THERMAL PHENOMENA IN THE FIELD OF A TROPICAL HURRICANE

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Abstract

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To investigate the mechanism by which the hurricane system is supplied with thermal energy originating from the strongly heated ocean surface, we first constructed a simplified scheme of the hydrodynamic field of such a hurricane, permitting integration of the equations of motion by quadratures and the derivation of approximate analytical expressions for some parameters of the system [(1)]. On the basis of that work we shall attempt to trace the thermal phenomena in the system and to verify how close to reality our conception is of the thermobaric field of a hurricane over the extent from the outer boundary of the entire field to the outer boundary of the "eye of the hurricane."

On the basis of the computed curve 1 in Fig. 1 of [(1)], one may conclude that the pressure drop from the outer boundary of the entire field ($r = R$) to $r_1 \approx\approx \frac{1}{3}R$ is comparatively small: about 2 mb in the particular case considered. The main, decisive pressure drop occurs over the interval from r_1 to r_0 (in our approximation, to the outer boundary of the "eye of the hurricane" —here it amounts to 84 mb).

According to the literature, one may assume that, in hurricanes of the scale considered, the upper boundary of the disturbed field lies at the level of the isobaric surface $p = 100$ mb, where all components of the wind velocity become zero over the interval from r_0 to $r = r_1$. Consequently, the indicated isobaric surface must coincide with an equipotential surface.

As is known, the geopotential Φ is expressed by the formula

$$\Phi = R_{\text{B}}\overline{T}_{\text{B}} \ln\left(p_{\text{lower}}/p\right), \tag{1}$$

where the specific gas constant for air is $R_{\text{B}} = 2.87 \cdot 10^6$ erg/g • deg. The virtual temperature $\overline{T}_{\text{B}}$ is averaged for the air column from Φ_{lower}, where the pressure is p_{lower}, to Φ, where the pressure becomes equal to p.

Passing from the expression of Φ in absolute units to the dynamic height H, expressed in dynamic meters, one may write [(2)]

Fig. 1

Figure 1: Fig. 1

$$H = 66.1\,\overline{T}_{\mathrm{B}} \log\left(p_{\mathrm{lower}}/p\right), \tag{2}$$

(log denotes the common logarithm). On the basis of hypsometric tables [3], this surface lies at a height of 18,640 m. There we also find that this corresponds to $H = 18\,210$ dynamic meters; $\log p = \log 100 = 2$.

Substituting the numerical values found into (2), we obtain

$$\overline{T}_{\mathrm{B}} = \frac{276}{\log p_{\mathrm{lower}} - 2}. \tag{3}$$

Here p_{lower} is the pressure at ocean level at the corresponding distance r from the center of the "eye." It is expressed by formula (7) and curve 1 in Fig. 1 of our work [1].

Substituting into formula (3) the values of p_{lower} from [1], we find the values of the virtual temperatures $\overline{T}_{\mathrm{B}}$, averaged vertically up to the upper boundary of the hurricane field, and then the differences between the values of $\overline{T}_{\mathrm{B}}$ at distance r from the axis of symmetry and at distance r_1, where the vertical component of the veloc-

the wind velocity passes through zero: instead of ascending flows, descending flows appear.

On the basis of (2), one may, with sufficient accuracy for our purposes, assume that these differences of virtual temperatures are equal to the differences of molecular temperatures T; they make it possible to judge how the vertically averaged temperature rises over the interval from $r = r_1$ to $r = r_0$. The calculated law of increase of air temperature in the most important part of the field of a tropical hurricane is shown by curve 1 in Fig. 1. As we see, the sharp pressure drop, calculated in (1), corresponds to an increase of the averaged temperature by 12° near the outer boundary of the "eye of the hurricane."

Fig. 1

Of course, this is only an approximate value, since the form of the pressure-drop curve in the immediate vicinity of this boundary is admittedly inaccurate.

But even such an approximate calculation has led us to a figure that closely agrees with the data of direct soundings of the field of tropical hurricanes. This once again emphasizes the decisive role of some kind of "core" in the field of a tropical hurricane, occupying on the ocean surface a circle with radius about 1/3 of the radius of the outer boundary of the entire field. In the particular case chosen, r_1 is approximately about 2° of a meridian arc. But the diameter of this core, $2r_1 \approx 4^\circ$, coincides with the width of the belt covered in the Atlantic Ocean by the North Equatorial Current. This is precisely why, over the stretch from Cape Verde (on the western coast of Africa) to the Lesser Antilles, this warm current serves as a guide in the displacement of the most powerful hurricanes from east to west.

Let us try to estimate what contribution to the heat budget of the hurricane core is made by that layer of air which, according to our work (1), lifts in the ascending flow a large amount of water vapor, releasing heat upon condensation.

In work (1), the expression for the tangential component v of the wind velocity, found from natural measurements in a theoretical work (4) as applied to a very limited region of the field, was extrapolated very far. Using modern photographs of cloud formations in the system of strong tropical hurricanes (taken from artificial Earth satellites), one may regard as practically constant the angle between the tangential component v and the full wind-velocity vector ($\alpha = 18^\circ$), and therefore, with sufficient justification, one may rely on the expression for the radial component u of the wind velocity in (1),

$$u = a - b\ln(r/r_0) \text{ m/sec} \tag{4}$$

and on the expression, following from it, for the vertical component w,

obtained in (1), as applied to the upper boundary of the surface layer of thickness $h = 500$ m:

$$w = (u - b)h/r \text{ m/sec.} \tag{5}$$

Let us single out, in the plane under study, a ring of radius r and width dr. Through its area there is carried upward each second, from below, an amount of heat released in the condensation of vapor,

$$2\pi r w f \lambda\, dr,$$

where f denotes the vapor content, in g/m^3, in the air above the ocean surface, and λ is the latent heat of condensation.

