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Abstract

Full Text

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A NEW CASE OF INTEGRABILITY OF THE EQUATIONS OF MOTION OF A HEAVY RIGID BODY HAVING A FIXED POINT

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The Euler-Poisson equations for the problem of the motion of a heavy rigid body in the case when the center of gravity lies on the first principal axis, and the gyrostatic moment is orthogonal to the third axis, have the form

$$\begin{aligned}
&A\,dp/dt = (B-C)qr + \lambda_2 r,\\
&B\,dq/dt = (C-A)rp - \lambda_1 r + \Gamma v_1,\\
&C\,dr/dt = (A-B)pq + \lambda_1 q - \lambda_2 p + \Gamma v_2,\\
&dv_1/dt = rv_2 - qv_3, \quad dv_2/dt = pv_3 - pv_1, \quad dv_3/dt = qv_1 - pv_2.
\end{aligned} \tag{1}$$

Here v_1, v_2, v_3 are the components of the unit vector in the direction of the force of gravity; Γ is the product of the weight of the body and the distance from the fixed point to the center of gravity; $\lambda_1, \lambda_2, 0$ are the gyrostatic moment; the remaining notation is standard.

The known integrals of these equations are:

$$Ap^2 + Bq^2 + Cr^2 - 2\Gamma v_1 = 2E,$$

$$(Ap + \lambda_1)v_1 + (Bq + \lambda_2)v_2 + Crv_3 = k/\Gamma,$$

$$v_1^2 + v_2^2 + v_3^2 = 1.$$

Let us subject the moments of inertia to the condition

$$A = 18C(B-C)/(10B-9C),$$

and, instead of λ_1, λ_2, introduce the parameters u and ε, putting

$$\lambda_1 = \frac{u}{9}\,\frac{25B^2 - 54BC + 27C^2}{B^2C(10B-9C)}\,n\cos\varepsilon, \quad \lambda_2 = -\frac{u}{3}\,\frac{(B-C)^2}{B^2C^2}\,n\sin\varepsilon.$$

Equations (1) then have, in this case, a solution in which the principal variables p, q, r, v_1, v_2, v_3, expressed as functions of the new variable σ, are as follows:

$$p = -u\frac{B-C}{ABC}\left(\frac{1}{B}\frac{5B-3C}{10B-9C}n\cos\varepsilon + 2\cos\sigma\right),$$

$$q = \frac{u}{3BC}\left(\frac{B-C}{BC}n\sin\varepsilon - 2\sin\sigma\right),$$

$$r = \frac{u}{C}\left(r_0 + r_1\cos\sigma + r_1'\sin\sigma + r_2\cos 2\sigma + r_3\cos 3\sigma + r_3'\sin 3\sigma\right)^{1/2},$$

$$v_1 = -\frac{u^2}{\Gamma}\frac{2B-3C}{36BC^2}\left(\chi_0 + \chi_1\cos\sigma + \chi_1'\sin\sigma + \chi_2\cos 2\sigma + \chi_3\cos 3\sigma + \chi_3'\sin 3\sigma\right),$$

$$v_2 = -\frac{u^2}{\Gamma}\frac{2B-3C}{36BC^2}\left(\chi_0 + \chi_1\cos\sigma + \chi_1'\sin\sigma + \chi_2'\sin 2\sigma + \chi_3\cos 3\sigma + \chi_3'\sin 3\sigma\right),$$

$$v_3 = \frac{u}{\Gamma}\frac{1}{18C}\left[-\frac{B-C}{B^2C}h\cos\varepsilon + \frac{6(2B-3C)}{BC}\cos\sigma\right]\left(r_0 + r_1\cos\sigma + r_1'\sin\sigma + \right.$$

$$\left. + r_2\cos 2\sigma + r_3\cos 3\sigma + r_3'\sin 3\sigma\right)^{1/2},$$

where

$$r_0 = 2E_* - \frac{2B-3C}{27B^2C^3}(16B^2 + 13BC - 18C^2) +$$

$$+\frac{1}{162B^3C^2}\left[18\frac{(B-C)^2}{C} - \frac{(5B-3C)(35B-33C)}{10B-9C}\right]n^2\cos^2\varepsilon,$$

$$r_1 = -\frac{1}{36}\frac{(B-C)(50B-27C)}{B^3C^2}\,n\cos\varepsilon,$$

$$r_1' = \frac{1}{36}\frac{(B-C)(10B+9C)}{B^3C^2}\,n\sin\varepsilon,$$

$$r_2 = -\frac{1}{9}\frac{(2B-3C)(11B-6C)}{B^2C^2}, \qquad n^2 = \frac{4}{3}\frac{(2B-3C)(4B-3C)B^2}{(B-C)^2},$$

$$r_3 = -\frac{1}{12}\frac{(B-C)(2B-3C)}{B^3C^2}\,n\cos\varepsilon,$$

$$r_3' = -\frac{1}{12}\frac{(B-C)(2B-3C)}{B^3C^2}\,n\sin\varepsilon,$$

$$\varkappa_0 = \frac{3}{2}\frac{10B-9C}{BC} - \frac{(B-3C)(5B-3C)}{9B^3C(2B-3C)}\,n^2\cos^2\varepsilon,$$

$$\varkappa_1 = \frac{1}{3B^2C}\left(\frac{5B+3C}{2} + \frac{10B^2-15BC+9C^2}{2B-3C}\right)n\cos\varepsilon, \qquad \varkappa_1' = \frac{3}{2}\frac{B-C}{B^2C}\,n\sin\varepsilon,$$

$$\varkappa_2 = \frac{6(2B-C)}{BC}, \qquad \varkappa_3 = \frac{3(B-C)}{2B^2C}\,n\cos\varepsilon, \qquad \varkappa_3' = \frac{3(B-C)}{2B^2C}\,n\sin\varepsilon,$$

$$\chi_0 = -\frac{2(B-C)(B-3C)}{B^3C(2B-3C)}\,n^2\cos\varepsilon\sin\varepsilon, \qquad \chi_1 = -\frac{9}{2}\frac{B-C}{B^2C}\,n\sin\varepsilon,$$

$$\chi_1' = \frac{1}{3B^2C}\left(-\frac{5B+3C}{2} + \frac{10B^2-15BC+9C^2}{2B-3C}\right)n\cos\varepsilon, \qquad \chi_2' = \frac{8}{C},$$

$$\chi_3 = -\frac{3(B-C)}{2B^2C}\,n\sin\varepsilon, \qquad \chi_3' = \frac{3(B-C)}{2B^2C}\,n\cos\varepsilon,$$

$$E_* = \frac{4}{27}\frac{(B-2C)(4B-3C)}{BC^3} + \frac{1}{5184B^2}\left[3\frac{(B-3C)^2}{B^2C^2}\frac{10B-7C}{2B-3C} + 9\frac{5B-3C}{B^2C^2}\times\right.$$
$$\left.\times\frac{6B-7C}{10B-9C} - 2(34B-51C)(5B-3C)\frac{B-3C}{2B-3C} - 36\frac{(B-C)^2(8B-9C)}{B^2C^3}\right]n^2\cos^2\varepsilon$$

The dependence between σ and t is established by inverting the integral

$$t - t_0 = \frac{3(B-C)}{uB}\int_0^\sigma (r_0 + r_1\cos\sigma + r_1'\sin\sigma + r_2\cos 2\sigma +$$
$$+ r_3\cos 3\sigma + r_3'\sin 3\sigma)^{-1/2}\,d\sigma$$

The parameters $B, C, \Gamma, \varepsilon$ in this solution are independent.

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Note: Figure translations are in progress. See original paper for figures.