INDUCED ACCELERATION OF A MODULATED CURRENT IN AN INVERTED PARAMAGNETIC MEDIUM

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Abstract

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As is known, the energy that can be stored per unit volume of a substance as a result of inversion of the population of its levels is comparatively large. Thus, upon inversion of the optical levels of a dielectric, the corresponding energy density ($W = N_0\hbar\Omega$), for a density of active centers $N_0 \sim 10^{22}$ cm^{-3}, amounts to several hundred joules. Energy densities of the same order may be obtained as a result of population inversion in a paramagnetic medium.

It is of interest to study the prospects for using this energy for the direct acceleration of charged particles. The possibility of such acceleration follows from the following elementary considerations. Suppose that an inversely populated pair of levels is characterized by a sufficiently large lifetime θ relative to spontaneous emission. Let a sequence of charged bunches pass through the medium formed by inverted atoms, the repetition frequency of the bunches ω_M being close to the resonant transition frequency $\Omega \equiv (E_2 - E_1)/\hbar$. The radiation of the medium stimulated by this current, as will be shown below, accelerates the particles that cause this radiation.

A self-consistent system of equations describing the excitation of collective oscillations of a paramagnetic medium by a given external current $\mathbf{j}$, for $kT \ll \hbar\Omega$, may be written in the form

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t}(\mathbf{H} + 4\pi\mathbf{M}); \quad \operatorname{rot}\mathbf{H} = \frac{1}{c}\frac{\partial}{\partial t}\mathbf{E} + \frac{4\pi}{c}\mathbf{j}; \quad \partial\mathbf{M}/\partial t = \gamma[\mathbf{H}, \mathbf{M}]. \quad (1)$$

We are interested in nonlinear solutions of system (1), since the maximum amplitude of the field acting on the particles, as well as the time of induced emission T, are determined essentially by the nonlinear effect of the change in the population of the levels under the action of the radiation field. Below we shall find such solutions for a model problem in which the density of the driving current is determined by the expression

$$\mathbf{j} \equiv \mathbf{j}_0 \cos\Phi; \quad \Phi \equiv \omega_M t - k_\parallel z - k_\perp x;$$

$$\mathbf{j}_0 \parallel Oz; \quad k_\parallel \equiv \omega_M / V_0.$$

Assuming the inequality $k_\perp \ll k_\parallel$ to be satisfied and taking into account that, for sufficiently strong fields, $M_0 \equiv N_0\mu \ll H_0$, system (1) can be substantially simplified:

$$\left(\frac{\partial^2}{\partial\tau^2} + \mathbf{K}^2\right) h_x - 4\pi\Omega_M^2 m_x = 0;$$

$$\left(\frac{\partial^2}{\partial\tau^2} + \mathbf{K}^2\right) h_y - 4\pi\Omega_M^2 m_y = -K_\perp I_0 \sin\Phi; \qquad (2)$$

$$\frac{\partial m_x}{\partial\tau} + m_y = \frac{\rho}{4\pi} h_y (1 - \mathbf{m}_\perp^2)^{1/2};$$

$$\frac{\partial m_y}{\partial\tau} - m_x = -\frac{\rho}{4\pi} h_x (1 - \mathbf{m}_\perp^2)^{1/2},$$

where the dimensionless variables have been introduced: $\tau \equiv \Omega t$; $\Omega \equiv \gamma H_0$; $\Omega_M \equiv \omega_M/\Omega$;

$$\rho \equiv 4\pi(\mathbf{M}_0, \mathbf{H}_0) H_0^{-2}; \quad I_0 \equiv 4\pi j_0 \Omega^{-1} |\mathbf{M}_0|^{-1}; \quad \mathbf{m}_\perp \equiv (m_x, m_y, 0) = \mathbf{M}_\perp / |\mathbf{M}_0|;$$

$$\mathbf{h} \equiv (h_x, h_y, 0) = \mathbf{H}_\perp / |\mathbf{M}_0|; \quad K_\parallel \equiv k_\parallel c/\Omega; \quad K_\perp \equiv k_\perp c/\Omega.$$

Eliminating the fields h_x and h_y from the first pair of equations of system (2), we introduce the substitution $m_x + i m_y = m \exp[i(\Phi + \varphi)]$, where $m(\tau)$ and $\varphi(\tau)$ are the slowly time-varying amplitude and phase of the magnetic polarization $\mathbf{m}_\perp$. Then the equations determining the dependences $m(\tau)$ and $\varphi(\tau)$ take the form

$$\frac{\rho}{\Delta} \frac{\dot m}{(1 - m^2)^{1/2}} + 2\Delta \frac{d}{d\tau} \frac{m}{(1 - m^2)^{1/2}} = \frac{\rho}{8\pi} K_\perp I_0 \sin\varphi;$$

$$\left(\frac{\rho}{\Delta} + 2\Delta\right) \frac{m\dot\varphi}{(1 - m^2)^{1/2}} = \rho m \left[(1 - m^2)^{-1/2} - 1\right] + \frac{\rho}{8\pi} K_\perp I_0 \cos\varphi; \qquad (3)$$

$$\Delta \equiv 1 - \Omega_M,$$

where we have assumed that the wave vector $\mathbf{K}$ satisfies the dispersion equation of the linear theory

$$\mathbf{K}^2 = \Omega_M^2[1 + \rho/\Delta(\Omega_M)], \qquad \rho/\Delta > 0. \tag{4}$$