The air masses, moving with radial velocity in the direction toward the "eye of the hurricane," on average pass over the selected ring during a certain interval of time $dr/\bar{u}$, where $\bar{u} = \eta u$ denotes the radial velocity averaged over the vertical (at a distance r from the axis of symmetry of the hurricane system).

Let h_e denote the equivalent height of the entire layer of atmosphere encompassed by the hurricane (i.e., a layer of air of surface density δ, having the same mass as in nature). An elementary cylinder of thickness dr, cut out of this layer, must have the total heat capacity

$$2\pi r h_e \delta \cdot c_p \, dr.$$

Here c_p is the specific heat capacity of air at constant pressure.

On the basis of the foregoing, the averaged increase in air temperature $d\vartheta$, which the vertical flux from the surface layer h could produce throughout the entire layer h_e during the time $dr/\bar{u}$, is expressed as follows:

$$d\vartheta = -\frac{f\lambda h(u-b)}{\delta c_p h_e \eta u} \frac{dr}{r} = N\left(\frac{b}{u}-1\right)\frac{dr}{r}, \tag{6}$$

where

$$N = \frac{f\lambda}{\delta c_p \eta} \frac{h}{h_e}. \tag{7}$$

Substitute into (6) the expression for u from (4) and, for brevity, denote

$$\ln(r/r_0) = \xi.$$

After simple transformations, we obtain

$$\frac{d\vartheta}{N} = \frac{d\xi}{a/b-\xi} - d\xi. \tag{8}$$

Along the path from the cylinder of radius r_1, where ascending air currents begin, to a cylinder of some smaller radius r_2, the averaged temperature of the air in the layer up to the upper boundary of the hurricane field could rise by $\vartheta_2 - \vartheta_1$ under the action of the heat source considered (if there were no additional sources and losses of energy for the expansion of the air as pressure falls). This difference is determined by integrating equation (8):

$$\vartheta_2 - \vartheta_1 = N\left[(\xi_1 - \xi_2) - \ln\left(\frac{a/b-\xi_2}{a/b-\xi_1}\right)\right]. \tag{9}$$

To determine the numerical value of N, which first appeared in (6) and (7), we shall take the values usually obtained from measurements in the field of hurricanes: $f = eq = 0.9 \cdot 27.2 = 24.5$ g/m^3 at a surface-water temperature of 28°; $\lambda = 539$ cal/g; $\delta = 1250$ g/m^3; $c_p = 0.243$ cal/g • deg; on the basis of (4), one may set $\eta = 0.4$.

Taking into account that, at ocean level, an equivalent column of air of uniform density extending to a height of 8000 m would exert a pressure of 1000 mb, and that the pressure at the upper boundary of the hurricane field may be taken as

100 mb, we find $h_e = 7200$ m. The value $h = 500$ m has already been mentioned. Substituting all these numbers into (7), we obtain $N = 7.4^\circ$.

For this value of N, curve 2 shown in Fig. 1 has been calculated from equation (9). As we see, curves 1 and 2 are of the same type.

Thus, the contribution of condensation of vapor carried by ascending air currents from the 500-meter lower layer proved to be of the same order as the result of the algebraic summation of all terms of the heat balance of the hurricane core (from $r = r_0$ to $r = r_1$) and of the heat supplied by vapor condensation, and the expenditure of heat on the expansion of air in the region of sharply reduced pressure.

Until the dependence of the vertical component of the velocity w of the hurricane is obtained not only on r, but also on the coordinate z, it is impossible to determine all the missing components. But even now one can approximately calculate that part of the power of the heat source which falls on the investigated near-water layer of thickness $h = 500$ m within a circle of radius $r_1 = 225$ km.

Through an elementary ring of radius r and width dr, the following amount of latent heat released during condensation is carried upward through the surface under consideration, dQ cal/sec, namely

$$dQ = 2\pi r w f \lambda \, dr = 2\pi f \lambda h[(a - b) - b \ln(r/r_0)]dr. \tag{10}$$

Integrating (10), we find that the total amount of heat delivered upward from the near-water layer of the hurricane core will be

$$Q = 2\pi f \lambda h[a(r_1 - r_0) - b r_1 (r_1/r_0)]. \tag{11}$$

On the other hand, equation (5) at $r = r_1$ gives $w = 0$. Consequently, $\ln(r/r_0) = (a-b)/b$. Substituting this expression into (11), after transformations we obtain

$$Q = 2\pi f \lambda h(b r_1 - a r_0) \text{ cal/sec}. \tag{12}$$

Substituting here the known numerical values and passing from the power Q, expressed in calories per second, to the power W, expressed in kilowatts, the calculations give

$$W = 2 \cdot 10^{11} \text{ kW}.$$

It is evident that the hurricane wind mixes the upper layers of water along the path of the core in the ocean, and therefore, for the nourishment of a developing and established hurricane, it is necessary that not only at the very surface of the ocean, but also beneath it to a sufficient depth, there be maintained the water temperature required to support the thermal regime in the hurricane core. This explains the peculiarity of the main tracks along which hurricanes move, noted in work (5): only 4% of all tropical storms reach hurricane strength if the water temperature at a depth of 60 m differs from the surface temperature by more than 8.5°C.

The data on the temperature of surface waters in (6) testify still more clearly to the severity of the conditions for hurricane development. Namely, on the most important hurricane track, running westward along the parallel of Cape Verde, the temperature of the surface water at the end of August and beginning of September is only 2° higher than that characteristic of July. Meanwhile, according to hurricane statistics for 60 years, contained in work (6), the number of hurricanes at the August–September boundary exceeds by almost a factor of four the number observed in July.

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Note: Figure translations are in progress. See original paper for figures.