ON THE EVOLUTION OF MELTING ZONES IN THE THERMAL HISTORY OF THE EARTH

GEOPHYSICS

1969

SovietRxiv

Abstract

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UDC 550.3

GEOPHYSICS

Academician A. N. TIKHONOV, E. A. LYUBIMOVA, V. K. VLASOV

ON THE EVOLUTION OF MELTING ZONES IN THE THERMAL HISTORY OF THE EARTH

The appearance of a melting layer in the upper mantle of the Earth is a most important event in the thermal history of the Earth. The process of melting-out of relatively light sialic materials from the Earth' s mantle is associated with the history of formation of the Earth' s crust ([1,2]). Geological data testify to the existence of cycles of definite duration in the development of the observed features of the structure of the Earth' s crust ([3]).

It is known that the melting temperature increases with depth under the influence of high pressure at a rate of the order of 3°C/km. The physical conditions in the upper mantle are such that the temperature curve, determined by the heat of radioactive decay, must at some level intersect the melting curve. The possibility of formation of a melting zone about 2-3 billion years ago at depths of 150-600 km was shown in ([4]). Possible variations in the depths and time of formation of the primary melting zone, depending on the adopted parameters, were considered in ([5,6]), with allowance for absorption of the latent heat of melting. The melt that has formed must rise upward ([7]). The reason for this rise is the circumstance that the gradient of the melting temperature, substantially greater than the adiabatic gradient, leads to mixing in the melt layer and to the supply of relatively superheated material upward and of supercooled material downward ([2]). The essential role of convective mixing in this mechanism was emphasized in ([8]).

In the present work, by means of a computer, a model problem is solved on the influence of enhanced heat transfer in a molten layer on the character of displacement of this layer in the thermal history of the Earth, caused by the effect of radioactive decay. It will be shown that the controlling parameter of the process of motion of the layer is the ratio $\lambda_{\text{eff}}/\lambda$, where λ_{eff} is the heat-transfer coefficient in the molten layer, and λ is the coefficient of thermal conductivity in the solid medium outside the molten layer. Calculations of the temperature inside the Earth are carried out by solving the quasilinear heat-conduction equation for a spherically symmetric body containing internal heat sources and characterized by variable coefficients of heat transfer. The surface temperature is prescribed as constant, and the initial temperature as a certain smooth function $\varphi(r)$.

Let $u(r,t)$ be the unknown function giving the dependence of the temperature of the body on radius and time; R the radius of the body; $c(u,r)$ the heat capacity; $\rho(r)$ the density of the material of the body; $\lambda(u,r)$ the coefficient of thermal conductivity; $f(r,t)$ the heat-release function, specifying the amount of heat released by radioactive sources in 1 cm^3 in 1 sec at a distance r from the center of the body at time t. In dimensionless variables $r' = r/R$ and $t' = t/T$ we shall have:

$$c(u,Rr')\rho(Rr')\frac{\partial u}{\partial t'} - \frac{T}{R^2}\frac{1}{{r'}^2}\left\{{r'}^2\lambda(u,Rr')\frac{\partial u}{\partial r'}\right\} = f(Rr',Tt')T;$$

$$0 \le r' \le 1, \quad t' \ge 0; \tag{1}$$

under the additional conditions:

$$u(Rr', t_0/T) = \varphi(Rr'), \qquad u(1,t') = u_0 = \text{const},$$

$$\lambda(Rr',u)\frac{1}{R}\frac{\partial u}{\partial r'}\bigg|_{r'=0} = 0.$$

To solve this problem we use an implicit four-point difference scheme with a nonuniform spatial grid. The difference equation is written in the form [9]

$$c(Rr'_i, u_i^{j+1})\rho(Rr'_i)\,\frac{u_i^{j+1} - u_i^j}{\tau} =$$

$$= \frac{\chi}{r_{i'}^2}\frac{1}{\bar{h}_i}\left[\left(r'_i + \frac{h_{i+1}}{2}\right)^2 \lambda\left(\frac{u_{i+1}^{j+1} + u_i^{j+1}}{2},\, R\left(r'_i + \frac{h_{i+1}}{2}\right)\right)\frac{u_{i+1}^{j+1} - u_i^{j+1}}{h_{i+1}} -\right.$$

$$\left. - \left(r'_i - \frac{h_i}{2}\right)^2 \lambda\left(\frac{u_i^{j+1} + u_{i-1}^{j+1}}{2},\, R\left(r'_i - \frac{h_i}{2}\right)\right)\frac{u_i^{j+1} - u_{i-1}^{j+1}}{h_i}\right] +$$

$$+ f(Rr'_i, Tt'_{j+1})T, \tag{2}$$

where τ is the time step, $h_i = r'_i - r'_{i-1}$, $\bar{h}_i = (h_i + h_{i+1})/2$, $\Sigma h_i = 1$, $\chi = T/R^2$. We write the initial and boundary conditions as follows:

$$u(Rr'_i, t_0/T) = \varphi(Rr'_i), \qquad u_0 = u_1, \qquad u_M = 0, \tag{3}$$

where M is the number of steps in radius.

