Soviet-era science, translated into English

PHYSICS

1969

SovietRxiv

View the original and related papers at https://sovietrxiv.org/items/ru-196901.68141

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.

SovietRxiv

Abstract

Full Text

PHYSICS

V. RYBARSKA

CONFIGURATION REPRESENTATION OF THE INTERACTION OF NUCLEONS IN THE SUPERFLUID MODEL OF THE NUCLEUS

(Presented by Academician N. N. Bogolyubov on 8 VII 1968)

The Hamiltonian of the superfluid model of the nucleus consists, as is known ([1]), of a one-particle term and of the interaction between nucleons of the last unfilled shell. This interaction contains a pairing and a multipole-multipole part. In primarily quantized form

$$H = \sum_{i=1}^{A} H_i + \sum_{i,j=1}^{n} V_{ij}(\mathbf{r}_i\rho_i, \mathbf{r}_j\rho_j), \tag{1}$$

where $H_i = T_i + U_i(\mathbf{r}_i\rho_i)$; A is the number of nucleons in the nucleus; ρ_i is the spin variable; n is the number of nucleons in the last shell. We know explicitly only the multipole-multipole part, which consists of terms of the type

$$C_\lambda r_i^\lambda r_j^\lambda P_{\lambda\mu}(\cos v_i) P_{\lambda\mu}(\cos v_j).$$

The pairing interaction was introduced ([2]) by analogy with the model Hamiltonian for a superconductor, where initially all essential interactions between electrons and between electrons and ions were taken into account. The model Hamiltonian ([3]) was obtained as a result of neglecting, in momentum space, many matrix elements of the interaction on the basis of clear physical arguments. But for a superconductor as well, the question is meaningful as to what interaction in x-space corresponds to such an effective interaction truncated in p-space. This question is much more interesting for the nucleus, where we do not know the nature of the interaction between nucleons, and pairing with respect to the quantum numbers of the last shell expresses an assumption about the result of various interactions between nucleons that are unknown to us.

We start from the second-quantized Hamiltonian

$$H = \sum_{ij} T_{ij} a_i^+ a_j + \frac{1}{4} \sum_{ijkl} V_{ijkl} a_i^+ a_j^+ a_{ka} l, \tag{2}$$

SovietRxiv

where $T^*_{ij} = T_{ij}$, $V_{ijkl} = -V_{jikl} = -V_{ijlk} = V^*_{klij}$.

Passing to the x-representation gives

$$H = \int T(x_1x_2)\psi^+(x_1)\psi(x_2)\,dx_1\,dx_2 + \\ + \int V(x_1x_2x_3x_4)\psi^+(x_1)\psi^+(x_2)\psi(x_3)\psi(x_4)\,dx_1\,dx_2\,dx_3\,dx_4, \tag{3}$$

where $\psi(x) = \sum_i \varphi_i(x)a_i$, $\varphi_i(x) = \langle x \mid i\rangle$, $\mid i\rangle = a_i^+ \mid 0\rangle$, $\mid 0\rangle$ is the vacuum,

$$T(x_1x_2) = \sum_{ij} T_{ij}\varphi_i(x_1)\varphi^*_j(x_2),$$

$$V(x_1x_2x_3x_4) = \frac{1}{4}\sum_{ijkl} V_{ijkl}\varphi_i(x_1)\varphi_j(x_2)\varphi^*_k(x_3)\varphi^*_l(x_4). \tag{4}$$

$\int ...\,dx$ means integration over $d\mathbf{r}$ and summation over ρ ($\rho = +,-$). The state vector of the system $|\rangle$ can be represented in the coordinate representation by means of its projections

$$\langle 0|\rangle, \qquad \langle x_i|\rangle, \qquad \langle x_ix_j|\rangle, \qquad \langle x_ix_jx_k|\rangle, ... \tag{5}$$

The transition from secondary to primary quantization consists, in this representation, in the following:

$$\langle x_1x_2|H|\rangle = H\varphi(x_1, x_2), \tag{6}$$

where $\tilde{H}$ is the operator of primary quantization in the x-representation.

For the interaction we obtain

$$\langle x_1x_2|V|\rangle = \int \tilde{V}(x_1x_2x_3x_4)\,\varphi(x_3x_4)\,dx_3\,dx_4, \tag{7}$$

where

$$\tilde{V}(x_1x_2x_3x_4) = V(x_2x_1x_3x_4) - V(x_1x_2x_3x_4). \tag{8}$$

The interaction is local by definition if

$$\int \tilde{V}(x_1x_2x_3x_4)\,\varphi(x_3x_4)\,dx_3\,dx_4 = \hat{V}(x_1x_2)\,\varphi(x_1x_2), \tag{9}$$

SovietRxiv

where $\hat{V}(x_1 x_2)$ is an operator of multiplication by a function in x-space.

