STATIONARY MOTION OF A BEAM OF CHARGED PARTICLES WITH ACCOUNT TAKEN OF ITS OWN SPACE CHARGE

PHYSICS

1969

SovietRxiv

Abstract

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UDC 517.9:621.3.032.26

PHYSICS

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STATIONARY MOTION OF A BEAM OF CHARGED PARTICLES WITH ACCOUNT TAKEN OF ITS OWN SPACE CHARGE

(Presented by Academician A. N. Tikhonov, 16 V 1969)

1°. In the calculation of many electronic devices, especially in the case of high powers, there arises the problem of investigating the motion of a beam of charged particles in an external electromagnetic field with account taken of the beam' s own space charge. The same questions have to be solved in calculating various injectors of charged particles for accelerators and magnetic traps. For a sufficiently complicated field geometry, as possessed by real devices, the solution of the corresponding self-consistent problem is possible only by numerical methods. In the nonstationary case, the method of enlarged charges is often used to solve such problems ([1–3]). However, although a number of practically interesting results have been obtained here as well, a complete theoretical justification of this method has not yet been given.

For the investigation of stationary processes, iterative methods prove convenient. Various versions of these methods were used in works ([4–7]). In the present note, by the method of successive approximations, a numerical solution of a self-consistent axisymmetric problem is constructed in the case of stationary motion.

2°. Consider a domain T, bounded by a surface Σ, which is a right circular cylinder in which one of the bases is replaced by an arbitrary convex surface Σ_1, which is a surface of revolution about the axis of the cylinder. In what follows we shall conventionally call the surface Σ_1 the emitter. Introduce a cylindrical coordinate system with the z-axis coinciding with the axis of the cylinder.

Consider the following stationary process. Let a flux of charged particles with a prescribed distribution of velocities and current density on the emitter be continuously injected from the surface of the emitter into the domain T. We shall regard as known the value $u_0(p)$ of the potential Φ of the electric field on the surface Σ ($p \in \Sigma$) and the stationary magnetic field $\mathbf{H}(r, z)$ in the domain T. Assuming that the corresponding stationary regime exists, we pose the problem of determining the trajectories of the particles of the beam.

The trajectory and velocity of each particle of the beam are determined by the equations

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}, \qquad m\frac{d\mathbf{v}}{dt} = -e\nabla\Phi + \frac{e}{c}[\mathbf{v}\times\mathbf{H}] \tag{1}$$

and by the initial conditions

$$\mathbf{r}\big|_{p_1\in\Sigma_1} = \mathbf{r}_0(p_1), \qquad \mathbf{v}\big|_{p_1\in\Sigma_1} = \mathbf{v}_0(p_1). \tag{2}$$

For nonrelativistic velocities of motion, the magnetic field $\mathbf{H}(r,z)$ in equations (1) may be regarded as prescribed ([2]), while the potential of the electric field $\Phi(r,z)$ is determined from the solution of the boundary-value problem

$$\Delta\Phi = -4\pi\rho(M),\ M\in T; \qquad \Phi\big|_{p\in\Sigma} = u_0(p), \tag{3}$$

which contains the density $\rho(M)$ of the beam' s own space charge.

In order to close the system (1)–(3), it is necessary to specify an algorithm for determining the density $\rho(M)$. Consider the trajectories $r_i(z)$ and $r_{i+1}(z)$ of two cha-

particles with sufficiently close initial conditions. When these trajectories are rotated about the z-axis, they form surfaces of revolution. We shall call the region between the corresponding surfaces of revolution a current tube. It is natural to assume that, for sufficiently close initial conditions and under certain additional restrictions on the gradients of the electric and magnetic fields, the trajectories of particles located inside the current tube will not intersect its boundaries, and the total current through the transverse cross section of the current tube will remain constant. Then the density $\rho(M)$ in the current tube is determined by its geometry and by the initial conditions at the emitter. In this case, different current tubes may intersect one another. At the corresponding points of the region T, the total density $\rho(M)$ is determined as the sum of the densities in the intersecting current tubes. This algorithm for computing the function $\rho(M)$ makes it possible to close the system of equations (1)–(3).

