THE QUASIOPTICAL POTENTIAL IN SOME SIMPLE MODELS OF QUANTUM FIELD THEORY

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Abstract

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THE QUASIOPTICAL POTENTIAL IN SOME SIMPLE MODELS OF QUANTUM FIELD THEORY

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1. Recently, significant successes have been achieved along the path of a kinematic description of experimentally observed multiplets of elementary particles. Nevertheless, in the quantum theory of strong interactions of elementary particles, the elucidation of the nature of the dynamical mechanism of these interactions remains a problem. Along with the many powerful and effective methods for studying strong interactions that have been developed so far and are not connected with the use of perturbation theory, the study of simple models that make it possible to obtain the result of interest to us in closed form is of undoubted interest. In the present work the known method of the quasioptical potential ([1]), certain aspects of which have recently again been discussed ([2]), is applied to simple models of quantum field theory that admit an exact solution.

2. The equation describing the scattering of a scalar relativistic particle of mass μ by a very heavy particle, which we shall regard as fixed, can be written in the form

$$\left(-\frac{\partial^2}{\partial t^2} + \Delta_r - \mu^2\right)\Psi(t,\mathbf{r}) = \int_{-\infty}^{+\infty} dt' \int d\mathbf{r}'\, \widetilde{W}(t',\mathbf{r},\mathbf{r}')\Psi(t-t',\mathbf{r}). \quad (1)$$

The interaction $\widetilde{W}$ introduced by us between the light particle and the fixed source is nonlocal not only in space but also in time. The stationary function will satisfy the equation

$$(E^2 + \Delta_r - \mu^2)\Phi(\mathbf{r}) = \int W(E,\mathbf{r},\mathbf{r}')\Phi(\mathbf{r}')\, d\mathbf{r}' \quad (2)$$

with a nonlocal potential W, which will depend on the energy as a consequence of the nonlocality in time of the interaction $\widetilde{W}$, and will be its Fourier transform with respect to the time variable

$$W(E,\mathbf{r},\mathbf{r}') = \int_{-\infty}^{+\infty} \widetilde{W}(t,\mathbf{r},\mathbf{r}')e^{iEt}\,dt. \tag{3}$$

The interactions under consideration that are nonlocal in time must satisfy the principle of microscopic causality. It immediately follows from this that the potential W, defined by relation (3), will be a function analytic in the upper and lower half-planes of the complex variable E.

In the momentum representation, equation (2) is written in the form

$$(E^2 - \omega_q^2)\psi(\mathbf{q}) = \int V(E,\mathbf{q},\mathbf{q}')\psi(\mathbf{q}')\,d\mathbf{q}', \tag{4}$$

where $\mathbf{q}$ is the particle momentum, $\omega_q = \sqrt{q^2+\mu^2}$ is its energy.

In accordance with the results of ([3]), we shall henceforth assume,

that the potential $\tilde{V}(E,\mathbf{q},\mathbf{q}')$ is completely factorized,

$$\tilde{V}(E,\mathbf{q},\mathbf{q}') = V(E)v(q)v^*(q'), \tag{5}$$

and we shall call the function $V(E)$ the quasioptical potential. The functions $v(q)$ and $v^*(q')$ depend only on the modulus of the corresponding momentum, which is consistent with the scattering of scalar particles, in the models considered below, only in the S-state.

Equation (4), with relation (5) taken into account, takes the form

$$(E^2 - \omega_q^2)\psi(\mathbf{q}) = V(E)v(q)\int v^*(q')\psi(\mathbf{q}')\,d\mathbf{q}'.$$

Following the general scheme (3), we find its solution

$$\psi_p(\mathbf{q}) = \delta(\mathbf{p}-\mathbf{q}) + \chi_p(\mathbf{q}),$$

$$\chi_p(\mathbf{q}) = \frac{v^*(p)v(q)V(E)}{1+V(E)I(E)}\frac{1}{E^2-\omega_q^2}, \tag{6}$$

where

$$I(E) = \int \frac{|v(q)|^2\,d\mathbf{q}}{\omega_q^2 - E^2}, \qquad \operatorname{Re} E = \omega_p.$$

With the aid of the solution (6) found above, one can determine (through the quasipotential) the amplitude

$$T(\omega_p) = -2\pi^2 p|v(p)|^2 V(\omega_p)/[1 + V(\omega_p)I(\omega_p)], \tag{7}$$

which is related to the S-matrix by

$$S(\mathbf{p}, \mathbf{q}) = \delta(\mathbf{p} - \mathbf{q}) + 2i\delta(\omega_p - \omega_q)T(\omega_p).$$

It can also be shown that if the potential $V(E)$ is complex and its imaginary part is negative in the physical energy region $\mathrm{Re}\, E \geq \mu$, then the scattering matrix satisfies the condition $SS^+ < 1$, i.e., it is "subunitary," which corresponds to absorption of light particles into bound states or to the presence of inelastic scattering channels. The representation (7) obtained by us will be used below for the "reconstruction" of the quasipotential $V(E)$ from the amplitude T, and for establishing its analytic properties in the complex energy plane E.

