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Abstract

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PHYSICS

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RADIATION OF AN ELECTRON-POSITRON PAIR

(Presented by Academician V. L. Ginzburg on January 6, 1969)

In studying the influence of multiple scattering of particles in the atmosphere and of the Earth' s magnetic field on the radio emission of cascade atmospheric showers, the results of calculations of the radiation of a relativistic electron-positron pair may be of interest. The corresponding calculations, analogous to those previously carried out by the author for one particle in Ref. ([1]), are given below. In the present article the results of these calculations are reported.

For equal energies and lifetimes of the particles of the pair, the results of averaging the radiation intensity of waves of different polarizations per unit solid angle over all possible trajectories of the electron and positron caused by multiple scattering in the presence of a magnetic field have the form

$$W_{n\omega x,y} = \frac{e^2\omega^2}{2\pi^2 c}\,\mathrm{Re}\left\{\int_0^T dt \int_0^{T-t} d\tau\,[U'_{x,y}(\omega_H) + U'_{x,y}(-\omega_H)] - \left[\int_0^T U''_{x,y}(\omega_H)\,dt\right]\left[\int_0^T U''_{x,y}(-\omega_H)\,dt\right]^*\right\}, \tag{1}$$

where

$$U'_x(\omega_H) = \frac{\exp[\Omega'(\omega_H)]}{(1+\eta t\ \mathrm{th}\,\eta\tau)^3\,\mathrm{ch}^2\,\eta\tau}\left[2qt(1+\eta t\ \mathrm{th}\,\eta\tau) + \theta_x^2\right],$$

$$U'_y(\omega_H) = \frac{\exp[\Omega'(\omega_H)]}{(1+\eta t\ \mathrm{th}\,\eta\tau)^3\,\mathrm{ch}^2\,\eta\tau}\left[2qt(1+\eta t\ \mathrm{th}\,\eta\tau) + \left(\theta_y - \frac{\omega_H t}{\mathrm{ch}\,\eta\tau}\right)\left(\theta_y - \omega_H t\ \mathrm{ch}\,\eta\tau - \frac{\omega_H}{\eta}\,\mathrm{sh}\,\eta\tau\right)\right],$$

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$$U''_x(\omega_H) = \frac{\theta_x}{\operatorname{ch}^2 \eta t} \exp[\Omega''(\omega_H)], \qquad U''_y(\omega_H) = \frac{\eta\theta_y - \omega_H \operatorname{sh} \eta t}{\eta \operatorname{ch}^2 \eta t} \exp[\Omega''(\omega_H)], \tag{2}$$

$$\Omega'(\omega_H) = -\frac{i\omega\tau}{2}(2-\beta^2-\varepsilon) - \frac{\eta\theta^2 \operatorname{th}\eta\tau}{4q(1+\eta t \operatorname{th}\eta\tau)} + \frac{\omega_H\theta_y}{2q}\frac{\operatorname{ch}\eta\tau + \eta t \operatorname{sh}\eta\tau - 1}{\operatorname{ch}\eta\tau + \eta t \operatorname{sh}\eta\tau} - \frac{\omega_H^2}{4q\eta}\left(\eta t + \eta\tau - \frac{\eta t + \operatorname{th}\eta\tau}{1+\eta t \operatorname{th}\eta\tau}\right),$$

$$\Omega''(\omega_H) = -\frac{i\omega t}{2}(2-\beta^2-\varepsilon) - \frac{\eta\theta^2}{4q}\operatorname{th}\eta t + \frac{\omega_H\theta_y}{2q}\frac{\operatorname{ch}\eta t - 1}{\operatorname{ch}\eta t} - \frac{\omega_H^2}{4q\eta}(\eta t - \operatorname{th}\eta t),$$

$$\eta = (1+i)\sqrt{\omega q}, \qquad \omega_H = \frac{eH_\perp c}{E}, \qquad q = \frac{E_s^2}{4E^2}\frac{c}{L}, \qquad E_s = \sqrt{4\pi\cdot 137}\, mc^2.$$

Here $W_{n\omega x}$ and $W_{n\omega y}$ are the angular distributions of the spectral energy density of radiation of waves polarized, respectively, in the plane passing through the direction of motion of the γ-quantum producing the pair and the magnetic-field vector, and in the perpendicular plane (in the plane of deflection of the particles in the magnetic field); L is the radiation unit of length; e is the absolute value of the particle charge; $H_\perp$ is the magnitude of the projection of the magnetic-field intensity onto the plane perpendicular to the direction of motion of the γ-quantum; T is the lifetime of the particles ($T \sim L/c$); ε is the dielectric-permittivity of the atmosphere at the altitude of pair formation; θ is the angular vector between the direction of motion of the γ-quantum producing the pair and the direction of observation; θ_x and θ_y are the projections of this angle.

