ON A PARADOX IN PLASMA DIFFUSION IN TOROIDAL MAGNETIC TRAPS

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Fig. 1

Figure 1: Fig. 1

Abstract

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Academician R. Z. SAGDEEV, A. A. GALEEV

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It is known that the classical diffusion of a fully ionized plasma across a magnetic field occurs only as a result of collisions between particles of different kinds (between ions and electrons) ([1]). The absence of self-diffusion is also connected with the fact that the diffusion automatically proves to be ambipolar (independently of the magnitude of the electric field). All these features of diffusion are a consequence of the fact that in the interval of time between collision events the generalized momentum of individual particles is conserved, while collisions conserve the total momentum. In magnetic traps with complicated geometry the generalized momentum may not be an integral of the motion, and in such systems the diffusion coefficients may contain an explicit dependence on the collision frequencies of identical particles ([2]).

Fig. 1

In toroidal magnetic traps with axial symmetry ("Tokamak," "Levitron") the axial component of the generalized momentum is conserved, and the diffusion coefficient, in this case determined by the well-known Pfirsch–Schlüter formula, is proportional to the frequency of electron collisions with ions ([3]). However, the consideration in ([3]) is applicable only in the case of a sufficiently dense plasma (when the mean free path is smaller than the torus parameter). The opposite limiting case of a rarefied plasma, investigated in the authors' work ([4]), led to the following result: 1) the diffusion is not ambipolar until a certain equilibrium distribution of the electric field has been established in the plasma; 2) the coefficient of ambipolar diffusion explicitly depends on the collision frequency of particles of one kind (namely, electrons with electrons). It is not surprising that this could appear paradoxical and give grounds for criticism in a number of subsequent works ([5], [6]). Thus a rather contradictory situation arose. In this connection it seems instructive to clarify the physical meaning of the apparent paradox of self-diffusion obtained in ([4]).

Let us consider an axially symmetric magnetic field (see Fig. 1)

$$\mathbf{H} = H_0\{(1-\varepsilon\cos\vartheta)\mathbf{e}_\zeta - \Theta\mathbf{e}_\vartheta\}, \qquad \varepsilon = r/R; \qquad \Theta = ir/2\pi R. \tag{1}$$

For a small rotational transform ($\Theta \ll 1$) and small toroidality ($r/R \ll 1$), the conserved axial generalized momentum of an individual particle with charge e and mass m has the form ([4])

$$\frac{eH_0}{mc}\int_0^r \Theta(r)\,dr + v_\parallel \simeq \text{const}, \tag{2}$$

where $v_\parallel$ is the component of velocity parallel to the direction of the magnetic field.

Let us now consider a gas of such particles, described by a kinetic equa-

···in the drift approximation

$$\frac{\partial f_j}{\partial t} + \left\{-\frac{\mu H_0/m_j + v_\parallel^2}{\omega_{cj}R}\sin\vartheta\,\frac{\partial}{\partial r} + \left(-\Theta v_\parallel + \frac{c}{H_0}\frac{d\Phi_0}{dr} - \frac{\mu H_0/m_j + v_\parallel^2}{\omega_{cj}R}\cos\vartheta\right)\frac{\partial}{r\,\partial\vartheta} + \Theta\,\frac{\mu H_0}{m_{jR}}\sin\vartheta\,\frac{\partial}{\partial v_\parallel}\right\} f_j = \text{St} \tag{3}$$

Multiply equation (3) by $mv_\parallel$ and integrate over dv and $d\vartheta$. Then, in the same approximation as in (2), we obtain