In the case of a pairing interaction in unrestricted p-space,

$$\frac{1}{4}\sum_{ijkl} V_{ijkl} a_i^+ a_j^+ a_k a_l \tag{10}$$

reduces to

$$-G \sum_{p_1,\mathbf{p}_3} a_{p_1}^+ a_{-\mathbf{p}_1}^+ a_{-\mathbf{p}_3} a_{p_3}, \tag{11}$$

if $i \to \mathbf{p}_i \sigma_i$, and

$$V_{ijkl} = -\frac{1}{4} G \delta_{-\mathbf{p}_1,\mathbf{p}_2} \delta_{-\mathbf{p}_3\mathbf{p}_4}\, \Xi(\sigma_1\sigma_2)\, \Xi(\sigma_3\sigma_4), \tag{12}$$

where

$$\Xi(\sigma_i\sigma_j) = \left(\delta_{\sigma_i^+\sigma_j^-} - \delta_{\sigma_i^-\sigma_j^+}\right). \tag{13}$$

Correspondingly,

$$V(x_1x_2x_3x_4) =$$

$$= -\frac{G}{4} \sum_{\substack{\mathbf{p}_1,\mathbf{p}_3 \\ \sigma_1,\sigma_2,\sigma_3,\sigma_4}} \Xi(\sigma_1\sigma_2)\Xi(\sigma_3\sigma_4)\, \varphi_{p_1\sigma_1}(x_1)\varphi_{-\mathbf{p}_1\sigma_2}(x_2)\varphi^*_{p_3\sigma_3}(x_3)\varphi^*_{p_3\sigma_4}(x_4). \tag{14}$$

Let us assume, for simplicity, that the one-particle wave functions are plane waves. Substituting into (14), we obtain

$$V(x_1x_2x_3x_4) = -\frac{G}{4}\, \delta(\mathbf{r}_1 - \mathbf{r}_2)\delta(\mathbf{r}_3 - \mathbf{r}_4)\, \Xi(\rho_1\rho_2)\Xi(\rho_3\rho_4). \tag{15}$$

Equation (9) takes the form

$$\int \tilde{V}(x_1x_2x_3x_4)\, \varphi(x_3x_4)\, dx_3\, dx_4 = -\frac{G}{2}\, \delta(\mathbf{r}_1 - \mathbf{r}_2)\, \Xi(\rho_1\rho_2) \int \varphi(\mathbf{r}_3^-, \mathbf{r}_3^+)\, d\mathbf{r}_3. \tag{16}$$

It is clear that the locality condition is not fulfilled here. We are dealing with a nonlocal interaction. On the other hand, a local δ-like interaction between

SovietRxiv

particles with opposite projections of momenta and spins is effected by the potential

$$V(x_1x_2x_3x_4) = -\frac{G}{2}\delta(\mathbf{r}_1-\mathbf{r}_2)\delta_{\rho_1-\rho_2}(\delta(\mathbf{r}_1-\mathbf{r}_4)\delta(\mathbf{r}_2-\mathbf{r}_3)\delta_{\rho_1\rho_4}\delta_{-\rho_1\rho_3}-$$

$$-\delta(\mathbf{r}_1-\mathbf{r}_3)\delta(\mathbf{r}_2-\mathbf{r}_4)\delta_{\rho_1\rho_3}\delta_{-\rho_1\rho_4}). \tag{17}$$

The matrix element in p-space corresponding to this interaction takes the form

$$V_{p_1\sigma_1\mathbf{p}_2\sigma_2\mathbf{p}_3\sigma_3\mathbf{p}_4\sigma_4} = -\frac{G}{2}\delta_{\sigma_1,-\sigma_2}\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_3-\mathbf{p}_4)\left(\delta_{\sigma_1,\sigma_4}\delta_{\sigma_3,-\sigma_1}-\delta_{\sigma_1,\sigma_3}\delta_{\sigma_4,-\sigma_1}\right). \tag{18}$$

Substitution into the interaction Hamiltonian gives

$$-G\sum_{p_1,\mathbf{p}_3,\vec{\delta},\sigma} a^+_{p_10}a^+_{\vec{\delta}-\mathbf{p}_1,-\sigma}a_{-\mathbf{p}_3,-\sigma}a_{\vec{\delta}+\mathbf{p}_3,\sigma}. \tag{19}$$

Thus, in order to obtain the pair interaction (11), we must take into account only one term $\vec{\delta}=0$ from the sum over $\vec{\delta}$.

