THE ENERGY CONDITION FOR CRACK GROWTH IN ELASTIC-PLASTIC BODIES

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Fig. 1

Figure 1: Fig. 1

Abstract

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THEORY OF ELASTICITY

E. M. MOROZOV

THE ENERGY CONDITION FOR CRACK GROWTH IN ELASTIC-PLASTIC BODIES

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We shall show the possibility of using the energy criterion of equilibrium to solve problems of the theory of cracks in an ideal elastic-plastic body. The discussion will be carried out for the case in which the plastic deformation is concentrated in a narrow zone ahead of the crack edge ([1,2]). The thickness of this zone is of the order of the elastic displacements (Fig. 1a). Cracks with a thin plastic zone ahead of the edge are considered for the convenience of further analysis*, which is reduced to the solution of an elastic problem instead of an elastic-plastic one. This reduction is based on the fact that, in the linearized formulation, the thin plastic zone may be schematically replaced by an additional cut, along whose banks forces are applied that replace the action of the plastically deformed material (Fig. 1b).

We shall dwell in particular on the circumstance that, in the model under consideration, the region of plastic nonlinear effects (of size d, Fig. 1a) changes with the change in the external load and represents plastically deformed material, whose stressed and deformed state should be determined from the solution of the elastic-plastic problem. By assumption, the thickness of the plastic zone $2v(x)$ in the symmetric problem is sufficiently small to permit a linearized formulation of the problem, but at the same time it is large in comparison with the interatomic distance; consequently, in this scheme the stresses on the surface of the additional cut differ from the cohesion forces of interatomic interaction. Thus the scheme under consideration differs essentially from the scheme presented in works ([3]), which proceeds from the presence, in the region $d \ll l$, of interatomic interaction forces depending, according to a known physical law, on the distance $2v(x)$, comparable with the interatomic distance; it is understood here that in the region $d \ll l$ the material between the crack surfaces is absent, and the region d itself does not change with the change in external load.

Fig. 2.

Figure 2: Fig. 2.

Fig. 1

* A more general case is also possible, when the plastic zone ahead of the crack tip is not thin (Fig. 2). Here, however, this case is not considered.

In the formulation of the problem proposed above, narrow and long plastic zones ($d \sim l$) in front of the crack tip were observed experimentally in tension of plates with a crack ([4]).

The size d of the plastic zone is not bounded by any limits, and for a sufficiently small crack length the onset of general yielding in the given cross-section of the body is possible, for which $d \to \infty$.

The equilibrium criterion expressing the law of conservation of energy under an actual or possible increment of the crack area may be written in the form ([5,6])

Fig. 2.

$$\delta W + \delta U_0 = \delta A, \tag{1}$$

where δA is the mechanical work of the external forces, δW is the volume potential energy of elastic deformation of the body, and δU_0 is the fracture energy.

In our case, the expenditure of energy for the creation of new rupture surfaces, i.e., the fracture energy, is in fact determined by the energy of plastic deformation δW_p, i.e., $\delta U_0 = \delta W_p$. This fracture energy differs from the fracture energy of an elastic body in that here δU_0 is wholly determined by the expenditure of energy for the work of plastic deformation, whereas for a brittle body, by definition, $d = 0$; therefore for a brittle body δU_0 is part of the internal energy, not necessarily associated with the work of the cohesive surface forces acting on the edges of the already formed rupture, which may be determined through the energy flux at the crack tip, determined by the singular elastic solution ([5]). Therefore, in contrast to an ideal elastic brittle body, the fracture-energy density γ for the model under consideration cannot, generally speaking, be regarded as a material constant; in this case the quantity $\gamma = \delta W_p / \delta S$ (the energy of plastic deformation per unit area of the newly formed crack area) depends on the manner of application of the external loads, on the shape and dimensions of the body, and in particular on the crack dimensions.

To the variational condition (1) three additional conditions should be added: 1) To determine the size d of the plastic zone in front of the crack edge (for example, smooth closing of the boundaries of the plastic zone at its end or, what is the same, continuity of the stresses at this end). 2) To determine the stresses σ_{0i} on the surface of the additional cut (for example, either the solution of an independent elastic-plastic problem for the neighborhood of the crack edge using known plasticity conditions (^{7}), or the prescription of this stress, which may be different, in particular, constant and equal to the yield limit). 3) To fix the limiting value of δW_p, which is necessary (^{7,8}) for the study of cracks capable of propagating; otherwise this will be an elastic-plastic problem for a stationary cut (for example, equality of the greatest separation between the boundaries of the plastic zone to some material constant δ_k, or experimental data).

Let a body with a crack be given, in equilibrium under the action of prescribed loads q_i on the surface of the body Σ and on the crack surface S. The work of the external forces is

$$\delta A = \int_{\Sigma+S} q_i \delta u_i \, d\sigma, \tag{2}$$

where u_i are the components of the displacement vector; the numerical index indicates the number of the state introduced below.