$$nm\,du_\parallel/dt + ne\Theta H_0(v_r/c) = F_\parallel. \tag{4}$$

Here the contribution to the right-hand side $F_\parallel$, which has the form of a friction force, arises only from collisions between particles of different kinds. Expression (4) may be regarded as a generalization of (2). It is seen from this that, in the case of a fully ionized plasma, the diffusion flux consists of two parts: an ambipolar part, associated with cross-collisions of ions and electrons, and, additionally, diffusion associated with a change in the mean longitudinal velocity. Obviously, this latter term vanishes in the regime of quasistationary diffusion. But in the process of establishing a quasistationary regime, self-diffusion is also possible. Let us first consider the ions, since their self-diffusion is considerably greater than the electron self-diffusion. For the moment we shall neglect cross-collisions, in order to isolate self-diffusion in its pure form. Suppose that the trap is filled with ions having a Maxwellian distribution. Let us switch off collisions for a time. Then, over a time of the order of several periods of motion of the trapped particles $\left(\tau_i \sim r\sqrt{R/r}/v_{Ti}\Theta\right)$, a distribution will be established for the latter that resembles the plateau in the problem of nonlinear Landau damping ([4]). For what follows, it is essential that this plateau $f(v_\parallel)$ inevitably has a slope arising because of the density and temperature gradients. Now let us take into account collisions of these particles with passing particles having velocities $\gg \sqrt{r/R}\,v_{Ti}$ (and therefore retaining a distribution close to Maxwellian). Let us assume temporarily that the plasma is so rarefied that there is no radial self-consistent electric field. The presence of a slope in the plateau for the trapped particles leads to the appearance, as a result of collisions, of a friction force between the passing and trapped particles. The passing particles will be set into motion along the minor axis of the torus (along ζ).

It now follows immediately from (4) that a change in the mean velocity of the ions must lead to expansion of the ion component of the plasma. In other words, "self-diffusion" of ions occurs. The coefficient of such nonambipolar ion diffusion was calculated by us earlier ([4]). Such self-diffusion ceases when the friction between passing and trapped ions disappears. Before self-diffusion ceases, the ions have time to be displaced by some distance. Let us find it.

It is interesting to note that, if the displacement of the ions due to "self-diffusion" is found from the condition of conservation of the total generalized momentum, then we arrive at a very paradoxical result: the energy of the magnetic field of the current arising in the plasma greatly exceeds the energy released owing to the expansion of the plasma.

This paradox can be eliminated if one takes into account that the expansion of the plasma occurs under conditions in which the changing magnetic field H_ϑ induces an electric field along the minor axis of the toroid. Therefore the total generalized momentum of the particles is no longer conserved.

We shall therefore use the following system of particle equations of motion and Maxwell equations:

$$\partial H'_\vartheta/\partial r = 4\pi e n u_\parallel/c, \tag{5}$$

$$-\frac{\partial E_\parallel}{\partial r} = -\frac{1}{c}\frac{\partial H'_\vartheta}{\partial t}, \tag{6}$$

$$m\frac{\partial u_\parallel}{\partial t} = -\frac{e v_r}{c}(H_\theta + H'_\theta) + eE_\parallel, \tag{7}$$

where $u_\parallel$ is the directed velocity of the ions; H'_θ is the correction to the main magnetic field arising because of the ion current; $E_\parallel$ is the longitudinal electric-field strength.

This system reduces to one equation:

$$\frac{c^2}{\omega_{pi}^2}\frac{\partial^2}{\partial r^2}\left(m_i\frac{\partial u_\parallel}{\partial t} - \frac{e}{c}v_r(H_\theta + H'_\theta)\right) = m_i\frac{\partial u_\parallel}{\partial t}. \tag{8}$$

It follows from this that there occurs, as it were, an increase of the effective mass of the ion, and the displacement of ions due to self-diffusion is of the order

$$\Delta r \sim \beta r/\Theta^2, \tag{9}$$

where $\beta = 8\pi n T_i/H^2$.

With such an expansion, the energy released from the plasma is sufficient to sustain the ion current.

The described picture of relaxation is valid only in the case of a very rarefied plasma, when the expansion of one component of the plasma across the magnetic field does not lead to the appearance of a self-consistent electric field. This could take place for

$$\lambda_D^2 \gg r^2\beta/\theta^2 \tag{10}$$

(λ_D is the Debye radius).

