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HYDROMECHANICS

1969

SovietRxiv

Abstract

Full Text

UDC 523.8+534.222.2

HYDROMECHANICS

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ON THE APPLICATION OF DIMENSIONAL ANALYSIS TO QUESTIONS OF THE MOTION OF INTERPLANETARY GAS DURING SOLAR FLARES*

(Presented by Academician L. I. Sedov, 10 IX 1968)

1. As is known ([1–3]), solar chromospheric flares arise and proceed in a comparatively small volume (the area of a flare occupies about 0.1% of the area of the solar disk; the height of the flare layer is of the order of 10^9 cm). The time of development of the processes in the flare source, as a rule, has a value of $3 \cdot 10^2$–$4 \cdot 10^3$ sec. The total energy E_0 released in the chromosphere during flares varies, in order of magnitude, in the range 10^{29}–10^{34} erg.

Let us consider the processes of motion of the medium outside the flare source, where the details of the process of origin and development of the flare no longer have essential significance. The propagation of disturbances from a solar flare occurs in the solar corona and the interplanetary medium.

From works ([1,4,5]), devoted to theoretical and experimental questions of determining the parameters of the interplanetary gas, one may conclude that for the gas density ρ_1 and the velocity of the undisturbed solar wind v_1 within a certain solid angle $\chi < 2\pi$ issuing from the Sun, and in the region between the Sun and the Earth' s orbit, the following approximate dependences may be adopted:

$$\rho_1 = Ar^{-\omega}, \qquad v_1 = Br^{\omega-2}. \tag{1}$$

Here r is the distance from the Sun; A and B are quantities which we shall regard as constants, independent of the angles θ, φ of a spherical coordinate system with center at the Sun and of the time t. Specific values of the quantities A, B, and ω may be found, for example, on the basis of data from rocket measurements and radar measurements.

2. We shall use the hydrodynamic approximation for the description of the motion of the medium. If the influence of the initial gas pressure p_1, electromagnetic forces, as well as viscosity and thermal conductivity, is

neglected, then the system of the principal characteristic parameters for the motion of the gas has the form

$$r, \theta, \varphi, t, E_0, A, B, \omega, \gamma, g_\odot, R_\odot, l, \Omega, \tag{2}$$

where γ is the effective adiabatic exponent of the gas; Ω is the mean angular velocity of rotation of the Sun; $g_\odot$ is the acceleration of gravity at the surface of the Sun; $R_\odot$ is the radius of the Sun; l is the characteristic linear size of the flare source. The quantities A, B, and ω may take different values depending on the time of year and on the state of the interplanetary medium, while the energy E_0 is different for different flares (by the energy E_0 here and below is meant that part of the total flare energy which participates in the motion of the gas). Therefore it is meaningful to determine those principal dimensionless parameters which characterize the phenomenon under consideration and to consider questions of recalculating the functions characterizing the motion of the gas when the parameters of the medium and the energy E_0 are varied.

* The principal results of this article were reported by the author at the Third All-Union Congress on Theoretical and Applied Mechanics (25 I–1 II 1968, Moscow).

Since gravitation and the Sun's own rotation have little effect on the propagation of disturbances during flares, the parameters Ω and $g_\odot$ may be disregarded in rough estimates of the characteristics of the moving gas. From the dimensional parameters E_0, A, B one can form the following quantities with the dimensions of length and time: $r^* = (E_0/AB^2)^{1/(\omega-1)}$–the kinetic characteristic length, $t^* = (r^*)^{3-\omega}/B$–the kinetic characteristic time. Then, for any dimensionless characteristic of the flow (for example, the density $g = \rho/\rho_1$), on the basis of the π-theorem of dimensional theory (6), we can write

$$g = g\left(r/r^*, t/t^*, \theta, \varphi, \gamma, \omega, a_1, a_2\right), \tag{3}$$

where $a_1 = R_\odot/r^*,\ a_2 = l/r^*\ (a_2 \ll a_1)$.

It follows from formula (3) that, for fixed γ, ω, a_1, a_2, dimensionless functions of the form (3) will describe a class of flows for different parameters E_0, A, and B. When the characteristic dimensions of the region of motion are much greater than $R_\odot$, one may neglect the influence of the finiteness of the solar radius on the gas motion in some neighborhood of the leading front of the disturbances, i.e., the influence of the parameter a_1, and consequently also a_2, may be neglected.

If, within the solid angle under consideration, the solar-wind velocity is assumed to be directed along the radius, and the conditions of energy release correspond to spherical symmetry of the flow, then the gas flow within the solid angle $\varkappa$ under consideration may be regarded as spherically symmetric.

