MICROSCOPIC THEORY OF THE DIRECT JOSEPHSON CURRENT

PHYSICS

1969

SovietRxiv

Abstract

Full Text

UDC 537.312.62+536.48

PHYSICS

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MICROSCOPIC THEORY OF THE DIRECT JOSEPHSON CURRENT

(Presented by Academician N. N. Bogolyubov on 12 VI 1968)

Spatially inhomogeneous problems in the theory of superconductivity are conveniently solved in the formalism of equations for Gor' kov' s Matsubara functions in the coordinate representation ([1]). However, these equations are too complicated, one reason being that they also describe very small-scale variations of the order parameter—over distances of the order of atomic ones.

Taking such variations into account leads, as can be shown, to relative corrections to the values of macroscopic quantities of order T_c/ε_F, i.e., it is essentially beyond the accuracy of the modern microscopic theory of superconductivity. From what has been said it is clear that, after carrying out an averaging procedure for the order parameter, i.e. replacing it by a smoothed function that varies over distances of the order of the coherence length ξ_0, we would pass to simpler equations. Although this procedure apparently cannot be carried out in general form, it is feasible in specific problems.

Below we consider the case in which the spatial inhomogeneity is due to the potential of a barrier of width d separating identical superconductors. In such a system current states are possible under conditions of thermodynamic equilibrium (the first Josephson effect ([2])).

We shall start from the equations

$$(i\omega-\xi)G^{ik}_{pp'}+\sum_{i'p''}\Delta^{ii'}_{pp''}F^{i'k}_{p''p'}-\frac{1}{2}\sum_{i'p''}(\hat{p}v_s+v_s\hat{p})^{ii'}_{pp''}G^{i'k}_{p''p'}=\delta_{ik}\delta_{pp'},$$

$$(i\omega+\xi)F^{ik}_{pp'}+\sum_{i'p''}(\Delta^*)^{ii'}_{pp''}G^{i'k}_{p''p'}-\frac{1}{2}\sum_{i'p''}(\hat{p}v_s+v_s\hat{p})^{ii'}_{pp''}F^{i'k}_{p''p'}=0,$$

$$\Delta^*(r)=gT\sum_{\omega}\sum_{pp'}\sum_{ik}F^{ik}_{pp'}(\omega)\omega^i_p(r)\psi^{*k}_{p'}(r).$$

Here $G^{ik}_{pp'}, F^{ik}_{pp'}$ are the coefficients in the expansion of the Green functions in a system of one-particle wave functions in the presence of a barrier, having the form $\psi^i_p(r) = V^{-1/2}\exp ip_\perp r_\perp \chi^i_p(z)$; V is the volume of the system; $\xi = p^2/2m - \varepsilon_F$; $p = \pi n/L \to 0$; $i = 1, 2$ refer respectively to waves incident from the left and from the right. On the separation of the superfluid velocity see ([3]).

Taking into account the closeness of the essential momenta to the Fermi momentum p_0, put $|p| = p_0 + \xi/v_0$, $p = |p|\cos\vartheta$, $|p_\perp| = |p|\sin\vartheta$, where ξ is small. Slowly varying functions of momentum will be replaced by their values at $|p| = p_0$. In ξ we perform a Fourier transformation, introducing a variable t having the dimension of time. In the calculations one must take into account that only $\exp i(p-p')z$ should be retained, while $\exp i(p+p')z$, oscillating at atomic distances, should be discarded for the reasons set out above. To rewrite the system of equations (1) in the t-representation, note that

$\xi \to -i\dfrac{d}{dt}$, $\quad \delta_{p,p'} \to \dfrac{\pi}{L} v_0 \cos\vartheta\, \delta(t - t')$, and sums of the form $\sum_{i'p''} f^{ii'}_{pp''} h^{i'k}_{p''p'}$, where f and h in

in configuration space–arbitrary functions z, pass into the expressions $\sum_{i'} f^{ii'}(t) h^{i'k}(t, t')$, i.e., in this way the local character of the original equations for the Green' s functions in configuration space is restored.

The expressions for $f^{ik}(t)$ are easily calculated:

$$\begin{aligned} f^{11}(t) &= Df(tv_0\cos\vartheta) + R\{\theta(-t)f(tv_0\cos\vartheta) + \theta(t)f(-tv_0\cos\vartheta)\}, \\ f^{12}(t) &= -i\sqrt{DR}\,\theta(t)\{tv_0\cos\vartheta) - f(-tv_0\cos\vartheta)\}. \end{aligned} \tag{2}$$

Here D and R are the transmission and reflection coefficients. The expressions for $f^{22}(t)$ and $f^{21}(t)$ are obtained from (2) by symmetry with respect to inversion of z. The matrix elements of the momentum in the t-representation have the form

$$\hat{p}_{11}(t) = (D - R\,\mathrm{sign}\,t)p_0\cos\vartheta, \qquad \hat{p}_{12} = -2i\theta(t)\sqrt{DR}\,p_0\cos\vartheta. \tag{3}$$

