A NUMERICAL METHOD FOR THE ANALYSIS OF PLANE MECHANISMS

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Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

Abstract

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MECHANICS

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A NUMERICAL METHOD FOR THE ANALYSIS OF PLANE MECHANISMS

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The velocities and accelerations of points of plane mechanisms are determined mainly graphically—by the method of plans (^{1}). Analytical methods of kinematic analysis are applied chiefly in the study of the simplest four-link mechanisms (^{2},^{3}). For the study of more complicated mechanisms numerical methods are being developed (^{4}), but these methods are so complex that they can be implemented practically only with the aid of electronic digital computers (^{5}), since, in investigating a structural group consisting of $2k$ links, it is necessary to solve systems of $4k$ equations in $4k$ unknowns. Below a numerical method for the analysis of plane mechanisms is described; using it, in the study of a structural group consisting of $2k$ links, one need solve systems of no more than k equations.

Fig. 1

Fig. 2

Each link of a mechanism is connected with the remaining links by at least two kinematic pairs. Therefore it is always possible to determine the kinematic parameters not at once for all $2k$ links of an Assur group, but initially only for half of them. The subsequent determination of the kinematic parameters of the second half of the links of the Assur group will no longer present difficulty.

Suppose it is necessary to investigate the structural group whose kinematic diagram is shown in Fig. 1. This structural group belongs to the VI class of the 3rd order according to the classification of I. I. Artobolevskii, or to the III class of the 0th order according to Assur. Introduce a fixed coordinate system XY. In this coordinate system the coordinates, the components of the velocities, and the components of the accelerations of the external points A, B, and C are assumed known, and the lengths of all links are also known; consequently, the angles $\alpha_1 = \angle DAI$, $\alpha_2 = \angle FBE$, and $\alpha_3 = \angle GCH$ are also known. Denote by φ_1 the angle that the vector $\overline{AI}$ makes with the axis OX. Then the coordinates of the points I and D can be expressed in terms of the coordinates of point A and the angle φ_1 as follows:

$$\begin{aligned} X_I &= X_A + AI\cos(\varphi_1), \\ Y_I &= Y_A + AI\sin(\varphi_1), \\ X_D &= X_A + AD\cos(\varphi_1 + \alpha_1), \\ Y_D &= Y_A + AD\sin(\varphi_1 + \alpha_1). \end{aligned} \tag{1}$$

Denoting by φ_2 and φ_3 the angles made by the vectors $\overrightarrow{BE}$ and $\overrightarrow{CG}$ with the axis OX, one can write analogous expressions for the coordinates of the points E, F, G, and H, expressing them in terms of the coordinates of the points B and C and the angles φ_2 and φ_3.

Let us write the expression for the square of the distance between the points D and E:

$$(X_D - X_E)^2 + (Y_D - Y_E)^2 = DE^2. \tag{2}$$

Writing similar expressions for the squares of the distances between the points F and G and between the points H and I, and substituting into these equations the coordinate values from expressions of type (1), we obtain a system of 3 equations for determining the 3 unknown angles φ_1, φ_2, and φ_3. We shall not dwell on methods for solving this system; let us merely note that a solution of such systems of equations may be obtained, for example, by the method of successive approximations. Substituting the found values of the angles φ_1, φ_2, and φ_3 into formulas of type (1), we determine the coordinates of all internal points of the structural group.

Let us introduce notation. Let the vector $\mathbf{a}$ have components a_x and a_y. Then by $\mathbf{a}^*$ we shall denote the vector obtained from $\mathbf{a}$ by rotating it counterclockwise through an angle of 90°, i.e., the vector $\mathbf{a}^*$ has components $a_x^* = -a_y$ and $a_y^* = a_x$. Hence

$$\begin{aligned} &\mathbf{a}^* \cdot \mathbf{b} = a_x \cdot b_y - a_y \cdot b_x = \mathbf{a} \times \mathbf{b}, \\ &\mathbf{a}^* \cdot \mathbf{b}^* = a_x \cdot b_x + a_y \cdot b_y = \mathbf{a} \cdot \mathbf{b}, \\ &\mathbf{a}^* \cdot \mathbf{a} = 0 \end{aligned} \tag{3}$$

(the vector product in the first of formulas (4) has no generally accepted meaning).

Express the velocity of point D in terms of the velocity of the external point A and the angular velocity of link 1:

$$\mathbf{V}_D = \mathbf{V}_A + \omega_1 \overrightarrow{AD}^*. \tag{4}$$

Similarly, we express the velocities of the remaining internal points E, F, G, H, and I in terms of the velocities of the external points A, B, and C and the angular velocities of links 1, 3, and 5.

The relative velocity of points D and E is perpendicular to the straight line DE, since the points D and E lie on the same link. Therefore the scalar product of the relative velocity $\mathbf{V}_{DE} = \mathbf{V}_D - \mathbf{V}_E$ with the vector $\overrightarrow{DE}$ is zero. Expressing in this scalar product the velocities $\mathbf{V}_D$ and $\mathbf{V}_E$ by formulas of type (4), we arrive at the equation

$$\omega_1 \cdot \overrightarrow{AD} \times \overrightarrow{DE} - \omega_3 \cdot \overrightarrow{BE} \times \overrightarrow{DE} = (\mathbf{V}_B - \mathbf{V}_A) \cdot \overrightarrow{DE}. \tag{5}$$

Writing analogous equations that use the perpendicularity of the relative velocities $\mathbf{V}_{FG}$ and $\mathbf{V}_{HI}$ to the vectors $\overrightarrow{FG}$ and $\overrightarrow{HI}$, respectively, we obtain a system of 3 linear equations in the 3 unknowns ω_1, ω_3, and ω_5. Solving this system and determining the angular velocities of links 1, 3, and 5, we use formulas of type (4) to determine the velocities of all internal points.