The results are applicable when the following inequalities are satisfied:

$$1-\beta^2 \ll 1, \qquad |1-\varepsilon| \ll 1, \qquad qT \ll 1, \qquad \omega_H T \ll 1, \qquad \hbar\omega \ll E, \qquad \theta^2 \ll 1. \tag{3}$$

Let us note that the mean square of the multiple-scattering angle over the lifetime of the pair is larger by 3 orders of magnitude than the square of the initial opening angle of the particles; therefore the initial opening angle may be neglected, as was done from the very beginning of the calculations.

The first term in braces in (1) describes the sum of the radiation intensities of the electron and the positron; the second (the product of two integrals) describes their interference. The functions $U_{x,y}(\omega_H)$ are obtained by averaging over all

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possible trajectories of a particle deflected by the magnetic field in the positive direction, while $U_{x,y}(-\omega_H)$ are obtained for deflection in the negative direction of the y-axis. If, for definiteness, the y-axis is taken to be directed toward the deflection of the electron, then the functions $U_{x,y}(\omega_H)$ are electron functions, and $U_{x,y}(-\omega_H)$ are positron functions. For different energies of the electron and positron, the corresponding energies must everywhere be substituted in the electron and positron functions, and for different lifetimes the corresponding limits of integration as well. In particular, by putting, for example, the positron lifetime equal to zero, we obtain the radiation of the electron (see also (13) ([1])). Replacing the positron functions by electron ones and changing the sign of the interference term gives the angular distribution of the radiation intensity of two electrons simultaneously emitted from one point with identical initial directions of motion. Taking into account the interference of the radiations of two particles emitted from different points in space and at different times leads to the appearance in the interference term of the exponential factor $\exp(i\omega t' - i\mathbf{k}\mathbf{r}')$, where t' is the time interval between the emissions of the particles; $\mathbf{k}$ is the wave vector, and $\mathbf{r}'$ is the radius vector from the emission point of one particle to the emission point of the other.

In the absence of a magnetic field, the interference term in formula (1) for $\theta_x = 0$ and $\theta_y = 0$ vanishes. Thus, in the direction of motion of the γ-quantum producing the pair, the radiation intensity is equal to twice the radiation intensity of one particle. In all other directions, the radiation intensity of the pair is less than twice the radiation intensity of one particle. In the presence of a magnetic field, the radiation intensity in some small neighborhood of the direction of motion of the γ-quantum producing the pair is greater than twice the radiation intensity of an individual particle, and as the magnetic-field strength increases it approaches a value four times greater than the radiation intensity of one charged particle.

The results obtained describe bremsstrahlung and magnetic bremsstrahlung radiation, and, for $\varepsilon\beta^2 > 1$, also the Vavilov–Cherenkov radiation generated in this case. Their detailed investigation can be carried out with the use of an electronic computer.

Here we shall restrict ourselves to considering some consequences of the results obtained in the simplest case, assuming the energies and lifetimes of the electron and positron to be equal. Expanding the integrands in powers of the mean square of the multiple-scattering angle and of the synchrotron frequency, integrating with respect to time, and retaining the first nonzero terms of the expansion, we obtain:

$$W_{\text{no}\,x} = \frac{e^2\omega^2T^3}{\pi^2ca^4}\Bigg\{\dot{q}\Bigg[2\alpha(\alpha - \sin\alpha) - 2\omega T\theta_x^2(2\alpha + \alpha\cos\alpha - 3\sin\alpha)$$
$$+ \omega^2T^2\theta^2\theta_x^2\left(\frac{\alpha^2}{3} + 2 + 2\cos\alpha - \frac{4}{\alpha}\sin\alpha\right)\Bigg] +$$

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$$+\omega_H^2\omega^2T^3\theta_x^2\theta_y^2\left(\frac{\alpha^2}{4}+\cos\alpha-\frac{2}{\alpha}\sin\alpha+\frac{2}{\alpha^2}-\frac{2}{\alpha^2}\cos\alpha\right)\Bigg\}, \tag{4}$$

