STIMULATED SCATTERING BY SURFACE WAVES

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Fig. 1

Figure 1: Fig. 1

Abstract

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STIMULATED SCATTERING BY SURFACE WAVES

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1. As is known ([1,2]), in light scattered from the boundary between two media with densities $\rho^{\mathrm{I,II}}$ and dielectric constants $\varepsilon^{\mathrm{I,II}}$, fields $E_{2,3} \sim \exp(i\mathbf{k}_{2,3}\mathbf{r} - i\omega_{2,3}t)$ arise at the combination frequencies $\omega_{2,3} = \omega_1 \mp \operatorname{Re}\Omega(q)$, with wave vectors $\mathbf{k}_{2,3} = \mathbf{k}_1 \mp \mathbf{q}$, where Ω and $\mathbf{q}$ are the frequency and wave vector of a surface (for example, gravitational, capillary, or Rayleigh) wave (s.w.), while ω_1 and $\mathbf{k}_1$ are the corresponding quantities for the incident wave. The scattered field is found from the boundary conditions at the surface $z = \zeta(x, y, t) = \zeta_{q\Omega}\exp(i\mathbf{q}\mathbf{r} - i\Omega t)$, where $\zeta_{q\Omega}$ is the amplitude of the s.w.:

Fig. 1

$$[\mathbf{n}, \mathbf{E}^{\mathrm{I}} - \mathbf{E}^{\mathrm{II}}]_{z=\zeta(x,y,t)} = 0, \qquad [\mathbf{n}, \mathbf{H}^{\mathrm{I}} - \mathbf{H}^{\mathrm{II}}]_{z=\zeta(x,y,t)} = 0. \tag{1}$$

Here $\mathbf{n}$ is the normal to the surface $z = \zeta(x, y, t)$, directed into the second medium. Assuming $k_1|\zeta| \ll 1$ and $q|\zeta| \ll 1$, in the approximation linear in ζ we find from (1) the fields $E_2 \sim \zeta^* E_1$ and $E_3 \sim \zeta E_1$, bilinear in the amplitudes of the s.w. and of the incident field. For nonmagnetic media ($\mu^{\mathrm{I}} = \mu^{\mathrm{II}} = 1$) and $\omega_1 \gg \Omega$,

$$\mathbf{E}_2^{R,T} = -\frac{1}{2} i\zeta^*(\varepsilon - 1)\left[C_2^{R,T} E_{1y}^T + \mathbf{B}_r^{R,T} H_{1y}^T\right],$$

where $\varepsilon = \varepsilon^{\mathrm{II}}/\varepsilon^{\mathrm{I}}$, and the indices R and T denote respectively the reflected and refracted waves (Fig. 1) (see also ([2])),

$$\mathbf{E}_3^{R,T} = -\frac{1}{2} i\zeta(\varepsilon - 1)\left[C_3^{R,T} E_{1y}^T + \mathbf{B}_3^{R,T} H_{1y}^T\right];$$

$$C_{2x}^{R,T} = -a_2 k_{2x} k_{2y}, \qquad C_{2y}^{R,T} = a_2\left(k_{2x}^2 - k_{2z}^R k_{2z}^T\right), \qquad C_{2z}^{R,T} = a_2 k_{2y} k_{2z}^{T,R},$$

$$B_{2x}^R = d_2\left[\varepsilon k_{1x} k_{2x} k_{2z}^R - k_{1z}^T\left(k_{2z}^R k_{2z}^T - k_{2y}^2\right)\right], \qquad B_{2y}^R = d_2 k_{2y}\left(\varepsilon k_{1x} k_{2z}^R - k_{2x} k_{1z}^T\right),$$

$$B_{2z}^R = -d_2\left[\varepsilon k_{1x}\left(k_{2x}^2 + k_{2y}^2\right) - k_{2x} k_{2z}^R k_{1z}^T\right], \qquad a_2 = \left(k_{2z}^T - \varepsilon k_{2z}^R\right)^{-1}, \qquad d_2 = a_2 c/\omega\varepsilon^{\mathrm{II}}. \tag{2}$$

The expressions for $\mathbf{B}_2^T$ are obtained from $\mathbf{B}_2^R$ by replacing $k_{2z}^T \leftrightarrow k_{2z}^R$ and omitting the factor ε before k_{1x}. The formulas for $C_3^{R,T}$ and $\mathbf{B}_3^{R,T}$ are obtained from the expressions for $C_2^{R,T}$ and $\mathbf{B}_2^{R,T}$ by replacing the index 2 by 3.