Let us introduce the notation: $x = r/r^*$, $y = t/t^*$. For a spherically symmetric flow model, functions of the form (3) will depend only on two variable parameters x, y, i.e.,

$$g = g(x, y, \gamma, \omega). \tag{4}$$

Naturally, in a global consideration of the process of disturbance propagation during flares one should take into account the dependence of the initial density and of the components of the solar-wind velocity on the angles θ and φ, and possibly on the time t. In this case, the desired solution of the hydrodynamic problem will depend on a larger number of variables, and the functions entering this solution will have the form (3).

In what follows we shall consider only a spherically symmetric model of the motion of the medium. If the motion of the undisturbed solar wind is taken to be isothermal and the question is raised of taking account of the initial pressure p_1, then for the one-dimensional model we shall have:

$$p_1 = Cr^{-\omega}. \tag{5}$$

In this case we have a new dimensional constant C, which makes it possible to form a new characteristic length $r^0 = (E_0/C)^{1/(3-\omega)}$. One more dimensionless parameter, $a_3 = r^0/r^*$, will enter formulas of the form (4). Estimates made by us of the total initial energy of the solar wind in the volume within a certain solid angle show that, at distances of the order of an astronomical unit, the total initial kinetic energy of the undisturbed solar wind is approximately an order of magnitude greater than the total initial thermal energy of the gas (and greater than the initial energy of the magnetic field). We shall assume that the parameter a_3 may be neglected in a rough qualitative analysis of the phenomenon under consideration.

In order to estimate the order of magnitude of the kinetic characteristic length r^*, let us refer r^* to some radius r_a and, on the basis of the formulas for ρ_1 and v_1, eliminate AB^2 from the expression for r^*. Then we obtain $r^*/r_a = (E_0/k^*)^{1/(\omega-1)}$, $k^* = \rho_1(r_a)[v_1(r_a)]^2 r_a^3$.

In Fig. 1, for various ω, the dependences of r^*/r_a on E_0 are given for the case where $r_a = 1.5 \cdot 10^{13}$ cm (an astronomical unit), $k^* = (1.5)^3 \cdot 10^{31}$ g $\cdot$ cm^2/sec^2 (dashed lines), $k^* = 5(1.5)^3 \cdot 10^{32}$ g $\cdot$ cm^2/sec^2 (solid lines), and the flare energy E_0 varies from 10^{28} to 10^{35} erg. Here

curves 1 correspond to $\omega = 2$, curves 2 correspond to $\omega = 2.5$, curves 3 correspond to $\omega = 2.9$. The calculations carried out show that for large values of E_0 the quantity r^* may be greater than an astronomical unit.

3. The dependences of the sought functions on dimensionless parameters may be determined theoretically or experimentally from measurements

Fig. 1

Figure 1: Fig. 1

on space rockets during solar flares. In the theoretical and, especially, in the experimental determination of the gas parameters, the dependences on coordinates and time for the density, pressure, and gas velocity, and the time of arrival of the disturbance at a given point, can be found only for certain fixed values $E_0 = E_{01}$, $A = A_1$, $B = B_1$. Therefore let us consider the question of recalculating the data obtained for the case of other values of these constants E_{02}, A_2, B_2. One may proceed as follows. We find the dimensionless time y and the dimensionless coordinate x for the state E_{01}, A_1, B_1, and then, from the formulas $r = r^* x$, $t = t^* y$, find the values of the coordinate and time for the state E_{02}, A_2, B_2.

Fig. 1

For recalculating the time we find

$$y = t_{(1)}/t_1^*, \qquad t_{(2)} = t_2^* y = (t_2^*/t_1^*) t_{(1)}, \tag{6}$$

where

$$t_i^* = (r_i^*)^{3-\omega}/B_i, \qquad r_i^* = (E_{0i}/A_{iB} i^2)^{1/(\omega-1)} \quad (i = 1, 2).$$

For recalculating distances (the Eulerian or Lagrangian coordinate of a gas particle) we have:

$$x = r_{(1)}/r_1^*$$

$$r_{(2)} = r_2^* x = (r_2^*/r_1^*) r_{(1)}. \tag{7}$$

Here $t_{(i)}$, $r_{(i)}$ $(i = 1, 2)$ are the dimensional values of time and coordinate corresponding to the flare with parameters E_{0i}, A_i, B_i. Analogous formulas can also be written for recalculating the velocity, density, gas pressure, and other quantities. Thus, for the density ρ we have

$$g = \rho_{(1)}/\rho_{11}, \qquad \rho_{(2)} = g\rho_{12} = (\rho_{12}/\rho_{11})\rho_{(1)}. \tag{8}$$

Formulas (6)–(8) give (in the adopted flow model) the similarity laws for the process of gas motion caused by a flare. For a more complicated model of gas flow, formulas of the form (6)–(8) will hold only for fixed values of additional dimensional constant parameters, for example a_j $(j = 1, 2, 3)$.