$$W_{n\omega y}=\frac{e^2\omega^2T^3}{\pi^2c\alpha^4}\Bigg\{q\Bigg[2\alpha(\alpha-\sin\alpha)-2\omega T\theta_y^2(2\alpha+\alpha\cos\alpha-3\sin\alpha)$$
$$+\omega^2T^2\theta^2\theta_y^2\left(\frac{\alpha^2}{3}+2+2\cos\alpha-\frac{4}{\alpha}\sin\alpha\right)\Bigg]$$
$$+\omega_H^2T\Bigg[\alpha^2-2\alpha\sin\alpha+2-2\cos\alpha-\omega T\theta_y^2\left(\alpha+\alpha\cos\alpha-4\sin\alpha+\frac{4}{\alpha}-\frac{4}{\alpha}\cos\alpha\right)$$
$$+\omega^2T^2\theta_y^4\left(\frac{\alpha^2}{4}+\cos\alpha-\frac{2}{\alpha}\sin\alpha+\frac{2}{\alpha^2}-\frac{2}{\alpha^2}\cos\alpha\right)\Bigg]\Bigg\}, \tag{5}$$

where the following notation has been introduced:

$$\alpha=\frac{1}{2}\omega T(2-\beta^2-\varepsilon+\theta^2). \tag{6}$$

For $|\alpha|\ll 1$ and $|\alpha|\gg 1$, the results (4) and (5) are considerably simplified. Let us write them in these limiting cases together with the conditions of applicability obtained from comparison with the following terms of the expansion.

For $|\alpha|\ll 1$

$$W_{n\omega x}\simeq\frac{e^2\omega^2T^3}{\pi^2c}\left[q\left(\frac{1}{3}+\frac{1}{20}\omega^2T^2\theta^2\theta_x^2\right)+\frac{1}{36}\omega_H^2\omega^2T^3\theta_x^2\theta_y^2\right], \tag{7}$$

$$\frac{1}{56}\omega_H^2\omega T^3\left|\alpha+2\omega T\theta_y^2\right|\ll 1;$$

$$W_{n\omega y}\simeq\frac{e^2\omega^2T^3}{\pi^2c}\left[q\left(\frac{1}{3}+\frac{1}{20}\omega^2T^2\theta^2\theta_y^2\right)+\frac{1}{36}\omega_H^2T\left(9+\omega^2T^2\theta_y^4\right)\right], \tag{8}$$

$$\frac{1}{9}\omega_H^2\omega T^3\left|\frac{\alpha}{5}+\frac{1}{10}\omega T\theta_y^2+\frac{5\alpha}{224}\omega^2T^2\theta_y^4+\frac{1}{21}\omega^3T^3\theta_y^6\right|\ll 1+\frac{1}{9}\omega^2T^2\theta_y^4.$$

For both polarizations the inequality must also be satisfied

$$q\omega T^2\left|\frac{7\alpha}{45}-\frac{1}{18}\omega T\theta^2+\frac{\alpha}{87}\omega^2T^2\theta^4+\frac{1}{113}\omega^3T^3\theta^6\right|\ll\frac{2}{3}+\frac{1}{20}\omega^2T^2\theta^4. \tag{9}$$

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For $|\alpha| \gg 1$

$$W_{n\omega x} \simeq \frac{e^2 T}{\pi^2 c(2-\beta^2-\varepsilon+\theta^2)^2}\left[4q\left(2+\frac{1}{3}\omega^2 T^2\theta^2\theta_x^2\right)+\omega_H^2\omega^2 T^3\theta_x^2\theta_y^2\right], \quad (10)$$

$$\omega_H^2\omega T^3\left|\frac{1}{\alpha}+\frac{1}{12}\omega T\theta_y^2\right| \ll 1;$$

$$W_{n\omega y} \simeq \frac{e^2 T}{\pi^2 c(2-\beta^2-\varepsilon+\theta^2)^2}\left[4q\left(2+\frac{1}{3}\omega^2 T^2\theta^2\theta_y^2\right)+\omega_H^2 T\left(4+\omega^2 T^2\theta_y^4\right)\right], \quad (11)$$

$$\omega_H^2\omega T^3\left|\frac{6}{\alpha}+\frac{3}{2}\omega T\theta_y^2+\frac{5}{2\alpha}\omega^2 T^2\theta_y^4+\frac{1}{6}\omega^3 T^3\theta_y^6\right| \ll 4+\omega^2 T^2\theta_y^4.$$

Here the following inequality must also be satisfied:

$$q\omega T^2\left|\frac{24}{\alpha}-\frac{2}{3}\omega T\theta^2+\frac{4}{\alpha}\omega^2 T^2\theta^4+\frac{1}{9}\omega^3 T^3\theta^6\right| \ll 4+\frac{1}{3}\omega^2 T^2\theta^4. \quad (12)$$

The presence of a magnetic field leads to an increase in the intensity and to a change in the degree of polarization of the radiation of the electron–positron pair.

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References

1. V. E. Pafomov, ZhETF, **49**, 1222 (1965).

Note: Figure translations are in progress. See original paper for figures.

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