The heat-generation function is taken in the form of exponentials with a common factor $K_{\mathrm{U}}(t)$, denoting the uranium concentration referred to the moment t. The distribution of sources is assumed uniform before the onset of melting, and at the present time layered, corresponding to the division of the Earth into crust, mantle, and core. The thermal conductivity coefficient in the solid part of the Earth was taken as the sum of the phonon and radiative components $\lambda = \lambda_{ph} + \lambda_r(\varepsilon)$, in accordance with work [10]. The effective coefficient λ_{eff} inside the molten layer is poorly known, but may be estimated on the basis of the convection relations applied to terrestrial conditions [11] and expressed through the Rayleigh number: $\lambda_{\text{eff}}/\lambda = 0.205[\mathrm{Ra}]^{1/4} + 1$, whence it follows that for melt-layer thicknesses from 50 to 500 km the value of the ratio $\lambda_{\text{eff}}/\lambda$ varies from 2 to 10.

Equation (3) can be transformed to the form

$$A_i^{j+1} u_{i+1}^{j+1} - 2B_i^{j+1} u_i^{j+1} + C_i^{j+1} u_{i-1}^{j+1} = D_i^{j+1} \tag{4}$$

with coefficients

$$A_i^{j+1} = \frac{1}{h_{i+1}} \frac{\chi}{r_{i'}^2} \frac{1}{\bar{h}_i} \left(r_i' + \frac{h_{i+1}}{2} \right)^2 \lambda \left(\frac{u_{i+1}^{j+1} + u_i^{j+1}}{2},\, R\left(r_i' + \frac{h_{i+1}}{2} \right) \right);$$

$$2B_i^{j+1} = \frac{\chi}{r_{i'}^2} \frac{1}{\bar{h}_i} \left[\frac{1}{h_{i+1}} \left(r_i' + \frac{h_{i+1}}{2} \right)^2 \lambda \left(\frac{u_{i+1}^{j+1} + u_i^{j+1}}{2},\, R\left(r_i' + \frac{h_{i+1}}{2} \right) \right) + \right.$$

$$\left. + \frac{1}{h_i} \left(r_i' - \frac{h_i}{2} \right)^2 \lambda \left(\frac{u_i^{j+1} + u_{i-1}^{j+1}}{2},\, R\left(r_i' - \frac{h_i}{2} \right) \right) \right] + \frac{1}{\tau} c(Rr_i', u_i^{j+1}) \rho(Rr_i');$$

$$C_i^{j+1} = \frac{\chi}{r_{i'}^2} \frac{1}{\bar{h}_i} \frac{1}{h_i} \left(r_i' - \frac{h_i}{2} \right)^2 \lambda \left(\frac{u_i^{j+1} + u_{i-1}^{j+1}}{2},\, R\left(r_i' - \frac{h_i}{2} \right) \right);$$

$$D_i^{j+1} = -\frac{1}{\tau} c(Rr_i', u_i^{j+1}) \rho(Rr_i', u_i^{j+1}) u_i^j - f(Rr_i', Tt'_{j+1}) T$$

Obviously, $A_i^{j+1} > 0$, $C_i^{j+1} > 0$, $2B_i^{j+1} = A_i^{j+1} + C_i^{j+1} + c\rho/\tau$, i.e. $2B_i^{j+1} > A_i^{j+1} + C_i^{j+1}$.

At each time layer we obtained a system of coupled algebraic equations with coefficients depending on the as yet unknown solution of these equations and on the solution already obtained at the preceding time layer. The problem was solved by an iterative method. The system of algebraic equations was solved

Fig. 1 and Fig. 2

Figure 1: Fig. 1 and Fig. 2

by the well-known sweep method [12]. The grid of nodes was taken nonuniform: denser near the surface and becoming coarser toward the center. The refinement of the step reached 1.5 km. This made it possible to increase significantly the accuracy of the solution near the surface in comparison with other works, which

important for calculating the surface heat flux q and comparing it with the observed one. The time step was taken as constant, $\tau = 0.001$. The total calculation interval was assumed equal to the age of the Earth, $t^* = 4.5 \cdot 10^9$ years. When the melting effect was taken into account, a certain melting curve $u_1 = u_1(r)$ was specified in advance. The solution at the next j-th time layer was compared with the value of the melting curve. If, at some grid nodes at some moment in time, the temperature value exceeds the melting temperature at these points, then a melting zone is formed, and the para-

Fig. 1

Fig. 2

Fig. 1. Cyclicity of melting in the upper mantle when intensive heat transfer is included in the melt layer. $\lambda_{\text{eff}} = 0.05$ cal/cm • sec • deg, coefficient of absorption of radiant component $\varepsilon = 100$ cm^{-1}, uranium concentration $K_U = 0.5 \cdot 10^{-7}$ g/g, $t = 4.5 \cdot 10^9$ years corresponds to the present moment. Number of cycles 4; their period is of the order of 500 to 300 million years.

