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Abstract

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CYBERNETICS AND CONTROL THEORY

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ON SELF-ADJUSTING DECODING AUTOMATA

(Presented by Academician V. M. Glushkov on 12 XII 1968)

This paper considers decoding automata for memoryless encodings that self-adjust after any errors in the input sequence (consisting in the random replacement of one input letter by another), and also after certain (but not arbitrary) automaton failures and certain shifts of the input sequence. Conditions are given that are necessary and sufficient for an encoding to have a self-adjusting automaton (in this sense, generalized in comparison with ⁽¹⁾), one of which is a generalization of the synchronization property introduced by Levenshtein in ⁽²⁾.

The terminology, notation, and some results concerning decoding automata here fully correspond to the article ⁽¹⁾.

1. In this section, by an automaton we shall mean a connected initial automaton without outputs with a finite input alphabet (see ⁽³⁾).

Definition 1. An automaton $\mathfrak{A} = (S, s_0, B, \delta)$ (where S is the set of states, s_0 the initial state, B the input alphabet, and δ the transition function) will be called **semibounded** if there exists a number $l \geq 0$ such that for any word $\beta \in B^l$ and any words $\beta', \beta'' \in \bigcup_{n=0}^{\infty} B^n$, from the equality of lengths $\lambda(\beta') = \lambda(\beta'')$ there follows the equality

$$\delta(s_0, \beta' \beta) = \delta(s_0, \beta'' \beta), \tag{1}$$

where $\delta(s, \gamma)$, for any $s \in S$, $\gamma \in \bigcup_{n=0}^{\infty} B^n$, denotes the state to which the automaton passes from state s after the word γ is applied to the input.

If it is necessary to emphasize the value of the number l mentioned in the definition, we shall speak of an l -semibounded automaton. It is clear that an l -semibounded automaton is also l' -semibounded for any $l' > l$, and that, in addition to equality (1), for it the equality

$$\delta(s, \beta' \beta) = \delta(s, \beta'' \beta) \tag{2}$$

also holds for any $s \in S$, $\beta \in B^l$, $\beta', \beta'' \in \bigcup_{n=0}^{\infty} B^n$, provided only that $\lambda(\beta') = \lambda(\beta'')$.

A special case of semibounded automata is formed by those for which equality (1) is satisfied without the restriction $\lambda(\beta') = \lambda(\beta'')$ on the words β' and β'' , i.e., for arbitrary β' and β'' . Such automata, under the name of bounded (l -bounded) automata, have been studied in a number of works (see, for example, (4)) and have been used as decoding automata (see (1)). Whereas all bounded automata are finite, semibounded automata may, as will be clear from what follows, also be infinite; the conditions for finiteness of a semibounded automaton are given below.

The intuitive meaning of the concept of an l -semibounded automaton is that the state to which the automaton arrives in the course of its operation on an input word depends on no more than the last l letters of that word; therefore any error in the input sequence affects the operation of the automaton for no more than l clock cycles, after which the automaton self-adjusts and operates as if the error had not occurred. Such an automaton can

self-adjust after the random replacement of one state s by another s' (a failure), if s and s' are attainable from s_0 by words of the same length. After shifts of the input sequence, a semibounded automaton, generally speaking, does not self-adjust. In contrast to this, the bounded automata studied earlier do self-adjust completely (after any shifts and errors in the inputs and after any failures).

In an entirely analogous way a partial l -semibounded automaton is defined.

Definition 2. For the automaton $\mathfrak{A} = (S, s_0, B, \delta)$, denote by $\delta_r(x_1 \dots x_r)$ the function $B^r \rightarrow S$ equal to $\delta(s_0, x_1 \dots x_r)$ ($r = 1, 2, \dots$), and let $\delta_0(\Lambda) = s_0$.

The functional sequence

$$\bar{\delta} = [\delta_0, \delta_1(x_1), \delta_2(x_1 x_2), \dots, \delta_r(x_1 \dots x_r), \dots] \quad (3)$$

will be called the **complete transition characteristic** (c.t.c.) of the automaton \mathfrak{A} .

Proposition 1. An automaton \mathfrak{A} is l -semibounded if and only if its c.t.c. has the form

$$\bar{\delta} = [\delta_0, \delta_1(x_1), \dots, \delta_l(x_1 \dots x_l), \delta_{l+1}(x_2 \dots x_{l+1}), \dots], \quad (4)$$

i.e., if each function in the c.t.c. depends on no more than its last l arguments.

Theorem 1. If an automaton is l -semibounded, then the following assertions are equivalent:

- a) the automaton \mathfrak{A} is finite;

- b) the c.t.c. of the automaton \mathfrak{A} is ultimately a periodic sequence of functions of l' variables;
- c) for the automaton \mathfrak{A} there exists a number $l > l'$ and a number $m \geq 1$ such that, for any $\beta \in B^l$ and any $\beta', \beta'' \in \bigcup_{n=0}^{\infty} B^n$, from the relation

$$\lambda(\beta'') = \lambda(\beta') \pmod{m}$$

there follows equality (1).

Definition 3. An automaton possessing property c) from Theorem 1 will be called (l, m) -semibounded.

The intuitive meaning of this definition is that such an automaton self-adjusts after any errors in the input sequence, after any shift of it by km units ($k = 1, 2, \dots$), and after such failures when the state s is replaced by a state s' attainable from s by a word of length km for some $k = 1, 2, \dots$

Let us note that l -bounded automata are nothing other than $(l, 1)$ -semibounded automata, and conversely.

