

# THE GENERATION OF A SURFACE WAVE NEAR A ROUGH INTERFACE BETWEEN TWO MEDIA

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**Abstract**

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*PHYSICS*

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**THE GENERATION OF A SURFACE WAVE  
NEAR A ROUGH INTERFACE BETWEEN  
TWO MEDIA**

*(Presented by Academician L. M. Brekhovskikh, 17 IV 1969)*

The generation of intense grazing spectra in the scattering of a plane wave by a periodically uneven surface (the “Wood phenomenon” <sup>(1)</sup>) has been studied for acoustic waves in <sup>(2-4)</sup>. Below, by the perturbation method, the existence is shown and the basic features are considered of an analogous phenomenon for a statistically uneven interface between two liquid media. The usual perturbation method is not applicable here: one must take into account the attenuation of waves primarily scattered in the grazing direction, caused by their multiple scattering by surface irregularities along the propagation path. The idea of allowing for such attenuation was apparently first expressed in <sup>(5)</sup> for the case of a compliant surface. The method set forth below is its generalization to the case of an interface between two media.

1. Let the boundary be described by a statistically homogeneous function  $z = \zeta(\mathbf{r})$ ;  $\langle \zeta \rangle = 0$ . Let the upper and lower media be characterized respectively by densities  $\rho_1$  and  $\rho_2$  and sound velocities  $c_1$  and  $c_2$ . A wave  $\exp(i\mathbf{k}_1\mathbf{r} - ik_z z)$  is incident on the interface from above. We seek the total pressure in the form <sup>(6)</sup>  $p_{1,2} = \langle p_{1,2} \rangle + \psi_{1,2}$ , where  $\langle \psi_{1,2} \rangle = 0$ .

In the first approximation of the perturbation method we have on the plane  $z = 0$  <sup>(7)</sup>

$$\psi_1 - \psi_2 = (\partial \langle p_2 \rangle) = (\partial \langle p_2 \rangle / \partial z - \partial z) \zeta(\mathbf{r}), \quad (1)$$

$$m \partial \psi_1 / \partial z - \partial \psi_2 / \partial z =$$

$$= (\partial^2 \langle p_2 \rangle / \partial z^2 - m \partial^2 \langle p_1 \rangle / \partial z^2) \zeta(\mathbf{r}) - (\nabla \zeta, \nabla [\langle p_2 \rangle - m \langle p_1 \rangle]),$$

where  $m = \rho_2 / \rho_1$ . Introducing the representation

$$\psi_{1,2} = (2\pi)^{-1} \int \exp(i\chi\mathbf{r}) w_{1,2}(\chi, z) d\chi,$$

$$w_1 = d_1(\chi) \exp(i\chi_z z), \quad w_2 = d_2(\chi) \exp(-i\chi'_z z), \quad (2)$$

$$\chi_z = \sqrt{k^2 - \chi^2}, \quad \chi'_z = \sqrt{k^2 n^2 - \chi^2},$$

$k = \omega/c_1$ ,  $n = c_2/c_1$ , and for real  $\chi$ ,  $\text{Re } \chi_z, \chi'_z \geq 0$ , we obtain

$$w_1(\chi, 0) - w_2(\chi, 0) = A(\chi), \quad (3)$$

$$m \frac{dw_1}{dz}(\chi, 0) - \frac{dw_2}{dz}(\chi, 0) = B(\chi),$$

where  $A$  and  $B$  are the Fourier transforms of the right-hand sides of (1).  $A$ ,  $B$  may be interpreted as the spectral amplitudes of sources on the plane  $z = 0$ , respectively of dipole and monopole types, which create the scattered field. To account for the attenuation of scattered waves, we assign to the plane  $z = 0$  the reflection coefficients  $V_1$  for plane waves incident from above, and  $V_2$  for waves incident from below:

$$V_1 = (\chi_z - k\eta)(\chi_z + k\eta)^{-1}, \quad V_2 = (\chi'_z - kn\eta')(\chi'_z + kn\eta')^{-1},$$

$\eta, \eta'$  are effective conductivities depending on the parameters of the media and of the irregularities. As is known (see, for example, (8)), a unit-amplitude monopole placed on such a plane produces the field

$$(2\pi)^{-1} \int \exp(i\kappa\mathbf{r} + i\kappa_z z) \frac{i(1 + V_1)}{\kappa_z} d\kappa$$

in the upper medium, and the field

$$(2\pi)^{-1} \int \exp(i\kappa\mathbf{r} - i\kappa'_z z) \frac{i(1 + V_2)}{\kappa'_z} d\kappa$$

in the lower one. It follows from this that the contributions of monopoles with amplitude  $B$  to the scattered field will be proportional to  $iB(1 + V_1)/\kappa_z$  and  $iB(1 + V_2)/\kappa'_z$ . For dipoles we similarly find that the contributions are proportional to  $A(1 - V_1)$  and  $-A(1 - V_2)$ . Thus,