2. The nonlinear effect under consideration* consists in taking into account the inverse influence of the scattered and incident fields on the motion of the boundary. It may become significant at high intensities of the incident field. The wave of light pressure arising at the boundary, $p_{\text{sv}} \sim \exp(i\mathbf{qr} - i\Omega t)$, bilinear in the amplitudes of the incident and scattered waves ($p_{\text{sv}} \sim E_1 E_2^* + E_3 E_1^*$), in turn pumps the surface oscillations. Solving the lin-

* For a preliminary report on the authors' results of this work, see ([4]). Independently and somewhat earlier, this effect was considered in ([3]).

the linearized equations of motion of an incompressible fluid $\rho\,\partial\mathbf{v}/\partial t = -\nabla p + \rho\mathbf{g}$, $\operatorname{div}\mathbf{v} = 0$, taking into account the forces acting from the electromagnetic fields (5), with boundary conditions at $z = \zeta$: $p^{\mathrm{II}} - p^I - \alpha(\partial^2/\partial x^2 + \partial^2/\partial y^2)\zeta = p_{\text{rad}}$, $p_{\text{rad}} = \Pi_{nn}^I - \Pi_{nn}^{\mathrm{II}}$, $\partial\zeta/\partial t = v_n$, where $p \equiv p' - E^2(8\pi)^{-1}\rho\,\partial\varepsilon/\partial\rho$, Π_{nn} is the normal component of the Maxwell stress tensor, p', $\mathbf{v}$, α are the pressure, velocity, and coefficient of surface tension, and $\mathbf{g}$ is the acceleration of gravity, we find the Fourier component of the deflection ζ, which is expressed in terms of the Fourier component of the light pressure:

$$\zeta_{q\Omega} = |q|(p_{\text{rad}})_{q\Omega}/(\rho^I + \rho^{\mathrm{II}})[\Omega_0^2(q) - \Omega^2], \tag{3}$$

$\Omega_0^2(q) = q(\rho^I + \rho^{\mathrm{II}})(g(\rho^{\mathrm{II}} - \rho^I) + \alpha q^2)$ is the usual dispersion law for capillary-gravity waves. The amplitude of the light-pressure wave at frequency Ω is

$$p_{\text{rad}} = i\zeta q P e^I |E_1^i|^2/8\pi, \tag{4}$$

$$
\begin{aligned}
P = \frac{(\varepsilon-1)^2}{4q}\{ & T_\perp^2\cos^2\varphi\,(C_{3y}^T - C_{2y}^{T*}) + \varepsilon^I T_\parallel^2 \sin^2\varphi\times \\
& \times [Z_x(B_{3x}^T - B_{2x}^{T*}) + \varepsilon Z_z(B_{3z}^T - B_{2z}^T)+ \\
& + 2q_x(\varepsilon-1)Z_{xZ}z] - \sqrt{\varepsilon^I}T_\perp T_\parallel \frac{\sin 2\varphi}{2}[-B_{3y}^T - B_{2y}^{T*} + Z_x(C_{3x}^T - C_{2x}^{T*})+ \\
& + \varepsilon Z_z(C_{3z}^T - C_{2z}^{T*}) + 2q_y(\varepsilon-1)Z_z]\},
\end{aligned}
\tag{5}
$$

where $T_\perp = E_{1y}^T/E_{1y}^i$, $T_\parallel = H_{1y}^T/H_{1y}^i$ are the Fresnel coefficients, φ is the angle between the vector E_1^i of the incident wave and the y axis, $Z_{x,z} = E_{1x,z}/H_{1y}$; the amplitude of the incident wave is denoted by the index i.

The dispersion equation is found from (3) and (4), including in it the damping due to the small viscosity η of the fluid:

$$
\Omega(q) = \pm\Omega_0(q) - 2iq^2\frac{\eta^I + \eta^{\text{II}}}{\rho^I + \rho^{\text{II}}} \mp \frac{iq^2 P\varepsilon^I |E_1^i|^2}{16\pi(\rho^I + \rho^{\text{II}})\Omega_0(q)}. \tag{6}
$$

In the special case of transverse polarization of the incident light ($\varphi = 0$) and $\mathbf{q}$ lying in the plane of incidence (for $\varepsilon^I = 1,\ \rho^I = 1$), expression (6) coincides with the dispersion equation of work (3). At an incident-field intensity greater than the threshold,

$$
\varepsilon^I\frac{|E_j^i|^2}{8\pi} > \varepsilon^I\frac{E_0^2}{8\pi} = \frac{4(\eta^I + \eta^{\text{II}})\Omega_0(q)}{|\operatorname{Re} P|} \tag{7}
$$

the surface waves are excited and forced combination scattering on them occurs. Analysis of the threshold reduces to studying the quantity $\operatorname{Re} P$ (5), proportional to the light pressure.

