

**ON AN UPPER BOUND
FOR THE LENGTHS OF
SHORTEST
HOMOGENEOUS
EXPERIMENTS FOR
RECOGNIZING THE
FINAL STATE FOR
ALMOST ALL
AUTOMATA**

MATHEMATICS

1969

SovietRxiv

View the original and related papers at <https://sovietrxiv.org/items/ru-196901.58098>

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.

Abstract

Full Text

UDC 519.95

MATHEMATICS

A. D. KORSHUNOV

ON AN UPPER BOUND FOR THE LENGTHS OF SHORTEST HOMOGENEOUS EXPERIMENTS FOR RECOGNIZING THE FINAL STATE FOR ALMOST ALL AUTOMATA

(Presented by Academician S. L. Sobolev on 29 IV 1968)

In the present note we consider the problem of recognizing the final state of an automaton ⁽¹⁾, which consists of the following. Suppose that a completely specified noninitial automaton is given, described as a “black box,” and that the experimenter can obtain information about its states only by applying input letters and observing output letters. On the basis of these observations the experimenter must determine the state in which the automaton under consideration is found, i.e., the state of the automaton after the last input letter has acted on it. This final state, generally speaking, can be determined even in the case when the initial state of the automaton remains unknown. The procedure for determining the final state of an automaton after introducing a suitable finite sequence of input letters and observing the results at the output of the automaton is called an experiment for recognizing the final state, or simply an experiment. An experiment is called homogeneous if the experimenter chooses the input sequence before the beginning of the experiment and does not change it in the course of the experiment. If, however, the input sequence is such that each successive input letter is chosen by the experimenter as a function of the preceding input and output letters, then the experiment is called inhomogeneous.

By $u(A)$ and $e(A)$ we shall denote, respectively, the length of the shortest homogeneous and the shortest inhomogeneous experiments (the term shortest has its obvious meaning) for recognizing the final state of an automaton A .

Let $R_1(m, n, k)$ be the set of all distinct reduced automata (i.e., automata with pairwise distinguishable states) of Mealy ⁽²⁾ with input letters x_1, x_2, \dots, x_m , output letters y_1, y_2, \dots, y_n , and states q_1, q_2, \dots, q_k . By $R_2(m, n, k)$ we denote the subset of the set $R_1(m, n, k)$ consisting of Moore automata, i.e., such automata in which the output depends only on the state and does not depend on the current input. The set of those automata from $R_1(m, n, k)$ in which any pair of distinct states, under the action of each input letter, passes into distinct

states will be denoted by $R_3(m, n, k)$. Automata from $R_3(m, n, k)$ are called permutation automata ^(3,4).

The study of the lengths of shortest experiments (both homogeneous and inhomogeneous) for recognizing the final state of automata from the classes $R_1(m, n, k)$ and $R_2(m, n, k)$ is the subject of the works ^(1,3,5-7).

The most definitive results were obtained by Hibbard ⁽⁶⁾. He proved the following assertions:

- 1) For any automaton $A \in R_1(m, n, k)$

$$u(A) \leq k(k-1)/2;$$

at the same time, in the class $R_1(m = k-1, 2, k)$ there exists at least one permutation automaton A such that $e(A) \geq k(k-1)/2$.

- 2) For any automaton $A \in R_2(m, n, k)$

$$u(A) \leq (k-1)(k-2)/2 + 1;$$

at the same time, in the class $R_2(m = k-2, 2, k)$ there is at least one permutation automaton A such that

$$e(A) \geq (k-1)(k-2)/2 + 1.$$

The following question arises. Does there exist a sequence of such subclasses $\{\tilde{R}_i(m, n, k)\}_{k=1}^{\infty}$, $i = 1, 2, 3$, such that $\tilde{R}_i(m, n, k)$ contains "almost all" automata* from $R_i(m, n, k)$, and for any automaton $A \in \tilde{R}_i(m, n, k)$ the length of the shortest homogeneous (or, in the worst case, inhomogeneous) experiment for recognizing the final state would be substantially less than the upper bound established by Hibbard.

A positive answer to this question follows from the following facts. It is known that if the degree of distinguishability ⁽⁸⁾ of an automaton $A \in R_1(m, n, k)$ is equal to h , then

$$e(A) \leq \sum_{i=1}^h i + (k-h-1)h < kh.$$

At the same time it is known ^(4,9) that if $m = \text{const} \geq 2$ and $n = \text{const} \geq 2$, then for almost all automata $A \in R_i(m, n, k)$, $i = 1, 2, 3$, the degree of distinguishability does not exceed $\log_m \log_n k + 4$.

Thus, for any $m = \text{const} \geq 2$ and $n = \text{const} \geq 2$, for almost all automata $A \in R_i(m, n, k)$, $i = 1, 2, 3$, the relation

$$e(A) < k(\log_m \log_n k + 4)$$

holds.

It turns out that in fact the following stronger fact holds, for the formulation of which the present note has been written.

Theorem. *For any $m = \text{const} \geq 2$ and $n = \text{const} \geq 2$, for almost all automata $A \in R_i(m, n, k)$, $i = 1, 2, 3$, the length of the shortest homogeneous experiment for recognizing the final state satisfies the relation*

$$u(A) < 5 \log_n k.$$

Remark. An essential point in the proof of this theorem is the establishment of the fact that, for almost all automata from $R_i(m, n, k)$, $i = 1, 2, 3$, $m = \text{const} \geq 2$, $n = \text{const} \geq 2$, as homogeneous experiments (generally speaking, not shortest ones) for recognizing the final state, one can use one and the same input sequence of the form $x_1^r x_2 x_1^r$, where $x_1^r = x_1 x_1 \dots x_1$ (r times), and $r = [2.5 \log_n k] - 1$.

It is easy to see that any permutation automaton A has the following property: when the input and output sequences and the final state of this automaton are known, it is always possible to determine in which initial state the automaton A was.

From this fact and the theorem formulated above we obtain

Corollary. *For any $m = \text{const} \geq 2$ and $n = \text{const} \geq 2$, for almost all automata $A \in R_3(m, n, k)$, the length of the shortest homogeneous experiment for recognizing the initial state does not exceed $5 \log_n k$.*

Institute of Mathematics
Siberian Branch of the Academy of Sciences of the USSR

Received
12 IV 1968

REFERENCES

- ¹ E. F. Moore, in: *Automata*, IL, 1956.
- ² V. M. Glushkov, *Synthesis of Digital Automata*, Moscow, 1962.
- ³ S. Ginsburg, *J. Assoc. Comput. Math.*, 5, No. 3, 266 (1958).
- ⁴ A. D. Korshunov, *DAN*, 182, No. 2 (1968).
- ⁵ A. A. Karatsuba, *UMN*, 15, issue 3, 157 (1960).
- ⁶ T. H. Hibbard, *J. Assoc. Comput. Math.*, 8, No. 4, 601 (1961).
- ⁷ Z. Bavel, *Switching Circuit Theory and Logical Design*, N. Y., 1964, p. 234.
- ⁸ N. E. Kobrinskii, B. A. Trakhtenbrot, *Introduction to the Theory of Finite Automata*, Moscow, 1962.
- ⁹ A. D. Korshunov, *Discrete Analysis*, issue 10, 39 (1967).

* We say that almost all automata of the class $R_i(m, n, k)$, $i = 1, 2, 3$, are contained in $\widetilde{R}_i(m, n, k)$ if the ratio of the number of automata in $\widetilde{R}_i(m, n, k)$ to the number of automata in $R_i(m, n, k)$ tends to one as $k \rightarrow \infty$.

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.