viscosity, and the volumetric viscosity of plastic flow (Fig. 1), respectively.

The system of 7 tensor equations (1), (2), and (3) describes the regularities of deformation under shear, uniaxial, uniform and nonuniform biaxial and triaxial compressions (extensions) in a medium whose rheological properties are represented by the B.–Sh.–B. model with variable coefficients.

Thus, the B.–Sh.–B. model and its deformation equations satisfy all the requirements set forth above for a generalized rheological model of rocks.

The construction of deformation equations with variable rheological coefficients, carried out here for the normal and tangential components of the tensors σ_{ik} and ε_{ik} of the most complex rheological medium–the B.–Sh.–B. medium–significantly expands the possibilities of using the methods of theoretical rheology in the mathematical description of deformation processes of real solids (and, in particular, rocks) and can be used for a wide range of problems in the mechanics of deformable media, including problems of geophysics.

The significance of the system of equations (1), (2), (3) for solving geophysical problems is as follows. The determination of temporal changes in the tensors σ_{ik} and ε_{ik} in the Earth' s interior can be carried out with allowance for temporal changes in the rheological coefficients. This makes it possible to study the influence of temporal changes in the rheological coefficients on the time course of stresses and strains in the Earth, and, in earthquake focal zones, also on the recurrence of earthquakes. With the aid of the system of equations (1), (2), (3), changes in the Earth' s rheological coefficients with time can be found, and the laws of dependence of the rheological coefficients on the parameters of the state of the Earth' s material can also be established (for specified temporal changes in σ_{ik} and ε_{ik} and in the state parameters).

The introduction of variable coefficients into the deformation equations makes it possible to outline the following three directions of theoretical research. The first direction is the study of temporal changes in the parameters of the state of the Earth's material for specified laws of dependence of the rheological coefficients on these state parameters and for specified $\sigma_{ik}(t)$ and $\varepsilon_{ik}(t)$. The second direction is a mathematical description of the processes of avalanche failure of rocks; the possibility of such calculations is based on an expanded conception of the viscosity of rocks as a certain "quasi-viscosity" of the relative sliding of surfaces of discontinuities during avalanche failure, with the coefficient of viscosity and the strength of the rocks assumed to be quantities that decrease rapidly with time. Finally, the third direction of possible research is the application of the results of atomic-molecular studies of temporal changes in various defects of solids and of their relationships with rheological coefficients to the calculation of temporal changes in $\sigma_{ik}(t)$ and $\varepsilon_{ik}(t)$, which makes it possible to use the results of atomic-molecular studies of a solid body to explain the processes of deformation of rocks observed on a macroscale.

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Note: Figure translations are in progress. See original paper for figures.