Without disturbing the stressed and deformed state of the elastic part of the body, we mentally remove the plastically deformed volume in front of the crack edge (the first state). Then there remains an elastic body with a cut, whose surface includes the crack surface S and

the surface of the boundary of the plastic zone Ω, on which the stress σ_0, identified here with the yield limit, is distributed. In this case (^{8}, ^{9})

$$\delta W_p = \int_{\Omega} |\sigma_{0i} \delta u_{i1}^{*}| \, d\sigma, \tag{3}$$

where the displacement u_{i1}^{*} is determined with allowance for the third of the indicated additional conditions.

For the first state we have

$$\delta W = \frac{1}{2} \int_{\Sigma+S} q_i \delta u_{i1} \, d\sigma + \frac{1}{2} \delta \int_{\Omega} \sigma_{0i} u_{i1} \, d\sigma. \tag{4}$$

Let us introduce, according to (^{10}), a second state–the same body with loads q_i on Σ, with a cut $S+\Omega$, on whose surface stresses p_i act, ensuring deformation of the body as a continuous one. The bodies in both states are identical and elastic, which makes it possible to write Betti' s theorem, from which we obtain

$$\int_{\Sigma} q_i u_i \, d\sigma = \int_{\Sigma+S} q_i u_i^0 \, d\sigma - \int_{S+\Omega} p_i u_i \, d\sigma.$$

We use this relation to reduce formulas (2) and (4) to another form (^{11}).

The displacements u_{i1} are represented, on the basis of the superposition principle of the linear theory of elasticity, in the form of the sum of the displacements u_i^0 of the body without a crack, under the action of the load q_i on Σ (the body of the second state), and the displacements u_i of the body without the load q_i on Σ, with a crack, on whose surface the stresses $q_i - p_i$ act on S and $\sigma_{0i} - p_i$ act on Ω*.

Substituting expressions (2), (3), and (4) into condition (1), we obtain**

$$2\int_\Omega |\sigma_{0i}\delta u_i^*|\,d\sigma + \delta\int_S p_i u_i\,d\sigma - \delta\int_S q_i(2u_i^0 + u_i)\,d\sigma +$$

$$+\delta\int_\Omega (p_i + \sigma_{0i})u_i\,d\sigma - \int_\Sigma q_i \delta u_i^0\,d\sigma = 0. \tag{5}$$

In this form the energy criterion of equilibrium, in combination with the equations of the theory of elasticity, is suitable for solving specific problems of crack theory. We indicate the following types of problems solved with the aid of condition (5). 1) Determination of the limiting (critical) equilibrium state of a body with a crack by varying the area of the crack under constant external load; in particular, for the two-dimensional problem $\delta = (\partial/\partial l)\delta l$ (in this case $q_i = \text{const},\ \delta u_i^0 = 0$). 2) Determination of slow subcritical crack growth with increasing load; the varied state coincides with the actual equilibrium state, in which the external loads have another value; for the two-dimensional problem

$$\delta = \left(\frac{\partial}{\partial l} + \frac{\partial}{\partial q}\frac{dq}{dl}\right)\delta l$$

(q is the parameter of the external load, $q = q(l)$). 3) Determination of the number of cycles until the crack reaches the critical state under repeated static loading. 4) The possibility of applying approximate computational methods for determining the indicated characteristics in bodies of finite dimensions.

Condition (5), for $\Omega = 0$ and with the introduction of the experimentally determined quantity γ, passes into the elastic solution (^{11}).

We give the results of solving, by condition (5), certain plane and spatial problems.

* The minus sign before p_i denotes the direction of the vector opposite to its direction in the continuous body.

** Here the vectors p_i, σ_{0i}, and u_i are mutually oppositely directed; therefore the corresponding terms must be less than zero.

Curve 1 in Fig. 3 reflects the critical stress in tension of an infinite plane with a single rectilinear crack ($\lambda = p/\sigma_0$, p is the applied stress, $\zeta = l/c$, l is half the crack length, $c = \pi E\delta_k/8(1-\nu^2)\sigma_0$, E, ν are the modulus of elasticity and

Fig. 3

Figure 3: Fig. 3

Fig. 4

Figure 4: Fig. 4

Poisson' s ratio). Lines 2 show, for this case, the slow growth of a crack from some initial length to the critical one—the maxima of these curves lie on the curve of critical loads. Curve 3 shows the critical stress in tension of a space with a disk-shaped (circular in plan) crack ($\zeta = l/c$, l is the crack radius). Curves 4 show the character of the slow growth of a disk-shaped crack at a stress smaller than the critical one.

Fig. 3

The results obtained coincide with those that follow from the δ_k-theory $^{(2)}$, both for the critical stress and for slow crack growth. A description of the latter case was obtained with the corresponding extension of the formulation of the δ_k-theory. In Fig. 4 one can see the dependence of the crack length under repeated static tension of a plate, which makes it possible to compute the number of cycles to complete fracture. A sequence of two loading regimes is shown ($\lambda_{\text{max}} = 0.7$ in the first regime and 0.45 in the

Fig. 4

second) at the same cycle asymmetry coefficient $r = \lambda_{\text{min}}/\lambda_{\text{max}}$. It was assumed that upon unloading the crack length remains unchanged, and complete fracture is determined by the attainment by the stress of the critical value $^{(12)}$.

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Moscow Engineering Physics Institute

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