3°. To solve the system (1)–(3), an iterative method was used. Let n be the iteration number. Then

$$\Delta\Phi^{(n)} = -4\pi\rho^{(n)}(M), \quad \rho^{(0)}(M)\equiv 0; \quad \Phi^{(n)}\big|_{p\in\Sigma} = u_0(p); \tag{4}$$

$$\frac{d\mathbf{r}^{(n+1)}}{dt} = \mathbf{v}^{(n+1)}, \quad m\frac{d\mathbf{v}^{(n+1)}}{dt} = -e\nabla\Phi^{(n)} + \frac{e}{c}\left[\mathbf{v}^{(n+1)}\times\mathbf{H}\right],$$

$$\mathbf{r}^{(n+1)}\big|_{p_1\in\Sigma_1} = \mathbf{r}_0(p_1), \quad \mathbf{v}^{(n+1)}\big|_{p_1\in\Sigma_1} = \mathbf{v}_0(p_1); \tag{5}$$

$$\rho^{(n)}(M) = \sum_i \rho_i^{(n)}(M, \mathbf{v}_i^{(n)}, S_i^{(n)}(M)). \tag{6}$$

In formula (6), the summation is carried out over the current tubes intersecting at the point M; $S_i^{(n)}(M)$ is the transverse cross section, obtained at the n-th iteration, of the i-th current tube at the point M. Thus, at each step of the iterative process one must solve the boundary-value problem (4) with the given function $\rho^{(n)}(M)$, and the Cauchy problem for the system (5) with the given functions $\Phi^{(n)}(M)$ and $\mathbf{H}(M)$. In this process the function $\rho^{(n)}(M)$, needed for the next iteration step, is also determined. The convergence of this iterative process, under certain additional restrictions on the current density at the emitter, was proved in [8]. In solving practical problems, as a rule, 4–5 iterations were sufficient.

The numerical solution of the boundary-value problem (4) was carried out using the iterative scheme of the method of variable directions [9] on a rectangular grid in the plane (r, z). The choice of the iteration parameter ensures sufficiently rapid convergence of the method. The accuracy of the results obtained in this way is determined by specifying the grid spacing in the r and z directions. In calculating a number of specific problems, it proved expedient to choose a nonuniform step in the z direction. The results obtained in the calculations had an accuracy of the order of 0.1%. To determine values at the grid nodes (r_k, z_l) of problem (4), interpolation was performed of the results obtained for the current tubes when solving problem (5). In doing so, it was natural to introduce a restriction on the maximum values of $\rho^{(n)}(M)$. In addition, it turns out to be convenient in equations (5) to take, as the independent variable, not t, but z. The resulting equations were numerically integrated by Euler' s method with a step ensuring an accuracy not worse than 0.5%.

4°. The algorithm considered makes it possible to solve two types of problems:

1. Determination of the trajectories of charged particles and of the spatial distribution of the potential in the stationary regime for prescribed values of the velocities and current density at the emitter.
2. Determination of the limiting current density for which, for a given accelerating potential, a stationary regime with zero velocities at the emitter is established (the Langmuir problem). For the indicated problems, a universal program was written in the algorithmic language ALGOL-60. The calculations were carried out on BESM-4 and BESM-6 computers. In solving problems of the first

of this type, in particular, a significant influence of the space-charge density on the focusing properties of an electrostatic system was established. Thus, it

Fig. 1. Trajectories in the stationary regime for the limiting current. The dashed lines show the trajectories of the zeroth approximation, in which the solutions of the corresponding one-dimensional Langmuir problems were taken as the initial density $\rho^{(0)}(M)$.

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turned out that in a number of cases a current density at the emitter of the order of $50\,\mu\text{A/mm}^2$ substantially disrupts the focusing of an H_1^+ ion beam at a retarding potential of the electrostatic field of 10 kV.

In solving the Langmuir problem, the limiting current density is determined from the additional boundary condition at the emitter $\partial\Phi/\partial n\big|_{\Sigma_1} = 0$. The resulting mathematical problem is then ill-posed, and in order to find its stable solution one must use regularization methods [10]. As regularizing conditions, the requirements were imposed that the current density at the emitter be constant and that the condition $\partial\Phi/\partial n\big|_{\Sigma_1} = 0$ be satisfied only at a finite number of points. A typical region for which the problem was solved is shown in Fig. 1. The problem was solved for various values of the boundary potential $u_0(P)$. From the results obtained it follows, in particular, that the well-known Langmuir law [11]

$$j_n = C\Phi_a^{3/2}/a^2,$$

where C is a normalization constant, a is the length of the retarding gap, and Φ_a is the potential difference across the gap, is satisfied to an accuracy of about 5% if the length of the accelerating gap and the value of the potential are determined from the values of the potential on the system axis near the emitter. However, these values differ significantly from the boundary values of the potential and can be found only as a result of a complete solution of the problem.

Fig. 1. Trajectories in the stationary regime for the limiting current. The dashed lines show the trajectories of the zeroth approximation, in which the solutions of the corresponding one-dimensional Langmuir problems were taken as the initial density $\rho^{(0)}(M)$.

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Received
25 IV 1969

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Note: Figure translations are in progress. See original paper for figures.