3. Let us consider a model of field theory studied in detail in ([4]). We restrict ourselves here to the case in which the C-particle ([4]) is absent and the indicated model reduces to the well-known Lee model ([5]), whose Hamiltonian we write in the form

$$H = (m_V - \delta m_V)V^+V + m_{NN}^+N + \int \omega_k a_k^+ a_k \, d\mathbf{k} +$$

$$+ g Z_V^{-1/2} (2\pi)^{-3/2} \int u(k)\{V^+ N a_k + a_k^+ N^+ V\} \, d\mathbf{k}, \tag{8}$$

where $u(k) = f(k)(2\omega_k)^{-1/2}$; $f(k)$ is a real function (a cutoff in momentum); $V^+(V)$, $N^+(N)$, and $a_k^+(a_k)$ are the creation (annihilation) operators of fixed fermions V and N with masses m_V and m_N, and of a boson θ with mass μ, momentum $\mathbf{k}$, and energy $\omega_k = \sqrt{k^2 + \mu^2}$, with $m_N < m_V < m_N + \mu$.

In what follows, the cutoff $f(k)$ is assumed to be such that $0 < Z_V \leq 1$. The Hamiltonian (8) admits only the processes $V \leftrightarrow N + \theta$. Let us first consider $N\theta$-scattering. It is easy to obtain ([4,5]) the following expression for the amplitude

$$T(\omega_p) = -2\pi^2 g^2 p f^2(p)(2\pi)^{-3} h^{-1}(\omega_p), \tag{9}$$

$$h(\omega_k) = (\omega_k - m)\left\{1 + g^2(2\pi)^{-3}(\omega_k - m)\int \frac{u^2(q)\, d\mathbf{q}}{(\omega_q - m)^2(\omega_q - \omega_k)}\right\}.$$

Setting $v(p) = f(p)(\omega_p - m)^{-1}$ in representation (7) for T, we obtain

$$T(E) = -\frac{2\pi^2 p f^2(p)}{(\omega_p - m)^2}\frac{V(E)}{1 + V(E)J(E)}. \tag{10}$$

It is convenient to introduce the inverse potential ([3])

$$V(E) = g^2(2\pi)^{-3}(\omega_p - m)\frac{1}{1 + g^2 U(E)}. \tag{11}$$

Substituting (11) into (10) and comparing with (9), we obtain

$$g^2 U(E) = -g^2 \frac{E - m}{(2\pi)^3}\int_\mu^\infty \frac{k f^2(k)\, d\omega_k}{(\omega_k - m)^2(\omega_k + E)}. \tag{12}$$

It follows from expression (12) that, since $\omega_k \geq \mu > m$, $U(E)$, and consequently also $V(E)$, will be functions analytic in the entire E-plane with a cut along the left-hand real semiaxis $-\infty < \operatorname{Re} E \leq -\mu$.

4. Let us now consider the scattering of a θ-particle by a V-particle in the Lee model, which was studied in detail in ([4]). In order to simplify cumbersome formulas, we shall assume $m_V = m_N = 0$, which does not change the essence of the conclusions. We write the expression for the amplitude of $V\theta$-scattering in the following form ([4,6])

$$T(\omega_p) = +\frac{2\pi^2}{(2\pi)^3}\frac{p f^2(p)}{\omega_p} g^2 \left[\frac{1 - \omega_p C(\omega_p)}{1 + \omega_p C(\omega_p)} + \beta(\omega_p)\right]^{-1}. \tag{13}$$

Here

$$C(\omega_p) = \frac{1}{\pi}\int_\mu^\infty dx\, \frac{\operatorname{Im}[1 - \beta(x)]}{x(x - \omega_p)|1 - \beta(x)|}\;{}^2\;\frac{\beta(\omega_p - x)}{1 - \beta(\omega_p - x)}; \tag{14}$$

$$\beta(\omega_p) = -g^2 \frac{\omega_p}{4\pi^2}\int_\mu^\infty \frac{k f^2(k)\, d\omega_k}{\omega_k^2(\omega_k - \omega_p)}. \tag{15}$$