Let us consider the shell model with a spherical oscillator potential. Let there be nucleons of one kind in the last shell, and let pairing forces act between them for particles with opposite projections m and σ. The one-particle wave functions have the form

$$\varphi_{nlm\sigma}(r\nu\varphi) = \frac{1}{r}R_{nl}(r)Y_{lm}(\nu\varphi)\delta_{\sigma\rho}, \tag{20}$$

where $Y_{lm}(\nu,\varphi)$ are spherical functions,

$$R_{nl}(r) = \sqrt{\frac{2(2\nu)^{l+3/2}(n-1)!}{(\Gamma(n+l+1/2))^3}}\, r^{l+1}e^{-\nu r^2}L^{l+1/2}_{n+l-1/2}(2\nu r^2), \tag{21}$$

$\nu = M\omega/2\hbar$, M is the nucleon mass, ω is the frequency of oscillations, and L^k_n is a Laguerre polynomial.

In this case, pairing in the corresponding shell of quantum numbers gives

$$-G\sum_{\substack{2n_1+l_1=N_0\\2n_3+l_3=N_0\\|m_1|=0,1,\ldots,l_1\\|m_3|=0,1,\ldots,l_3}} a^+_{n_1l_1m_1^+}a^+_{n_1l_1-m_1^-}a_{n_3l_3-m_3^-}a_{n_3l_3m_3^+}. \tag{22}$$

SovietRxiv

Hence

$$V(x_1x_2x_3x_4) =$$

$$= -\frac{G}{4} \sum_{\substack{2n_1+l_1=N_0,\\ 2n_3+l_3=N_0\\ |m_1|=0,1,\ldots,l_1\\ |m_3|=0,1,\ldots,l_3\\ \sigma_1\sigma_2\sigma_3\sigma_4}} \varphi_{n_1l_1m_1\sigma_1}(1)\varphi_{n_1l_1-m_1\sigma_2}(2)\varphi_{n_3l_3-m_3\sigma_3}(3)\varphi^*_{n_3l_3m_3\sigma_4}(4)\times$$

$$\times\Xi(\sigma_1\sigma_2)\Xi(\sigma_3\sigma_4). \tag{23}$$

Using the summation formula for spherical functions, we arrive at

$$\nu(x_1x_2x_3x_4) = -\frac{1}{4}G \sum_{2n_1+l_1=N_0} \frac{(-1)^{l_1}(2l_1+1)}{4\pi}\frac{1}{r_1}\frac{1}{r_2}R_{n_1l_1}(r_1)R_{n_1l_1}(r_2)P_{l_1}(\nu_{12})\times$$

$$\times \sum_{2n_3+l_3=N_0} \frac{(-1)^{l_3}(2l_3+1)}{4\pi}\frac{1}{r_3}\frac{1}{r_4}R_{n_3l_3}(r_3)R_{n_3l_3}(r_4)\Xi(\rho_1\rho_2)\Xi(\rho_3\rho_4). \tag{24}$$

It is seen that the expression obtained for $V(x_1x_2x_3x_4)$ leads to a nonlocal equation for the two-particle wave function, and that the interaction between particles depends on the form of the shell-model potential. Moreover, $V(x_1x_2x_3x_4)$ cannot be summed and will not lead us to $\delta(\mathbf{r}_1 - \mathbf{r}_2)$, as was the case in the unrestricted p-space.

Thus, we have come to the conclusion that the pairing $(nlm\sigma, nl-m-\sigma)$, acting only in the last unfilled shell N_0, cannot be assigned a local two-particle short-range potential in x-space.

I express my gratitude to Prof. V. G. Solov'ev and to the members of the nuclear theory group in Dubna for discussion and comments.

Institute of Theoretical Physics
Wrocław University
Wrocław, Poland

Received
2 VII 1968

SovietRxiv

CITED LITERATURE

1. V. G. Solov' ev, *The Structure of Complex Nuclei*, Moscow, 1966.

2. V. G. Solov' ev, ZhETF, **35**, 823 (1958); Nucl. Phys., **9**, 655 (1958-1959); S. A. Belaev, Mat. Fys. Medd. Dan. Vid. Selsk., **31**, No. 11 (1959).

3. J. Bardeen, L. Cooper, J. Schrieffer, Phys. Rev., **108**, 1175 (1957); N. N. Bogolyubov, V. V. Tolmachev, D. V. Shirkov, *A New Method in the Theory of Superconductivity*, Publishing House of the Academy of Sciences of the USSR, 1958.

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.