This condition ensures resonance between the current and the wave excited by it in the medium, as a result of which, at the initial stage, the field amplitude grows linearly with time:

$$\varphi_L = 0, \qquad \varphi_L = -\pi/2 \quad (m_L > 0);$$

$$m_L = \frac{1}{8\pi}\frac{\rho\Delta}{|\rho + 2\Delta^2|}\,K_\perp I_0 \tau;$$

$$h_y^L = -\frac{1}{2}\frac{K_\perp I_0 \tau}{(2 + \rho/\Delta^2)}\cos\Phi. \tag{5}$$

Hence, for the amplitude of the longitudinal field acting on the bunch, we find

$$e_z^L = -\frac{1}{2}\frac{K_\perp^2 I_0 \tau}{(2 + \rho/\Delta^2)}\cos\Phi. \tag{6}$$

In a medium with normal population of the levels, $\rho > 0$, and the field (6) always decelerates the particles. In the presence of population inversion of the levels ($\rho < 0$), reversal of the sign of the force acting on the bunch can occur only when $|\rho| > 2\Delta^2$. This latter inequality, as is readily seen from (4), coincides with the condition for reversal of the sign of the group velocity in the medium

$$K\,dK/d\Omega_M \simeq (\beta_\phi \beta_{\rm gr})^{-1} = 1 + \rho/2\Delta^2 < 0.$$

Assuming this inequality to be satisfied, we find the nonlinear solution of system (3):

$$\varphi = \arccos(r^3); \qquad \frac{dr}{d\tau} = \frac{1}{T}(1 - r^6)^{1/2};$$

$$r \equiv \frac{m}{m_{\max}}; \qquad m_{\max} \equiv \left(\frac{K_\perp I_0}{2\pi}\right)^{1/3} \ll 1; \qquad T \equiv \frac{8\pi(\rho + 2\Delta^2)}{|\rho|\Delta(2\pi K_\perp^2 I_0^2)^{1/2}}. \tag{7}$$

From these relations it is seen that the principal nonlinear effect stabilizing the growth of the field amplitude is the change in the population of the levels and the resulting disruption of synchronism between the bunch and the wave. This effect leads to a periodic alternation of the processes of growth and decrease of the field amplitude, with a characteristic period of order T. In this case the maximum amplitude of the longitudinal field is

$$(e_z)_{\max} = -K_\perp (h_y)_{\max} = \frac{4\pi\Delta}{\rho}\left(\frac{K_\perp^4 I_0}{2\pi}\right)^{1/3}. \tag{8}$$

Strictly speaking, this relation is valid for $K_\perp \ll 1$. However, for estimating the maximum field we may put $K_\perp^2 = (1-\beta^{-2})+$

$+\rho/\Delta \sim 1$ ($\rho \sim \Delta$). Then from (8) we find (in dimensional units)

$$(E_z)_{\max} = 4\pi|M_0|\left(\frac{2j_0}{\Omega|M_0|}\right)^{1/3}. \tag{9}$$

For $M_0 \sim 10^3$, $H_0 \sim 10^5$ gauss, and $j_0 \sim 10^3$ A/cm^2, we find from this $m_{\max} \simeq 0.2$, $E_{\max} \simeq 750$ kV/cm. The corresponding period duration turns out to be $T \sim 10^3$ ($\sim 10^{-8}$ s). Since in this case $\mu \equiv eE_m\lambda/m_0c^2 \ll 1$, the threshold energy above which the particles are trapped in acceleration is small:

$$\gamma_c \equiv (2\mu)^{-1} \sim 20 \qquad (\mathcal{E}_c \sim 10 \text{ MeV}).$$

Since the maximum field (9) is comparable with the field of Bohr (polarization) losses, in order to weaken the latter the acceleration may be carried out in a channel whose radius a is determined from the condition $a < \gamma\lambda_0$ ($\lambda_0 \equiv m_0c^2/eH_0$). In this case the maximum field increases as a result of focusing of the field induced by the beam near the channel axis ($v_{\text{gr}} < 0$). An estimate of the maximum permissible external radius R_m of the paramagnetic cylinder can be obtained from the condition $R_m \equiv |\beta_{\text{gr}}|T\lambda_0$. For the parameters chosen above, $R_m \sim 15$ cm.

In conclusion, we note that the elementary mechanism of the interaction considered above reduces to Cherenkov acceleration of a modulated current in an inverted paramagnetic. The effect of reversal of the sign of the Cherenkov field acting on a particle in a nonequilibrium medium was first discovered by V. I. Veksler ([1]). Reversal of the sign of the Cherenkov field in an inverted dielectric was noted by V. P. Silin ([2]).

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Note: Figure translations are in progress. See original paper for figures.