Fig. 2. Cyclicity of melting in the upper mantle when heat transfer in the melting zone is increased to $\lambda_{\text{eff}} = 0.1$ cal/cm • sec • deg, $\varepsilon = 100$ cm^{-1}, number of cycles 16; their period is about 140 million years.

meters of the problem change abruptly. During this period, absorption of latent heat, decrease in density ρ, and intensification of heat transfer are taken into account. The results of the calculation are presented in Figs. 1, 2, 3. Along the vertical axis is plotted the depth at which melt arises; along the horizontal axis, the time of its occurrence. With the given heat generation, determined by the uranium concentration $K_U = 0.5 \cdot 10^{-7}$ g/g, the initial melting layer is formed at a depth of about 400 km during the time interval $(1.8—2.4) \cdot 10^9$ years. If, at this moment, intensive heat transfer with $\lambda_{\text{eff}} = 0.05$ or 0.1 cal/cm • sec • deg is included, then the melt layer begins to move upward (Fig. 2). Excess heat from the lower boundary is transported to the upper one, partially melting it. The thickness of the layer changes with time. The lower boundary catches up with the upper one, and at some level (30–100 km) beneath the surface the layer stops and disappears. During this time, in the lower layers, in the place of the departed melt, molecular thermal conductivity is restored, and thermal energy begins to accumulate until the melting temperature is again reached, and the process is resumed. Thus, a cycle of expenditure and accumulation of energy

Fig. 3. Shape of the melting cycles of the upper mantle

Figure 2: Fig. 3. Shape of the melting cycles of the upper mantle

arises. The duration of the cycle is determined by the value $\lambda_{\text{eff}}/\lambda$. If $\lambda_{\text{eff}}/\lambda$ is close to 1, cycles do not arise. The melting zone is stabilized for a long time at the level of 150–400 km. If $\lambda_{\text{eff}} = 0.05$ cal/cm • sec • deg, then 4 cycles arise over the entire period of the thermal history of the Earth, with an average duration of 500 million years (Fig. 1).* If $\lambda_{\text{eff}}/\lambda$ is close to 10, with $\lambda_{\text{eff}} = 0.1$ cal/cm • sec • deg, then 16 cycles arise with a duration of about 140 million years (Fig. 2). Thus, switching the heat-transfer coefficient from λ to λ_{eff} in the melt layer leads to the appearance of cycles. The switching moment depends, in particular, on factors determining the conditions for the loss of convective stability-

···If the molecular thermal conductivity of the upper solid layer is increased, for example by increasing the contribution of the radiative component (with absorption $\varepsilon = 10$ cm^{-1}), then the upper edge of the molten layer can reach a depth of 30 km (Fig. 3). With such an approach of the melting zone to the surface, the heat flux increases by 30-40%. In this case the ratio $\lambda_{\text{eff}}/\lambda$ is somewhat less than 10, and the number of cycles is smaller than in Fig. 2, being equal to 13 with a period of 170 million years.

Thus, the controlling factor in the evolution of melting zones is the increase in heat transfer within the molten layer. All other parameters may vary strongly, but the result will not change qualitatively. The limiting number of cycles is of the order of 20. The average duration of a cycle is of the order of 100-170 million years.

Fig. 3. Shape of the melting cycles of the upper mantle for model parameters $\lambda = 0.07$ cal/cm · s · deg and $\lambda_{\text{eff}} = 0.1$ cal/cm · s · deg and a decrease of ε to 10 cm^{-1} in the solid upper layer. Number of cycles 13, their period 170 million years.

According to the calculations performed, the molten layer at the present time is located at a level of 150-300 km, which is generally consistent with the position of the *LV*-layer of reduced seismic-wave velocities in the upper mantle.

The main result of the work consists in taking into account the switching of the heat-transfer coefficient in the course of the evolution of melting zones. A number of factors have been schematized, which naturally may lead to some changes in quantitative estimates. If the number of cycles were known with sufficient certainty, conclusions could be drawn about the structure and magnitude of the thermal parameters at depth.

The possibility of multiple melting of the upper mantle, as was said, was noted by A. P. Vinogradov ([1], [8]).

The demonstrated cyclicity of melting may make it possible to approach the hypothesis of "thermal cycles" ([13]) from new positions.

Received
13 VI 1969

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Note: Figure translations are in progress. See original paper for figures.