The angular velocities of the remaining 3 links can be found by the formula

$$\omega_2 = \overrightarrow{DE} \times \mathbf{V}_{ED} / |DE|^2, \tag{6}$$

which is easily obtained from expression (4).

Express the acceleration of point D in terms of the acceleration of point A and the angular acceleration of link 1:

$$\mathbf{a}_D = \mathbf{a}_A - \omega_1^2 \cdot \overrightarrow{AD} + \varepsilon_1 \cdot \overrightarrow{AD}^*. \tag{7}$$

Similarly, we express the accelerations of the remaining internal points E, F, G, H, and I in terms of the accelerations of the external points A, B, and C and the angular accelerations of links 1, 3, and 5.

Let us now write the expression for the acceleration of point E in terms of the acceleration of point D

$$\mathbf{a}_E = \mathbf{a}_D + \mathbf{a}_{ED}^n + \mathbf{a}_{ED}^t. \tag{8}$$

In this expression the normal relative acceleration is completely known to us, $\mathbf{a}_{FD}^n = -\omega_2^2 \cdot \overrightarrow{DE}$, while for the tangential relative acceleration $\mathbf{a}_{tED}$ it is known only that it is perpendicular to the vector $\overrightarrow{DE}$. Multiplying equation (8) scalarly by the vector $\overrightarrow{DE}$, after substituting into the latter the expressions for the accelerations of points E and D from formulas of type (7), we obtain the equation

$$\varepsilon_1 \cdot \overrightarrow{AD} \times \overrightarrow{DE} - \varepsilon_3 \cdot \overrightarrow{BE} \times \overrightarrow{DE} = \left(\mathbf{a}_B - \mathbf{a}_A + \omega_1^2 \cdot \overrightarrow{AD} + \omega_2^2 \cdot \overrightarrow{DE} - \omega_3^2 \cdot \overrightarrow{BE}\right) \cdot \overrightarrow{DE}. \tag{9}$$

Writing two more equations analogous to equation (9), we obtain a system of three equations in three unknowns $\varepsilon_1, \varepsilon_3$, and ε_5. Having determined these angular accelerations and substituted them into formulas of type (7), we find the accelerations of all internal points.

Knowing the accelerations of all internal points, it is not difficult to find the angular accelerations of the remaining three links by the formula

$$\varepsilon_2 = \overrightarrow{DE} \times \mathbf{a}_{ED} / |DE|^2. \tag{10}$$

The presence of sliding pairs in the mechanism does not complicate the computations. The calculation procedure remains the same; only instead of formulas (4) and (7) one must use formulas (11) and (12), respectively,

$$\mathbf{V}_{B2} = \mathbf{V}_{B1} + \lambda \cdot \overrightarrow{CD}, \tag{11}$$

$$\mathbf{a}_{B2} = \mathbf{a}_{B1} + 2\omega_1 \cdot \lambda \cdot \overrightarrow{CD}^* + \mu \cdot \overrightarrow{CD}. \tag{12}$$

The meaning of the notation in formulas (11) and (12) is clear from Fig. 2. The velocity $\mathbf{V}_{B1}$ and the acceleration $\mathbf{a}_{B1}$ are expressed in terms of the velocity $\mathbf{V}_A$ and the acceleration $\mathbf{a}_A$ by the usual formulas (4) and (7). In the case where the guide axis of the sliding pair is fixed relative to the frame, all calculations are considerably simplified.

In solving certain problems it is necessary to find the coordinates of the instantaneous center of velocities and the instantaneous center of accelerations of any link. Let us write the corresponding formulas for link 2 (Fig. 1):

$$\mathbf{r}_{pv} = \mathbf{r}_D + \frac{1}{\omega_2} \cdot \mathbf{V}_D^*, \tag{13}$$

$$\mathbf{r}_{pw} = \mathbf{r}_D + \frac{\omega_2^2}{\varepsilon_2^2 + \omega_2^4} \cdot \mathbf{a}_D + \frac{\varepsilon_2}{\varepsilon_2^2 + \omega_2^4} \cdot \mathbf{a}_D^*, \tag{14}$$

where $\mathbf{r}_{pv}$ and $\mathbf{r}_{pw}$ are the radius vectors, respectively, of the instantaneous center of velocities and of the instantaneous center of accelerations.

The radius of curvature ρ and the coordinates of the center of curvature $\mathbf{r}_\rho$ of the trajectory of a certain point D, which at the given instant has velocity $\mathbf{V}_D$ and acceleration $\mathbf{a}_D$, can be determined from the formulas

$$\rho_D = |\mathbf{V}_D|^3/\mathbf{V}_D \times \mathbf{a}_D, \tag{15}$$

$$\mathbf{r}_\rho = \mathbf{r}_D + \rho_D \cdot \mathbf{V}^*_D/|\mathbf{V}_D|. \tag{16}$$

The overwhelming majority of real mechanisms contain only two-link groups ($2k = 2$; mechanisms of class II according to

I. I. Artobolevsky). The kinematic investigation of such mechanisms is reduced to solving a number of equations, each of which is a first-degree equation with one unknown.

Having determined the numerical values of the kinematic parameters of all the links of the mechanism, carrying out a force analysis by an analogous numerical method no longer presents difficulties.

The method set forth may also be used in solving, on an electronic digital computer, various problems of mechanism synthesis by optimizing the parameters of the mechanism with respect to geometrical, kinematic, and dynamic conditions.

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Note: Figure translations are in progress. See original paper for figures.

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