Setting in (7) $v(p) = f(p)\omega_p^{-1}$, introducing the inverse potential analogously to how it was done in the preceding section, and comparing with representation (13) for the amplitude T, we obtain

$$g^2 U(E) = [1 - EC(E)]/[1 + EC(E)] - 1 + \beta(E).$$

According to relation (16), the function $\beta(-E)$, and consequently $U(E)$ and the quasipotential $V(E)$, will have an unphysical cut on the left-hand real semiaxis $-\infty < \operatorname{Re} E \leq -\mu$. On the right-hand real semiaxis we have

$$g^2 \operatorname{Im} U(E) = -2E \frac{\operatorname{Im} C(E)}{|1 + EC(E)|^2}, \quad (16)$$

i.e., the imaginary part of $U(E)$ will be determined by the imaginary part of $C(E)$.

It is easy to show that for $\operatorname{Re} E \geq 2\mu$ one must have $\operatorname{Im} C(E) \geq 0$, and therefore $\operatorname{Im} U(E) \leq 0$. From this it is seen that in the physical energy region ($\operatorname{Re} E \geq 2\mu$) $\operatorname{Im} V(E) \leq 0$, which corresponds to the presence of an inelastic channel of $V\theta$-scattering, namely, scattering into the state of one N-particle and two θ-particles.

5. In conclusion, let us consider the model proposed earlier ([7]) in connection with the problem of "ghost" states. The Hamiltonian of the model has the form

$$H = \int \omega_k \left(a^+_{k,+} a_{k,+} + a^+_{k,-} a_{k,-} \right) dk +$$

$$+ g_0 (2\pi)^{-3/2} \int u(k) \left\{ \left(a^+_{k,+} + a_{k,-} \right) A + A^+ \left(a_{k,+} + a^+_{k,-} \right) \right\} dk,$$

where $a_{\mathbf{k},\pm}$ $\left(a^+_{\mathbf{k},\pm}\right)$ are the annihilation (creation) operators of a light Fermi particle with charge ± 1, momentum $\mathbf{k}$, mass μ, and energy $\omega_k = \sqrt{k^2 + \mu^2}$; the operators $A^+(A)$ transform a neutron (proton) into a proton (neutron). In contrast to the Lee model considered in the preceding sections, the present model has the property of crossing symmetry. Therefore it is sufficient to consider scattering processes only on one charge state of the source, say, on the proton. The scattering amplitudes of light particles with charges ± 1 on the proton can be determined exactly (7) and are equal to

$$T^\pm(E) = T(\pm E) = -\frac{2\pi^2 p f^2(p) g^2}{(2\pi)^3 (\pm E)} \frac{1}{1 + g^2 I_1(E)}; \quad (17)$$

$$I_1(E) = \frac{E^2}{(2\pi)^3} \int \frac{f^2(k)\, dk}{\omega_k^3 (\omega_k^2 - E^2)}.$$

Substituting $v(p) = f(p) \omega_p^{-1}$ into representation (7), introducing the inverse potentials $U^\pm(E)$, and comparing with (17), we obtain

$$U^\pm(E) = \mp \frac{E}{(2\pi)^3} \int \frac{f^2(k)\, dk}{\omega_k^3 (\omega_k \pm E)}; \quad (18)$$

$$V^\pm(E) = \pm g^2 E (2\pi)^{-3} [1 + g^2 U^\pm(E)]^{-1}. \quad (19)$$

The potentials obtained have singularities in nonphysical regions: the potential $V^+(E)$ on the left real half-axis $-\infty < \mathrm{Re}\,E \leq -\mu$, and the potential $V^-(E)$, crossing-symmetric with respect to it, on the right real half-axis $+\mu \leq \mathrm{Re}\,E < +\infty$. Therefore, for physical values of the energy the unitarity condition is satisfied exactly, which corresponds to the absence of inelastic channels in scattering in the given model.

6. Summing up, one may draw the following conclusions. In the models considered, the exact solutions obtained for the scattering problem make it possible to reconstruct the quasioptical potential and to clarify its analytic properties. The most interesting characteristic of the quasioptical potential, besides its complete factorizability in these models with a fixed source, is its dependence on energy, which is associated with the nonlocality in time of the original interaction.

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Note: Figure translations are in progress. See original paper for figures.