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Corresponding Member of the USSR Academy of Sciences B. B. KADOMTSEV, O. P. POGUTSE

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Abstract

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DISSIPATIVE INSTABILITY ON TRAPPED PARTICLES IN A DENSE PLASMA

As was pointed out in papers ([1,2]), for the confinement of a rarefied plasma in magnetic traps, instability on trapped particles can pose a great danger. Neglecting collisions, this instability develops on groove-like perturbations of particles with small longitudinal velocity, trapped between the magnetic mirrors of an inhomogeneous toroidal field (for the further development of the theory, see ([3–11])). As the collision frequency increases, the effect of particle transfer from trapped to passing particles (and back) becomes ever more substantial, and the instability takes on a dissipative character ([2,3]). For the instability considered in ([2,3]), the presence of trapped ions is very important, so that it would be appropriate to call it an instability on trapped ions. In the present work it will be shown that, at a sufficiently high frequency of electron collisions, a new dissipative instability on trapped particles can develop, for which the presence or absence of trapped ions is entirely immaterial and which for this reason should be called an instability on trapped electrons. This instability is a variant of drift-dissipative instability, in which the mechanism of dissipation leading to the excitation of drift waves is the transfer of electrons from trapped to passing, owing to collisions.

Restricting ourselves, for definiteness, to axisymmetric systems with a strong longitudinal magnetic field of the Tokamak type, we shall use the following notation: a is the minor radius, R the major radius of the torus; r is the current radius—the distance from the magnetic axis; $\varepsilon = r/R$ is the toroidicity; B_z is the longitudinal and B_ϑ the azimuthal magnetic field; $q = q(r) = \dfrac{B_z r}{B_\vartheta R}$ is the safety factor against helical instability; $\theta = \dfrac{r^2}{Rq^2}\dfrac{dq}{dr}$ is the shear; n is the equilibrium density of electrons and ions; T_e is the electron and T_i the ion temperature; φ is the potential of the electric-field perturbation. Since the ion temperature is completely inessential for the instability under consideration, for simplicity we shall take it to be zero.

To obtain the dispersion equation determining the frequency of small quasineutral oscillations of the plasma, it is sufficient to find expressions for the perturbations of the density of ions n_i' and electrons n_e' and then set them equal to one another. As for n_i', for the oscillations of the drift-wave type considered by us, with phase velocity along the magnetic field considerably greater than the thermal velocity of the ions, this expression is well known (see, for example, ([12])):

$$n_i' = \left(\frac{\omega_*}{\omega} + \frac{k^2}{m_i\omega^2}T_e\right)\frac{en}{T_e}\varphi, \tag{1}$$

where m_i is the ion mass; ω is the oscillation frequency; ω_* is the drift frequency, equal to $\omega_* = -k_y\frac{cT_e}{eBn}\frac{dn}{dr}$; $k_y = \frac{lq}{R}$ is the component of the wave vector $\mathbf{k}$ perpendicular to $\mathbf{B}$ and $\mathbf{r}$; l is an integer; $k_\parallel$ is the longitudinal component of $\mathbf{k}$.

As for n_e', it consists of two terms

$$n_i = \frac{en}{T_e}\varphi + n_t, \tag{2}$$

where the first takes into account the passing particles distributed according to Boltzmann, while n_t' represents the density perturbations of trapped electrons. To find n_t' one may use the procedure set forth in ([2,3]). Taking into account that the main contribution to n_t' is made by "inner" trapped electrons, far from the boundary of transition into passing ones, we shall include collisions in the τ-approximation, replacing the collision term in the equation for the distribution function of trapped particles f_t' by $-\nu_{ef}f_t'$. In doing so the effective frequency ν_{ef} should be regarded as a function of v: first, because of the differential character of the collision term, with allowance for the fact that for trapped particles $v_\parallel \sim \sqrt{\varepsilon}\,v$, a factor $1/\varepsilon v^2$ appears; and second, in a dense plasma the "tail" electrons with large v play the greatest role, for which one more factor $1/v$ appears in the Landau collision term. Thus, approximately,

$$\nu_{ef} = \frac{v_e^3}{\varepsilon v^3}\nu_e, \tag{3}$$

where $v_e = \sqrt{2T_e/m_e}$ is the thermal velocity of the electrons; ν_e is the mean frequency of electron collisions; $\varepsilon = r/R$.