$$w_1(\kappa, 0) = a_1 A(1 - V_1) - b_1 B i(1 + V_1)/\kappa_z,$$

$$w_2(\varkappa, 0) = -a_2 A(1 - V_2) - b_2 B i(1 + V_2)/\varkappa_z, \quad (4)$$

where  $a_{1,2}, b_{1,2}$  are coefficients independent of the irregularities. The quantities  $\eta$  and  $\eta'$  are found by the usual perturbation method. With the aid of this method, from (7) a system of equations determining  $\eta$  was obtained. Solving it and retaining terms of order  $\langle k^2 \zeta^2 \rangle$ , we obtain

$$k\eta = k\eta_0 + k\eta_1, \quad k\eta_0 = \varkappa'_z/m,$$

$$k\eta_1 = ia_1(1 - m^{-1})^2 - i(a_2 - m^{-1}a_3)(1 - m^{-1}) - \\ -i\delta n(a_4 - a_5)(1 - m^{-1})m^{-1} + i(\delta n)^2 m^{-2} a_6. \quad (5)$$

Here  $\delta n = 1 - n^2$ ,  $a_1 = -\int \Delta \cdot f \cdot \mathbf{q}(\mathbf{q} - \varkappa) d\mathbf{q}$ ,  $a_2 = \int \Delta \cdot f \cdot q_z^2 d\mathbf{q}$ ,  $a_3 = \int \Delta \cdot f \cdot q_z'^2 d\mathbf{q}$ ,  $a_4 = -k^2 \int \Delta \cdot q_z^2 d\mathbf{q}$ ,  $a_5 = k^2 \int \Delta \cdot \mathbf{q}(\mathbf{q} - \varkappa) d\mathbf{q}$ ,  $a_6 = k^4 \int \Delta \cdot d\mathbf{q}$ ,

$$\Delta = i(2\pi)^{-1}(q_z + m^{-1}q'_z)^{-1}g(\mathbf{q} - \varkappa), \quad q_z = \sqrt{k^2 - q^2}, \quad q'_z = \sqrt{k^2 n^2 - q^2},$$

$f = (\mathbf{q}, \varkappa) - k^2$ ,  $g(\mathbf{q})$  is the Fourier transform of the correlation function of the irregularities.

The quantity  $\eta'$  is obtained from  $\eta$  by replacing  $m, n, k$  by  $m^{-1}, n^{-1}, kn$ . As a result we have

$$\eta' = \eta'_0 + \eta'_1, \quad kn\eta'_0 = m\varkappa_z, \quad kn\eta'_1 = kn\eta_1(m^{-1}, n^{-1}, kn). \quad (6)$$

If the attenuation of the scattered waves is neglected, i.e., if in (4) one sets  $k\eta = k\eta_0$ ,  $kn\eta' = kn\eta'_0$ , then the expressions (4) must satisfy the relations (3). From this one determines  $a_1 = a_2 = 1/2$ ,  $b_1 = 1/2m$ ,  $b_2 = 1/2$ . Substituting them into (4), and then (4)–(6) into (2), and calculating  $A, B$  for

$$\langle p_1 \rangle = \exp(i\mathbf{k}_\perp \mathbf{r}) [\exp(-ik_z z) + V_1 \exp(ik_z z)],$$

$$\langle p_2 \rangle = (1 + V_1) \exp(i\mathbf{k}_\perp \mathbf{r} - ik'_z z),$$

we obtain

$$\psi_{1,2} = \pm i(2\pi)^{-1} \int \{ [k_z - k'_z - (k_z + k'_z)V_1] \beta_{1,2} \mp$$

$$\mp(m-1)(1+V_1)[k^2 - (\mathcal{N}, \mathbf{k}_\perp)] \mp k^2 \delta n(1+V_1)\} T_{1,2} d\mathcal{N},$$

$$T_1 = [m\mathcal{N}_z + \mathcal{N}'_z + mk\eta_1(m, n, k)]^{-1} \xi(\mathcal{N} - \mathbf{k}_\perp) \exp(i\mathcal{N}\mathbf{r} + i\mathcal{N}_z z), \quad (7)$$

$$T_2 = [m\mathcal{N}_z + \mathcal{N}'_z + kn\eta_1(m^{-1}, n^{-1}, kn)]^{-1} \xi(\mathcal{N} - \mathbf{k}_\perp) \times$$

$$\times \exp(i\mathcal{N}\mathbf{r} - i\mathcal{N}'_z z), \quad \beta_1 = \mathcal{N}'_z, \quad \beta_2 = m\mathcal{N}_z,$$

$\xi$  is defined by the relation  $\zeta(\mathbf{r}) = (2\pi)^{-1} \int \exp(i\mathcal{N}\mathbf{r}) \xi(\mathcal{N}) d\mathcal{N}$ .