The relations given make it possible to write down directly the equations for the Green' s functions in the t-representation. They must be supplemented by relations expressing the order parameter $\Delta(z)$ and the current $j(z)$ through the Green' s functions. After simple transformations we obtain

$$\overset{*}{\Delta}(z) = \pi N(0) g T \sum_\omega \int_0^1 dx\{DF^{11}_\omega(t,t) + R(F^{11}_\omega(-t,-t) + F^{11}_\omega(t,t)) +$$

$$+DF^{22}_\omega(-t,-t) + i\sqrt{DR}(F^{21}_\omega(-t,-t) - F^{12}_\omega(-t,-t))\}; \tag{4}$$

$$j(z)=-2ev_0\pi N(0)T\sum_\omega\int_0^1 dx\cdot x\{D(G^{11}_\omega(t,t)-G^{22}_\omega(-t,-t))+$$

$$+R(G^{11}_\omega(t,t)-G^{11}_\omega(-t,-t))+i\sqrt{DR}(G^{12}_\omega(-t,-t)-G^{21}_\omega(-t,-t))\}. \quad (5)$$

Here $t=z/v_0x<0$; the normalization of the Green' s functions has been chosen so that the coefficient of $\delta(t-t')$ in the equations for them is equal to unity.

Let us emphasize that, since in passing to the t-representation an averaging over atomic distances was simultaneously performed, expressions (2)–(5) turned out to depend only on the reflection and transmission coefficients. This is quite natural, since under averaging only the asymptotics of the wave functions proved essential. No restrictions had to be imposed on the magnitude of D.

The equations in the t-representation are simpler than the original system (1) in the following respects: a) the order of each equation is lower by one; b) in the equations the limiting transition $V\to\infty$ has been carried out (in system (1) this could not be done because of the singular character of the matrix elements); c) the order parameter changes over distances of order ξ_0, while changes over atomic distances are excluded.

A complete calculation of the current can be performed for small D. In doing so it is necessary to take into account that the corrections to the Green' s functions associated with finite transparency consist of independent terms due to: a) the change in the matrix elements of the order parameter in the zeroth approximation with respect to states with finite D, as compared with their values at $D=0$; b) the change in the modulus $\Delta(z)$; c) a nonzero v_s. By the latter we mean the gradient of the continuous part of the phase; in addition, owing to averaging over atomic distances, a phase jump may arise, which is singled out separately, namely, in the zeroth approximation we set

$$\overset{0}{\Delta}(z)=\Delta(\theta(z)e^{i\varphi/2}+\theta(-z)e^{-i\varphi/2}),\quad \Delta=\text{const},\quad \varphi \text{ is the phase jump.}$$

Finding the indicated corrections to the Green' s functions and substituting them into (4) and (5), we obtain

$$j(z)=-ev_0N(0)\sin\varphi\, T\sum_\omega\frac{\Delta}{E^2}\int_0^1 dx\, xD(x)e^{-2|z|/v_0x}E+$$

$$+2ep_0xN(0)T\sum_\omega\frac{\Delta^2}{E^2}\int_0^1 dx\, x\int_{-\infty}^0 dz' v_s(z')\left(e^{-2E|z+z'|/v_0x}-e^{-2E|z-z'|/v_0x}\right); \quad (6)$$

$$\delta\widetilde{\Delta}(z) = \pi N(0) g \frac{T}{v_0} \sum_{\omega} \frac{\omega^2}{E^2} \int_0^1 \frac{dx}{x} \int_{-\infty}^{0} dz' \delta\widetilde{\Delta}(z') \left(e^{-2E|z+z'|/v_0 x} + e^{-2E|z-z'|/v_0 x} \right) -$$

$$-i\pi N(0) mgT \sum_{\omega} \frac{\Delta}{E} \int_0^1 dx \int_{-\infty}^{0} dz' v_s(z') \left(\mathrm{sign}(z+z') e^{-2E|z+z'|/v_0 x} - \right.$$

$$\left. - \,\mathrm{sign}(z-z') e^{-2E|z-z'|/v_0 x} \right) + \pi N(0) gT \sum_{\omega} \frac{\Delta}{E} \int_0^1 dx \times D(x) \left\{ \frac{\Delta^2}{E} \sin^2 \frac{\varphi}{2} - \right.$$

$$\left. - \sin^2 \frac{\varphi}{2} \left(1 + \frac{\Delta^2}{E^2} \right) e^{-2E|z|/v_0 x} - \frac{i}{2} \sin\varphi \, e^{-2E|z|/v_0 x} \right\}. \qquad (7)$$

$$E = \sqrt{\omega^2 + \Delta^2}.$$

Here $\widetilde{\Delta}(z)$ is the order parameter, from which the phase jump has been separated out. Since $\mathrm{Im}\,\delta\widetilde{\Delta}(z) = 0$, from (7) we obtain the law of current conservation, which serves to determine $v_s(z)$. At $z = 0$ we obtain the known expression for the Josephson current. As $z \to \infty$, $j \to ep_{sv}s(\infty)$, i.e., we obtain the usual expression for the supercurrent. The relation $j(0) = j(\infty)$ connects the phase jump with the superfluid velocity at infinity.

In conclusion, let us note that the t-representation method is a generalization, to the case of potentials that vary rapidly in space, of the method of averaging over quasiclassical trajectories proposed by de Gennes[4]. The averaging method was used in the problem of the Josephson effect in [5]; however, the basic relations associated with taking the barrier into account were written there intuitively, in the spirit of certain matching rules.

We express our gratitude to V. P. Galaiko and A. I. Larkin for discussions.

Received
7 VI 1968

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