With the above taken into account, the expression for n_e' has the form

$$n_e' = \frac{en}{T_e}\varphi - \sqrt{\varepsilon}\left\langle\frac{\omega-\omega_{e*}}{\omega+i\nu_{ef}}\right\rangle\frac{en}{T_e}\hat{K}\varphi. \tag{4}$$

Here $\sqrt{\varepsilon}$ before the second term is the fraction of trapped particles,

$$\omega_{e*} = -k_y\frac{cT_e}{eBf_e}\frac{\partial f_e}{\partial r} = -k_y\frac{cT_e}{eB}\left(\frac{1}{n}\frac{dn}{dr} + \left(\frac{m_ev^2}{2T_e} - \frac{3}{2}\right)\frac{1}{T_e}\frac{dT_e}{dr}\right) -$$

is the electron drift frequency; the angle brackets denote averaging over v with the Maxwellian function f_e, and K is an integral operator in the angular variable ϑ, which for smooth perturbations having a maximum on the outer circumference of the torus can approximately be replaced by unity (cf. ([3])). We note that in (4) we have neglected the small contribution from the magnetic drift.

Equating (1) and (4), we obtain the dispersion equation for determining the frequency and the growth increment of the oscillations. Since the contribution from trapped particles is small, it is sufficient to take into account only its imaginary part. Since the second term in (1) is also small, in the first approximation $\omega = \omega_*$, i.e., we have the ordinary drift wave. Substituting this value of ω into the small terms, we obtain:

$$\omega \simeq \omega_* + \frac{k_\parallel^2 T_e}{m_i \omega_*} + i\omega_* \sqrt{\varepsilon} \left\langle \frac{(\omega_{e*} - \omega_*)\nu_{ef}}{\omega_*^2 + \nu_{ef}^2} \right\rangle . \tag{5}$$

It is easy to see that

$$(\omega_{e*} - \omega_*) = -\frac{k_y c}{eB} \frac{dT_e}{dr} \left(\frac{m_e v^2}{2T_e} - \frac{3}{2} \right),$$

i.e., the angle bracket in (5) is proportional to the temperature gradient. Hence it follows that, if the temperature and density gradients are directed to the same side, then the increment $\gamma = \operatorname{Im}\omega$ is positive only under the condition that $\omega_* \lesssim \nu_e/\varepsilon$, and therefore it may be written approximately in the form

$$\gamma \simeq \varepsilon^{3/2} \frac{\omega_* \omega_{*T}}{\nu_e}, \tag{6}$$

where

$$\omega_{*T} = -k_y \frac{c}{eB} \frac{dT_e}{dr}.$$

Hence, using the order-of-magnitude relation $D \sim \gamma/k_\perp^2$, one can estimate the coefficient of turbulent diffusion D. For sufficiently large ν_e, when k_y, determined from the relation $\omega_* \sim \nu_e/\varepsilon$, is much larger than θ/ρ, where $\rho = \sqrt{(T_e/m_i)(m_i c/eB)}$ is the ion Larmor radius at the electron temperature, $k_\perp$ may be taken equal to k_y, and

$$D \sim \varepsilon^{3/2} \frac{c^2 T_e}{\nu_e e^2 B^2 n} \left| \frac{dn}{dr} \frac{dT_e}{dr} \right|.$$

Fig. 1. Dependence of the turbulent-diffusion coefficient on the frequency of electron collisions.

Figure 1: Fig. 1. Dependence of the turbulent-diffusion coefficient on the frequency of electron collisions.

We have inserted here the modulus sign, since for $\dfrac{dn}{dr}\dfrac{dT_e}{dr} < 0$ the increment γ is positive at ω_* somewhat larger than ν_e/ε, so that, in order of magnitude, D remains the same as before.

