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Abstract

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ON THE QUESTION OF OPTIMIZING COMPLEX LARGE-SCALE MULTICONNECTED SYSTEMS

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Optimization of complex multiconnected systems can be carried out by methods of mathematical programming ([1-3]). However, the practical solution of the problems obtained in this way encounters difficulties associated with large dimensionality; moreover, the specifics of multiconnected systems are such that mathematical programming problems have an arbitrary (in particular, complete) filling of the constraint matrix with nonzero elements, which makes the use of the ideas of block programming ineffective, while aggregation is often not permissible. Below we show an approach to solving large-scale mathematical programming problems arising in the optimization of complex multiconnected systems, which makes it possible, in an acceptable number of iterations, to obtain a solution both on a digital computer even with a small operating memory and on a network model.

1. Let the following problem be solved:

$$CX \to \max; \tag{1}$$

subject to

$$AX \leq B; \tag{2}$$

$$X \geq 0, \tag{3}$$

where $C = \{c_j\}$ is an n-dimensional row vector of the objective function; $X = \{x_j\}$ is an n-dimensional column vector of the unknown variables; $A = \{a_{ij}\}$ is a matrix of size $m \times n$; $B = \{b_i\}$ is an m-dimensional column vector with nonnegative components. The rows of system (2) are linearly independent.

Let us introduce the matrix A^*, each column and each row of which contain no more than one positive element. We shall call A^* the required form of the matrix. The positive elements of the matrix A^* will be called the elements of the pseudodiagonal.

Reduction of the matrix $A = \{a_{ij}\}$ to the required form can be accomplished if, in place of the elements of the pseudodiagonal A^* in the matrix $A = \{a_{ij}\}$, there are elements a_{sl} exceeding the remaining elements of the s-th row and the l-th column (i.e. $a_{sl} \geq \max_{j \neq l,\, i \neq s}\{a_{sj}; a_{il}\}$). In optimization problems for objects of multiconnected regulation this occurs when the "mutual influence" of the regulated parameters is less than the "self-influence." To reduce to the required form in this case, it is sufficient to carry out transformations as a result of which, from each element of the i-th row $(i = 1, \dots, m)$ of the matrix A, the number

$$a_m^i = \max_{\substack{j \\ j \neq l}} a_{ij}$$

will be subtracted. In doing so, it should be taken into account that the vector $B^* = \{b_i^*\}$ of the right-hand side of the constraints obtained after the transformation must have nonnegative components $b_i^* \geq 0$.

In some cases the required form of the constraint matrix can be obtained by reformulating the problem, for example, in a problem of optimal-

of controlling an oil-producing enterprise, formulated through influence coefficients with respect to flow rate. Rows that do not reduce to the required form can be separated out using known decomposition methods.

2. **Theorem 1.** *If the matrix A of problem (1)–(3) has been reduced to the form A^*, then, when solving the problem by the simplex method: a) each column of the matrix can be a pivot column only once; b) the pivot elements are always located on the pseudodiagonal.*

On the basis of part a) of Theorem 1, the columns of the matrix A^* that have been pivot columns need no longer be entered into the tableau; instead, columns having negative estimates of the constraint vectors may be introduced into the computer's operative memory, expressing them through the basis vectors. Hence, in accordance with the requirements determined by the capacity of the computer' s operative memory, the matrix A can be divided into parts (by columns), and at each stage of the solution only the set of vectors of the given part is used. Thus, the solution of the large-dimensional problem under consideration can be carried out by parts, independently of the form and degree to which the constraint matrix is filled with zero elements, and the optimum will be obtained in no more than n iterations of the simplex method.

Let us note that, using the ideas of work (4) concerning the simultaneous introduction into the basis, in one "large" iteration, of a set of vectors, one can construct the algorithm in such a way that the solution will be reached in one "large" iteration. In this case the matrix A^* will not be transformed in the process of the solution, which will make it possible to reduce the computational error considerably.

3. The solution of these problems can also be carried out on network models or on other devices modeling processes described by the system

$$A^* X^* = B'. \tag{4}$$

The constraints of the problem are:

$$B' \leq B^* \tag{5}$$

The objective function is:

$$L^* = C^* X^* \to \max, \tag{6}$$

where C^* is a row vector; X^* and B^* are column vectors with nonnegative components; C^*, X^*, B^* are obtained respectively from C, X, B after reducing A to A^*.

Let us show the process of solving the problem by the example of a network model of an oil reservoir operated by a system of wells. Currents at the corresponding nodes of the network I are equivalent to well flow rates, and voltages to drawdowns. We set the initial $I = B$. In this case it follows from the structure of A^* that conditions (5) will be satisfied automatically in the process of solution, i.e., no additional devices are required for modeling constraints (5). The solution procedure consists in determining the nodes whose disconnection of the imposed current gives a positive increment of the objective function (6) of the problem:

$$\Delta L^* = C^* \cdot \Delta X^* > 0,$$

and, on the basis of part a) of Theorem 1, a vector once removed from the basis can no longer be reintroduced into the basis; i.e., if it is expedient to disconnect the current of a given node ($\Delta L^* > 0$), then this current will remain equal to zero at the optimal point as well. Thus, for the practical solution of problems of optimizing the operating regime of an oil field on a network model, it is sufficient to check the expediency ($\Delta L^* > 0$) of disconnecting heavily watered wells, leaving disconnected those wells for which $\Delta L^* > 0$. The procedure described will make it possible to obtain the optimal regime easily and directly while modeling fields.

Let us note that in some cases it is possible in this way to solve on a network model also problems (3) of optimizing oil fields in an elastic regime. Let us also note that, in the presence of certain production or planning constraints that do not reduce to the required form, the solu-

can be obtained on the basis of the decomposition principle as a linear combination of the solutions of the problem considered for different objective functions.

4. Let us consider the question of solving linear programming problems in which the matrix of the constraining inequalities is symmetric. Problems of this class arise in the optimization of multiply connected objects. Preserving the symmetry property when solving problems on a digital computer would make it possible to halve the required amount of machine memory and the amount of computation. However, the difficulty lies in the fact that, when the problem is solved by finite methods of linear programming, the symmetry of the transformed matrix may be destroyed after the very first iteration.

Considering the conditions for preserving the symmetry property of the matrix $A = \{a_{ij}\}$ under transformations according to the recurrence formulas of modified Jordan eliminations, one can verify that the transformed matrix $A' = \{a'_{ij}\}$ will be symmetric in the absolute values of its elements if the pivot elements in the transformations always lie on the main diagonal. Taking into account item b) of Theorem 1, the following theorem can be proved.

Theorem 2. The matrices $A' = \{a'_{ij}\}$ obtained at the iterations of solving, by the simplex method, problem (1)–(3) with a symmetric matrix $A = \{a_{ij}\}$ are symmetric in the absolute values of their elements if $a_{ij} \leqslant 0;\ i = 1, \dots, n;\ j = 1, \dots, n;\ i \neq j$.

5. This approach can be generalized to the case of solving convex programming problems containing nonlinear separable functions only in the diagonal terms of the constraint matrix, if the linear part of the matrix can be reduced to the required form and the components of the objective function are nonnegative.

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Note: Figure translations are in progress. See original paper for figures.