3. The minimum threshold E_0 corresponds to small q: $\varepsilon^I E_0^2/8\pi \sim \eta\Omega(q)$, and is reached in the region of intermediate angles of incidence; as is seen from (2) and (5), P vanishes for grazing and normal incidence. For $q \ll k$, $P \sim \cos(\beta-\gamma)$, where $\gamma(\theta,\varphi) = \operatorname{arc\,tg}\left.\dfrac{\partial P}{\partial q_y}\middle/\dfrac{\partial P}{\partial q_x}\right|_{q=0}$, β is the angle between the vector $\mathbf{q}$ and the x axis; the minimum threshold corresponds to the propagation of surface waves in the directions $\beta = \gamma,\ \gamma+\pi$. For $\beta = \gamma$ the surface wave traveling in the direction γ is amplified; the Stokes and anti-Stokes waves with frequencies $\omega_{s,a} = \omega_1 \mp \Omega_0$ are scattered in the directions $\mathbf{k}_{s,a} = \mathbf{k}_1 \pm q\gamma$, where γ is the unit vector in the direction γ. The expression for the threshold is substantially simplified in the limiting cases $\varepsilon \gg 1$ and $|\varepsilon - 1| \ll 1$.

Fig. 2. Dependence of the light pressure on the angle of incidence θ and the angle of polarization φ

Figure 2: Fig. 2. Dependence of the light pressure on the angle of incidence θ and the angle of polarization φ

For $\varepsilon \gg 1$ and $\cos\theta \gg \varepsilon^{-1/2}$,

$$P = 2\sin\theta\sqrt{\cos^2\theta + \sin^2\theta\sin^2 2\varphi}\,\cos(\beta-\gamma), \qquad \gamma = \operatorname{arc\,tg}[\operatorname{tg} 2\varphi/\cos\theta].$$

(see Fig. 2). In the region of grazing angles of incidence $\psi = \pi/2 - \theta \ll 1$, for $\psi \sim \varepsilon^{1/2}$ the threshold is minimal:

$$P = 4\varepsilon\psi(1+\sqrt{\varepsilon}\psi)^{-3}\sin\varphi\sqrt{\sin^2\varphi + \psi^2(1+\sqrt{\varepsilon}\psi)^2\cos^2\varphi}\,\cos(\beta-\gamma),$$

$$\gamma = \operatorname{arc\,tg}[\psi(1+\sqrt{\varepsilon}\psi)\operatorname{ctg}\varphi],$$

with the smallest threshold corresponding to $\psi = (4\varepsilon)^{-1/2}$, $\varphi = \pi/2$, $\gamma = 0, \pi$: $(\varepsilon^1 E_0^2/8\pi)_{\min} = 27\eta\Omega_0(q)/4\sqrt{\varepsilon}$, i.e., the threshold decreases by a factor of $\sqrt{\varepsilon}$ in comparison with mean angles of incidence (see Fig. 2).

Analogous simple dependences are obtained in the limit $|\varepsilon - 1| \ll 1$. For lack of space we note only that for $\theta \sim 1$ the threshold increases by a factor $(\varepsilon-1)^{-2}$; for grazing angles ($\psi \sim \sqrt{\varepsilon - 1}$) the threshold is $\sim \eta\Omega_0(q)/\sqrt{\varepsilon-1}$ (see Fig. 2b).

4. For sufficiently large q there arises an analogue of total internal reflection—the conversion of one or several scattered waves into surface electromagnetic waves. For small q this is possible for grazing incidence at $q_x > q_{\text{tr}} = \omega\psi^2/2c$, in which case the wave E_3^R becomes a surface wave. The corresponding threshold is $\sim \eta\varepsilon^{-1}\Omega_0(q_{\text{tr}})[cq_{\text{tr}}/\omega]^{1/2}$ for $\varepsilon \gg 1$ and $\sim \eta(\varepsilon-1)^2\Omega_0(q_{\text{tr}})cq_{\text{tr}}/\omega$ for $|\varepsilon - 1| \ll 1$. For $|\varepsilon - 1| \ll 1$ and $q_x > q_{\text{tr}} = \omega(\psi^2 + \varepsilon - 1)/2c$, the wave E_3^T also becomes a surface wave; the threshold is $\sim \eta(\varepsilon-1)^{-3/2}\Omega_0(q_{\text{tr}})cq_{\text{tr}}/\omega$.

Fig. 2. Dependence of the light pressure on the angle of incidence θ and the polarization angle φ

5. Investigation of stimulated combination scattering by Rayleigh surface waves in an isotropic solid for small optoelastic constants leads to a dispersion equation of the form (6), in which $\Omega_0(q) = c_r q$, where c_r is the velocity of Rayleigh waves,^5 and the dissipative term is of the same order as for bulk acoustic waves,^5 and if it is written in the form $\eta_{\text{eff}}\phi q^2$, then the estimate for the threshold of stimulated combination scattering still has the form $E_0^2/8\pi \sim \eta_{\text{eff}}\phi\Omega_0(q)$. We do not give the explicit formulas

here because of the cumbersome form of the coefficients of order unity, which are functions of c_t/c_l. They do not affect the angular dependence or the magnitude of the threshold.

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Note: Figure translations are in progress. See original paper for figures.