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Abstract

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ON THE SELF-FOCUSING OF INTENSE LIGHT BEAMS

The present work is a continuation of the authors' work ([1]) and is devoted to a theoretical investigation of the phenomenon of self-focusing of intense light beams in media with a quadratic Kerr effect. In work ([1]) this phenomenon is studied for the case in which a monochromatic axially symmetric beam with a Gaussian transverse intensity distribution and a plane phase front is incident normally on the boundary ($z = 0$) of a nonlinear medium. In the present work this phenomenon is studied theoretically for the case in which the initial phase front is spherical, which corresponds to preliminary passage of the beam under consideration through a converging lens. The picture of the phenomenon in this case is clarified below.

The propagation of a monochromatic axially symmetric wave beam in a nonlinear medium with refractive index $n(E^2) = n_0(1 + \frac{1}{2}n_2|E|^2)$, where E is the complex amplitude of the electric-field oscillations, is described by the well-known equation

$$\frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} + 2ik\frac{\partial E}{\partial z} + n_2 k^2 |E|^2 E = 0, \tag{1}$$

where $k = \frac{\omega}{c} n_0$, ω is the frequency of the electric-field oscillations in the beam. In the case under consideration, equation (1) must be solved subject to the condition

$$E\big|_{z=0} = E_0 \exp\left(-\frac{1}{2}\frac{r^2}{a^2} - \frac{i}{2}\frac{kr^2}{R}\right). \tag{2}$$

Here a is the beam radius at the entrance into the nonlinear medium; R is the distance between the entrance plane ($z = 0$) and the point of convergence of the rays in the medium under consideration for $n_2 = 0$ (i.e., in the absence of nonlinearity). We note that in work ([1]) a numerical solution of problem (1)–(2) was obtained for $1/R = 0$. In the present work results are given for the numerical solution of this problem for a number of nonzero values of $1/R$.

Equation (1) can be brought to dimensionless form in the following way:

Fig. 1

Figure 1: Fig. 1

$$\frac{\partial^2 X}{\partial r_1^2} + \frac{1}{r_1}\frac{\partial X}{\partial r_1} + 2i\nu\frac{\partial X}{\partial z_1} + \nu^2|X|^2 X = 0, \tag{3}$$

where

$$r_1 = \frac{r}{a}, \quad z_1 = \frac{z}{R}, \quad X = \frac{E}{E_0}\frac{N}{\nu}, \quad N = \frac{E_0}{E_{\rm cr}}, \quad E_{\rm cr} = \frac{1}{\sqrt{n_2 (ka)^2}}, \quad \nu = \frac{ka^2}{R}$$

is the number of Fresnel zones in the plane $z = 0$. Accordingly, the boundary condition (2) will have the form

$$X\big|_{z_1=0} = \frac{N}{\nu}\exp\left[-\frac{1}{2}(1+i\nu)r_1^2\right]. \tag{4}$$

In solving this problem, equation (3) was approximated by an implicit difference scheme followed by application of difference marching. A description of the corresponding method and of the mathematical justifications is given in work ([2]).

The results of the numerical solution of problem (3)–(4), obtained for the values $\nu = 4; 8; 20$ and $N = 0.1; 0.3; 1; 2 \div 10$, lead to the following picture. If the quantity N (proportional to the square root of the initial beam power) is less than a certain critical value $N_1 \approx 2$, practically independent of ν, then the quantity $|X|^2$ has a single, and sharply pronounced, maximum at $r_1 = 0$, $z_1 = 1$. Thus, for $N < N_1$ the picture under consideration does not differ qualitatively from the picture of focusing in a linear medium. For $N > N_1$, an entire series of still more sharply pronounced maxima arises on the beam axis, determining a chain of focal points. The number of these points for a given N is finite and is practically the same for all ν. At the same time, their arrangement along the

Fig. 1

axis z_1 depends substantially on ν. For a given N, the entire series of focal points is located the more densely near the "linear focus" $z_1 = 1$, the larger ν is. At the same time, as ν increases, the longitudinal dimensions of the focal points also decrease.

If the value of ν is fixed and the value of N is increased, starting from N_1, then the first (initially the only) focal point will move in the direction toward the initial plane of the medium $z_1 = 0$. Upon passing through some subsequent value $N_2 > N_1$, a second focal point appears, and at a value of z_1 somewhat greater than unity (i.e., slightly beyond the linear focus $z_1 = 1$). As N increases, the second focal point (as well as the first) moves in the direction toward the initial plane $z_1 = 0$. Similarly, there exist values $N_3 < N_4 < N_5$ ..., upon passing through which a new focal point appears each time, approaching the boundary $z_1 = 0$. The general character of the arrangement of the focal points as a function of the values of the quantity N is shown in Fig. 1.

Analysis of the field off the axis shows that the mechanism of formation of focal points (for $N > N_1$) is the same as in the case of a beam with a flat initial phase front ([1]). The entire series of focal points is produced by the successive focusing of different annular zones into which the initial beam splits in the course of its propagation in the nonlinear medium. At the same time, in the interval where the focal points are located and beyond it, the transverse intensity distribution of the beam under consideration acquires a complicated annular structure, characterized by a series of maxima and minima of the quantity $|X|^2$ as a function of r_1.

Analysis of the results presented in the present work and in ([1]) shows that, under typical conditions, observation of self-focusing with preliminary passage of the beam through a lens may have the following advantage in comparison with self-focusing of a parallel beam. When a lens is used, observation of self-focusing can easily be carried out in practice with small excesses of the initial power

of the beam above the critical value, which in turn is essential for preserving the axial symmetry of a real beam during its subsequent propagation in a nonlinear medium.

Let us consider a numerical example. Let the initial radius of the beam be 0.5 mm; let the wavelength in the substance be $\lambda = 0.46 \cdot 10^{-4}$ cm; and let the length of the cell with the nonlinear substance be $l = 10$ cm. Then, for a parallel initial beam, the appearance of the first focal point within the volume of the cell will occur only if the initial power exceeds the critical value by more than a factor of 200. It is clear that, for such excesses of the initial power over the critical value, deviations from axial symmetry of the initial distribution in a real beam may themselves contain supercritical power, and thus individual regions of the beam may undergo independent self-focusing, which will naturally complicate the pattern of the arrangement of focal points and lead to a violation of the axial symmetry of the beam. At the same time, when a lens is used (with Fresnel number $v \lesssim 4$), focal points are formed near the linear focus as soon as the initial power exceeds the critical value. Therefore, if the linear focus lies inside the cell or not far from its exit face, then focal points in the volume of this cell will evidently be formed for small excesses of the initial power over the critical value.

Observation of self-focusing with preliminary passage of the beam through a lens may have one further advantage. For pulse durations of the driving laser on the order of 10^{-8} sec, the temporal variation of the initial power, corresponding to the envelope of the laser pulse, leads to quasistationary axial motion of the focal points [3]. It can be shown that the characteristic velocities of motion of these points in the case of self-focusing with a lens may be much smaller than without a lens, which in turn may facilitate observation of the time evolution of the positions of the focal points.

Let us also note that the results presented above make it possible to investigate the pattern of self-focusing not only for quasistationary, but also for ultrashort laser pulses that have first passed through a converging lens. The analogous question for a parallel initial beam was specially considered in Ref. [4]. In the case with a lens, the method of quantitative investigation and the main qualitative results do not differ from the case of a parallel beam. Therefore we shall not dwell further on this question.

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Note: Figure translations are in progress. See original paper for figures.