Of course, this condition is never satisfied in reality, and even an insignificant displacement of ions due to self-diffusion over a distance of order λ_D^2/r practically instantaneously leads to the appearance of a radial electric field of order

$$e\Phi'(r) \sim T_i n'/n.$$

The condition for the absence of ion self-diffusion is precisely what determines the equilibrium electric field. It can be shown that, in the general case of a shifted Maxwellian distribution of ions (i.e., in the presence of rotation), the equilibrium electric field is equal to

$$\frac{c}{\Theta H_0}\frac{d\Phi_0}{dr} = u_0 - u_\parallel; \qquad u_0 = \frac{v_{Ti}^2}{2\Theta\omega_{ci}}\left[\frac{d\ln n}{dr} - \left(\frac{3}{2} - \beta_i\right)\frac{d\ln T_i}{dr}\right]. \tag{11}$$

This electric field does not, generally speaking, reduce to zero the friction between passing and trapped electrons, and therefore a certain current of passing electrons arises, whose magnitude is ultimately determined by the balance of the friction forces acting on the passing electrons from the trapped electrons and from the ions.

This latter force is ultimately responsible for the additional contribution to diffusion (ambipolar), proportional to the frequency of electron-electron collisions (electron-ion collisions drop out of this term under the condition $1 > m_{iT}e/m_{eT}i > \varepsilon^{1/3}$).

Above we considered it necessary to dwell only on the physical mechanism of toroidal diffusion, in order to show the erroneousness of the ideas ([5]) based on a complete analogy with the case of ordinary diffusion of plasma across a magnetic field. For details of the calculations, see the preprint ([7]).* In connection with the successes in plasma stabilization in toroidal traps, the role of classical transport processes is increasing.

It is useful to give a summary of the coefficients of these processes, found in ([4,7]). The particle flux $\langle nv\rangle$ and the heat flux q_j for particles of species j in the limit of not very

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* There also is an analysis of the error contained in ([6]).

of a large temperature jump $T_i/T_e < \sqrt[3]{m_i/m_e}$ is determined by the expressions *

$$\langle nv\rangle = -\alpha_e D_e n\left[\frac{d\ln n}{dr} - \left(\frac{3}{2} - \beta_e\right)\frac{d\ln T_e}{dr} + \frac{T_i}{T_e}\left(\frac{d\ln n}{dr} - \left(\frac{3}{2} - \beta_i\right)\frac{d\ln T_i}{dr}\right)\right], \tag{12}$$

$$q_j = \beta_j\langle nv\rangle T_j - \gamma_j D_j n\,\partial T_j/\partial r, \tag{13}$$

where:

A. For $1/\varepsilon\tau_j < v_{Tj}\Theta^{e^{1/2}}/r$

$$\alpha_e = \frac{3\pi}{8\sqrt{2}}\left[1+\sqrt{2}-\ln(1+\sqrt{2})\right] \simeq 1.3; \qquad D_j = \sqrt{\varepsilon r_{cj}^2/\Theta^2\tau_j};$$

$$\beta_j = \frac{\delta_{je} + 1/\sqrt{2}}{\delta_{je} + \sqrt{2} - \ln(1+\sqrt{2})}; \qquad \beta_e = 1.11; \qquad \beta_i = 4.33; \qquad \tau_j = \frac{3m_j^2 v_{Tj}^3}{16\sqrt{\pi}\lambda e^4 n};$$

$$\gamma_j = \frac{3\pi}{8\sqrt{2}}\left\{\delta_{je}(2-\beta_j) + \frac{1}{2}\left(\frac{9\sqrt{2}}{4} - \beta_j\right)\right\}; \qquad \gamma_e = 1.41; \qquad \gamma_i = 0.6.$$

B. For $v_{Tj}\Theta/r > 1/\tau_{ij} > v_{Tj}\Theta^{3/2}/r$

$$\alpha_j = 4; \qquad B_j = \gamma_j = 3; \qquad D_j = \frac{\pi^{1/2}}{4}\varepsilon^2\frac{r_{cj}}{\Theta|r|}\frac{cT_j}{eH_0}.$$

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* The numerical factor $\alpha_e = 1.8$ in the analogous coefficient of Ref. (4) was incorrect.

Note: Figure translations are in progress. See original paper for figures.