**2.** Let  $n = 1$ ,  $m \neq 1$ . We compute  $\langle |\psi_{1,2}|^2 \rangle$  for a Gaussian correlation of the irregularities. Setting  $kl \ll 1$ , where  $l$  is the correlation radius, from (7) with to accuracy up to  $(k_\perp l)$  we find

$$\begin{aligned} \langle |\psi_1|^2 \rangle &\simeq \langle \xi^2 \rangle l^2 \int_0^\infty \left\{ 2k_z^2 |\chi_z V_1|^2 + \frac{1}{2} (m-1)^2 |1+V_1|^2 \left( k^4 + \frac{1}{2} \chi^2 k_\perp^2 \right) \right. \\ &\quad \left. - 2 \operatorname{Re} [k_z k^2 (m-1)(1+V_1)^* V_1 \chi_z] \right\} |T|^2 \chi d\chi, \quad (8) \\ T &= [(m-1)\chi_z + mk\eta_1(m)]^{-1} \exp\left(-\frac{1}{4} \chi^2 l^2 - 2z \operatorname{Im} \chi_z\right). \end{aligned}$$

Here

$$k\eta_1(m) \simeq -\frac{i\sqrt{\pi}}{2} \frac{(m-1)^2}{m(m+1)} \langle \xi^2 \rangle l^{-1} \chi^2 + \frac{1}{4} (\chi^2 + 2k^2) \frac{(m-1)^2}{m(m+1)} \langle k^2 \xi^2 \rangle k l^2$$

for  $\chi \lesssim k$ .

Dividing the region of integration in (8) into the intervals  $[k, \infty)$  and  $[0, k]$ , we represent  $\langle |\psi_1|^2 \rangle$  in the form of the contribution of inhomogeneous waves  $\langle |\psi_1|^2 \rangle_{\text{inhom}}$  and the contribution of homogeneous waves; the latter turns out to be  $(kl^2)^3$  times smaller than  $\langle |\psi_1|^2 \rangle_{\text{inhom}}$  near the boundary and therefore is of no interest in the present problem. Under the condition  $kz \langle k^2 \xi^2 \rangle k^2 l^2 \ll 1$  we have

$$\langle |\psi_1|^2 \rangle_{\text{inhom}} \simeq \frac{1}{2} |1+V_1|^2 \left( \frac{m-1}{m+1} \right)^2 k^2 \left( k^2 + \frac{1}{2} k_\perp^2 \right) \langle \xi^2 \rangle l^2 \times \left[ \ln |1 - \alpha^{-1}| + \frac{\pi\sqrt{\pi}}{2(kl)^3} \right] \exp(-2zk\alpha), \quad (9)$$

where  $\alpha = \langle k^2 \xi^2 \rangle (kl)^{-1} \sqrt{\pi} (m-1)^2 / 2(m+1)^2$ . Formula (9) gives the intensity of a weakly inhomogeneous surface wave with wave vector  $(k\sqrt{1+\alpha^2}, i\alpha k)$ . Its existence is evidently due to the elastic character of the effective impedance of the boundary ( $\text{Im } \eta_1 < 0$ ) (9). Analogously, one obtains the quantity  $\langle |\psi|^2 \rangle_{\text{inhom}}$ , which is expressed by formula (9) if  $z$  is replaced there by  $(-z)$ .

Let  $n \neq 1$ ,  $m \neq 1$ . For  $|\delta n| \simeq [\gamma \langle k^2 \xi^2 \rangle (kl)^{-1}]^2 \left(\frac{m-1}{m+1}\right)^2$  and  $\left|\frac{m-1}{m+1}\right| \gg \langle k^2 \xi^2 \rangle (kl)^{-1}$ , where  $\gamma = |m-1|$  for  $\psi_1$  and  $\gamma = |m^{-1}-1|$  for  $\psi_2$ , the contribution of inhomogeneous waves again reduces to a surface wave of type (9), but its intensity decreases to the intensity of homogeneous waves.

If  $m = 1$ ,  $n \neq 1$ , then in the approximation of small-scale irregularities from (5) we obtain, for  $\chi \lesssim \max[k, kn]$ ,

$$k\eta_1 \simeq i \frac{\sqrt{\pi}}{4} k(1-n^2)^2 \langle k^2 \xi^2 \rangle kl + \frac{k}{6} [|1-n^2|^{5/2} - (1-n^2)(1-n^3)] \langle k^2 \xi^2 \rangle (kl)^2.$$

In this case the impedance has a mass character, and surface waves are not formed.

If the condition  $kl \gg 1$  is satisfied, then, as the calculation shows, surface waves are also absent.

In summary, one may say that, when waves propagate in the presence of rough boundaries similar to those considered above, a certain low-frequency part of the scattered waves will be localized near these boundaries, and the energy density near the boundaries may be not small on the scale of the intensity of the scattered field.

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