Definition 4. Let arbitrary numbers $l, m \geq 1$ be given. Construct a special automaton $\mathfrak{A}_{l,m} = (S_{l,m}, s_0, B, \delta_{l,m})$ as follows:

- 1) $S_{l,m} = \{s_\beta; \beta \in \bigcup_{n=0}^{l+m-1} B^n\}$, where the s_β are symbols, and for words β' and β'' of the same length and with a common ending of length l , by definition we set $s_{\beta'} = s_{\beta''}$;
- 2) $s_0 = s_\Lambda$, where Λ is the empty word;
- 3) the transition function for $b \in B$ is determined by the condition

$$\delta_{l,m}(s_\beta, b) = \begin{cases} s_{\beta b}, & \text{if } \lambda(\beta b) < l, \\ s_\gamma, & \text{where } \beta b = \beta' \gamma, \quad \lambda(\gamma) = \text{rest } \frac{\lambda(\beta b) - l}{m} + l, \quad \text{if } \lambda(\beta b) \geq l; \end{cases}$$

(in other words, γ is the ending of length

$$\text{rest } \frac{(\lambda b)\beta - l}{m} + l$$

of the word βb).

The automaton $\mathfrak{A}_{l,m}$ will be called the **free (l, m) -semibounded automaton**. The justification for this name is given by the following.

Proposition 2. *The automaton \mathfrak{A} and any of its factor automata are (l, m) -semibounded, and every (l, m) -semibounded automaton is a homomorphic image of the automaton $\mathfrak{A}_{l,m}$.*

2. We shall now consider finite asynchronous automata $\mathfrak{A} = (S, s_0, B, A, \delta, \varphi)$ with output alphabet A and output function φ , i.e., such that the values of the output function may be words of length different from 1 in the alphabet A (see $(\bar{5}, 1)$). An asynchronous (and, possibly, partial) automaton will be called **semibounded** if it is such as an automaton without outputs.

Let alphabets $B = \{b_1, \dots, b_r\}$, $A = \{a_1, \dots, a_m\}$, and also a dictionary $V = \{v_1, \dots, v_m\}$ in the alphabet B , be given. A coding $K_{v_1 \dots v_m}^{a_1 \dots a_m}$ is called (see (1)) a mapping of the set of words in the alphabet A (messages) into the set of words in the alphabet B (codes), under which

$$a_{i_1} \dots a_{i_k} \rightarrow v_{i_1} \dots v_{i_k}.$$

Definition 5. Let $\beta \in \bar{B}(V)$. A deciphering

$$\xi * v_{i_1} * \dots * v_{i_k} * \eta$$

of the word β will be called an **m -deciphering** ($m \geq 1$), if there exists a $\gamma \in \bar{B}(V)$ for which three conditions are fulfilled: 1) $\gamma = \beta' \beta$, i.e., β is the ending of the word γ ; 2) $\lambda(\beta') \equiv 0 \pmod{m}$; 3) there exists such an initial deciphering of the word γ , of the form

$$*v_{j_1} * \dots * v_{j_s} * v_{i_k} * \dots * v_{i_{k_1}} * \eta,$$

where $\lambda(\xi) < \lambda(v_{j_s})$ (in other words, this initial deciphering of the word γ induces on the word β the given deciphering of it).

Definition 6. We shall say that the word β is an **m -ending** of the word γ , if $\gamma = \beta' \beta$ and $\lambda(\beta') \equiv 0 \pmod{m}$. The set of all m -endings of words from the set M will be denoted by \bar{M}^m .

Theorem 2. If the coding $K_{v_1 \dots v_m}^{a_1 \dots a_m}$ (abbreviated K) possesses the property of infinite mutual uniqueness, then the following assertions are equivalent:

- a) the coding K possesses a finite semibounded decoding automaton;
- b) (the \mathfrak{S} -property) for the coding K there exist numbers $l, m \geq 1$ such that, if

$$\beta b \in \bar{B}(V) \cap \left(\bigcup_{n=0}^{l+m-1} B^n \right)$$

and βb has an m -deciphering of the form

$$\xi * v_{i_1} * \dots * v_{i_k} * \dots * v_{i_{k_1}} * \eta,$$

where

$$\lambda(v_{i_{k+1}} \dots v_{i_{k_1}} \eta) = t_{\min},$$

then, first, every m -deciphering of the word βb has the form

$$\xi' * v_{j_1} * \dots * v_{j_s} * \dots * v_{j_{s_1}} * \eta',$$

where

$$\lambda(v_{j_{s+1}} \dots v_{j_{s_1}} \eta') = t_{\min},$$

and $v_{j_s} = v_{i_k}$, and, second, every word

$$\beta' b \in \bar{B}(V) \cap \left(\bigcup_{n=0}^{l+m-1} B^n \right)$$

such that $\lambda(\beta') = \lambda(\beta)$ and the endings of length l of β and β' are identical, has an m -deciphering of the form

$$\xi'' * v_{h_1} * \dots * v_{h_p} * v_{h_{p+1}} * \dots * v_{h_{p_1}} * \eta'',$$

where

$$\lambda(v_{h_{p+1}} \dots v_{h_{p_1}} \eta'') = t_{\min},$$

and $v_{h_p} = v_{i_k}$.

- c) (the m -synchronization property) for the coding K there exist numbers $m, T \geq 1$ such that

$$\bar{B}(V) \cap \bar{B}^m(V) \cap \left(\bigcup_{n=T}^{\infty} B^n \right) \subset B(V).$$

The proof of the theorem includes a method for constructing an (l, m) -semibounded decoding automaton for a coding possessing the \mathfrak{S} -property.

We note that the 1-synchronization property coincides with the synchronization property of Levenshtein ⁽²⁾.

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Note: Figure translations are in progress. See original paper for figures.

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