As ν_e decreases, the effective k_y decreases and may become smaller than $k_r \sim 1/\Delta$, where Δ is the width of localization of the perturbation in r. In this case, for an approximate determination of D and Δ, one may use the device proposed in (3), namely the introduction into the right-hand side of the dispersion equation (5) of an additional term $iD\Delta_\perp n'/n'$, which takes into account the effect of turbulent diffusion due to small-scale pulsations. The quantities D and Δ can then be found from the condition that the total increment vanish. For small k_y, in $\Delta_\perp n'$ one may retain only the term d^2n'/dx^2; $x = r - r_0$ is the distance from the localization point of the perturbation, where $k_\parallel = 0$. Near this point $k_\parallel^2 = k_y^2(\theta^2/r^2)x^2$, so that, according to (5), for n' we obtain a second-order equation having the form of the Schrödinger equation for a harmonic oscillator. Substituting, as a solution, $n' \sim \exp[-(\alpha/2)x^2]$ and equating to zero the coefficients of 1 and x^2, we obtain

$$D\alpha^2 = i\frac{\theta^2 T_e k_y^2}{r^2\omega_* m_i}, \qquad \alpha D = i(\omega - \omega_*) + \gamma. \tag{7}$$

Fig. 1. Dependence of the coefficient of turbulent diffusion on the frequency of electron collisions,

$$D_m \simeq \frac{\rho^2 c_s}{\theta R}, \qquad \nu_0 \simeq \frac{\theta c_s}{R}, \qquad \nu_1 \simeq \sqrt{\varepsilon\frac{T_i}{T_e}}\,\nu_0,$$

$$\nu_2 \simeq \left(\sqrt{\frac{m_i}{m_e}}\frac{\varepsilon}{\theta}\right)^{1/2}\nu_0.$$

Taking D to be real, one can find from this α, D, and ω. In order of magnitude $D \sim \nu_e\rho^2/\theta^2$, $\Delta \sim 1/\sqrt{\alpha} \sim \varepsilon^{1/4}\rho/\theta$. Approximately interpolating the dependence of D on ν_e for small and large ν_e, we obtain

$$D = \frac{\rho^2}{\theta^2}\left(\frac{r}{T_e}\frac{dT_e}{dr}\right)^2\frac{\nu_e}{1+\nu_e^2/\nu_0^2}, \tag{8}$$

where

$$\nu_0 = \frac{\theta c_s}{R\varepsilon^{1/4}}\sqrt{\left|\frac{d\ln n}{d\ln T_e}\right|}$$

is the frequency at which the transition occurs from the linear dependence $D \sim \nu_e$ to the inversely proportional dependence $D \sim 1/\nu_e$ of D on ν_e; $c_s = \sqrt{T_e/m_i}$ is the sound speed; $\rho = c_s/\omega_{Bi}$, $\omega_{Bi} = eB/m_i c$.

Along with diffusion, the instability considered must lead to turbulent electron thermal conductivity. Since $\nu_{ef} \sim v^{-3}$, the perturbation of the distribution function, whose integral over v gives the second term in (4), grows strongly with v. Therefore the perturbation of the electron temperature T_e' is approximately three times larger than the density perturbation n_f'. Correspondingly, the coefficient of thermal diffusivity $\chi \simeq 3D$. For a distribution function f_e having a non-Maxwellian "tail," the ratio χ/D may be still larger.

As the collision frequency ν_e increases, friction of passing electrons against ions begins to play a noticeable role, and the instability considered here

the instability continuously transforms into the drift-dissipative one ([12–14]). On the other hand, for small ν_e it transforms into the previously considered dissipative instability on trapped ions ([3]). In the expression for the coefficient of turbulent diffusion, this approximation can be taken into account by summing three coefficients for the different instabilities. According to ([3]) and ([8]), for the case $T_i < T_e$, in order of magnitude we have:

$$D \cong \left(\frac{T_i}{T_e}\right)^2 \varepsilon_i^{1/2}\frac{\rho^2 c_s^2}{\nu_e R^2} + \frac{\rho^2}{\theta^2}\frac{\nu_e}{1+\nu_e^2/\nu_0^2} + \frac{\rho^2\nu_e}{a}\left(\frac{m_e^2}{m_i^2}\frac{\nu_e a}{v_e}\frac{1}{\theta^2}\right)^{1/3}, \qquad (9)$$

where the first term corresponds to the instability on trapped ions, and the last to the drift-dissipative instability. The dependence of (9) on ν_e is presented in Fig. 1.

Let us note that the transition from the diffusion coefficient on trapped ions to the diffusion coefficient on trapped electrons occurs approximately at those values of ν_e above which, according to ([8]), the instability on trapped ions is stabilized. Since in this case the instability on trapped electrons considered here comes into play, the conclusion of work ([8]) on the stabilization of the less dangerous instability on trapped ions loses its relevance.

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Note: Figure translations are in progress. See original paper for figures.