4. Let us give the simplest example of dependences of the velocity, density, and gas pressure on the coordinate r and time t. We shall model the flare phenomenon by a spherically symmetric point explosion in a gas, neglecting the dimensions of the Sun, the motion of the interplanetary gas in the solar wind, the initial pressure, the influence of gravity, and magnetic fields. We shall assume that the initial gas density is

$$\rho_1 = Ar^{-\omega_1}, \qquad \omega_1 = (7-\gamma)/(\gamma+1). \tag{9}$$

In this case the system of determining parameters reduces to the constants E_0, A, and γ, and the gas motion is self-similar [6], with the solution of the gas-

the dynamic problem has the form:

$$\rho = \frac{\gamma+1}{\gamma-1}\rho_1\frac{r}{r_2}, \qquad p = \frac{2\delta^2\rho_1}{\gamma+1}\frac{r_2^2}{t^2}\left(\frac{r}{r_2}\right)^3, \qquad v = \frac{2\delta}{\gamma+1}\frac{r}{t}, \tag{10}$$

$$r_2 = \left(\frac{E_0}{\alpha A}\right)^{\delta/2} t^\delta, \qquad \delta = \frac{2}{(5-\omega_1)}, \qquad \alpha = \frac{8\pi(\gamma+1)}{3(\gamma-1)(3\gamma-1)^2}.$$

Here r_2 is the radius of the shock wave, which is the leading front of the disturbance.

The solution (10) was obtained by L. I. Sedov ([6]). In the particular case $\omega_1 = 2$, $\gamma = 5/3$, it was used in ([1]) to describe the motion of gas during flares. Let us note that if E_0 is the energy of a flare released within a solid angle χ, then, when using solution (10) to describe the motion within this solid angle, in the formula for α one should write 2χ instead of 8π. If we now put the formula for $r_2(t)$ into dimensionless form, referring the radius r_2 to r^*, and the time t to t^*, then we obtain

$$x_2 = \alpha^{-\delta/2} y^\delta. \tag{11}$$

Let r_a be the astronomical unit. Then for the time of arrival of the shock wave at the Earth' s orbit we have

$$t_a = r_a^{1/\delta}(\alpha A/E_0)^{1/2} \tag{12}$$

or, in dimensionless form:

$$x_{2a} = r_a/r^*, \qquad y_a = \alpha^{1/2} x_{2a}^{1/\delta}.$$

Let us note that formulas (11), (12) are valid not only for the dependence $\omega = \omega_1$ on γ indicated in relations (9), but also for any self-similar motions of the type considered [6]. If the quantities A, γ, ω are the same for two different flares, then it follows from formula (12) that the ratio of the squares of the arrival times of the disturbance at a given point of space is inversely proportional to the ratio of the energies of the solar flares, i.e. $t_{(2)}^2/t_{(1)}^2 = E_{01}/E_{02}$. Formula (12) makes it possible to determine the value of E_0 from known values of t_a, r_a, A. Thus, if $\omega = 2$, $t_a = 10^5$ sec, $A = 10^{-23} r_a^2$ g/cm, then we find $E_0 \sim 10^{33}$ erg. If, in the example under consideration, the motion of the gas in the solar wind is taken into account ($v_1 \neq 0$, $P_1 \neq 0$), then the problem will not be self-similar, and approximate analytical or numerical methods must be used to solve it.

5. When the influence of magnetic fields on the motion of interplanetary plasma is taken into account, the problem of determining the flow parameters becomes substantially more complicated, even in the approximation of ordinary magnetohydrodynamics. However, if the influence of the field on the motion of the medium is neglected and only the deformation of the field is determined, then here it is possible not only to carry out easily a dimensional analysis of the phenomenon, but also, on the basis of the results of [1,7], to indicate analytical dependences for the distribution of magnetic fields in space.

The author expresses his gratitude to L. I. Sedov for discussions and valuable advice.

Mathematical Institute named after V. A. Steklov
Academy of Sciences of the USSR

Received
1 IX 1968

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