ON THE OPTIMAL SYNTHESIS OF AUTOMATIC MACHINES

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Academician I. I. Artobolevskii, D. Ya. Il' inskii, I. I. Kapustin

ON THE OPTIMAL SYNTHESIS OF AUTOMATIC MACHINES

Increasing the reliability of automatic machines, achieved mainly by increasing their initial cost, leads, on the one hand, to a reduction in operating costs for the equipment itself and, on the other, to an increase in its actual productivity.

The cost price per unit of output, chosen as the objective function, may be represented in the form

$$C = \left[C_{\mathrm{p}} + \sum_{i=1}^{n} m_i C_{1i} + C_{1\mathrm{m}} + \sum_{i=1}^{n} m_i C_{2i} + C_{2\mathrm{m}}\right] \left(\frac{m}{T_{\mathrm{p}}} \eta_{\mathrm{is}}\right)^{-1}. \quad (1)$$

In the first factor of formula (1) there appear the monetary expenditures for manufacturing the product, referred to a unit of operating time. Here C_{p} is the "constant" part of the costs, i.e., costs not connected with the reliability of the machine; $\sum_{i=1}^{n} m_i C_{1i}$ is the total initial cost of a machine consisting of n different standard sizes or types of functional mechanisms or devices; the initial cost C_{1i} of each of the m_i identical i-th mechanisms or devices, corresponding to one or another level of their reliability, is a function (usually tabulated) of their specific duration of adjustment, i.e., $C_{1i} = f_{1i}(B_i)$; $C_{1\mathrm{m}}$ is the "initial" cost, i.e., the cost of the feedstock materials corresponding to the level of their suitability and being a function of the specific duration of adjustment, $C_{1\mathrm{m}} = f_{1\mathrm{m}}(qB_{\mathrm{m}})$; $\sum_{i=1}^{n} m_i C_{2i}$ is the total current cost, and its components (analogously to that indicated above) are $C_{2i} = f_{2i}(B_i)$; $C_{2\mathrm{m}}$ is the current cost, corresponding to the level of suitability of the feedstock materials, associated with the removal from the machine of unsuitable feedstock materials, etc., and constituting (analogously to that indicated above) $C_{2\mathrm{m}} = f_{2\mathrm{m}}(qB_{\mathrm{m}})$.

The first factor of formula (1) is interpreted as a nonlinear function of $n+1$ variables

$$C = f(B_1, B_2, \dots, B_n, B_{\mathrm{m}}),$$

which may be written as the sum of $2(n+1)$ functions, each of which is a function of only one variable, i.e.,

$$C = C_{\mathrm{p}} + m_1 f_{11}(B_1) + m_2 f_{12}(B_2) + ... + m_n f_{1n}(B_n) \\ + m_1 f_{21}(B_1) + m_2 f_{22}(B_2) + ... + m_n f_{2n}(B_n) + f_{1\mathrm{m}}(B_{\mathrm{m}}) + f_{2\mathrm{m}}(B_{\mathrm{m}}).$$

Functions that are written in this form are known as separable.

In the second factor of formula (1), in addition to the given quantities T_{p} and m, there appears the overall utilization coefficient η_{is}, which may be expressed through the utilization coefficient η_{nsl}, characterizing nonrandom off-cycle time losses, and the utilization coefficient η_{sl},

characterizing random off-cycle losses of time, i.e.

$$\eta_{\mathrm{use}} = \left(\frac{1}{\eta_{\mathrm{nsl}}} + \frac{1}{\eta_{\mathrm{sl}}} - 1 \right)^{-1}.$$

In accordance with work (1), the coefficients η_{nsl} and η_{sl} are

$$\eta_{\mathrm{nsl}} = \left(\frac{1}{\eta_{\mathrm{in}}} + \frac{1}{\eta_{\mathrm{rem}}} + \frac{1}{\eta_{\mathrm{feed}}} + \frac{1}{\eta_{\mathrm{serv}}} - 3 \right)^{-1},$$

$$\eta_{\mathrm{sl}} = \left(\frac{1}{\eta_{\mathrm{adj}}} + \frac{1}{\eta_{\mathrm{mat}}} - 1 \right)^{-1}.$$

From the standpoint of production costs for manufacturing the product, ensuring the stability of the work rhythm, and creating conditions for rational maintenance and operation of the equipment, nonrandom and random downtimes due to material losses that are identical in duration are far from equivalent. Therefore it is advisable to specify the minimum permissible value not only of the general utilization coefficient η_{use}, but also of the utilization coefficient η_{nsl}, which characterizes nonrandom losses of time.

The utilization coefficient η_{nsl} can be determined on the basis of probabilistic-statistical data concerning the corresponding off-cycle time expenditures. Thus, if η_{use} and η_{nsl} are known, the minimum permissible value $[\eta_{\mathrm{sl}}]$ will be:

$$[\eta_{\mathrm{sl}}] = \left(\frac{1}{\eta_{\mathrm{use}}} - \frac{1}{\eta_{\mathrm{nsl}}} + 1 \right)^{-1}.$$

But, on the other hand,

$$\eta_{\mathrm{sl}} = \left(\frac{1}{\eta_{\mathrm{adj}}} + \frac{1}{\eta_{\mathrm{mat}}} - 1 \right)^{-1} = \left(1 + \sum_{i=1}^{n} m_{iB} i + q B_{\mathrm{m}} \right)^{-1} = (1 + B_{\mathrm{tot}} + q B_{\mathrm{m}})^{-1}.$$

Consequently, the specific duration of adjustment of the entire machine B_{tot}, together with the specific adjustment qB_{m} caused by the arrival of substandard input materials, must be

$$B_{\text{tot}} + qB_{\text{m}} \leq [B], \quad \text{where } [B] = \{1 - [\eta_{\text{sl}}]\}/[\eta_{\text{sl}}].$$

In order that the actual productivity of the machine be no lower than the specified one, the following condition must be satisfied: at the specified operating rate, determined by the duration of the working cycle T_{r}, the reliability indicators of the functional mechanisms and the quality indicators of the input materials must satisfy the last equality-inequality. The tendency to choose more reliable mechanisms inevitably leads to an increase in the initial cost of the machine. Hence follows the problem: to minimize the monetary expenditures C, referred to a unit of operating time, for manufacturing the product, under the condition that the utilization coefficient of the machine operating with a specified working cycle T_{r} will be no lower than the specified $[\eta_{\text{use}}]$.

Since the reliability-cost indicator dependences for functional mechanisms differing in purpose have different characters, the formulated problem is essentially a problem of optimally distributing the reliability indicators of the entire machine among the functional mechanisms comprising it.

We arrive at the problem of nonlinear programming:

$$\sum_{i=1}^{n+1} m_{iB} i \leq [B],$$

$$m_{n+1} = q; \quad B_i > 0; \quad B_{n+1} = B_{\text{m}}; \quad m_i = 1, 2, \dots; \quad i = 1, 2, \dots, n, n+1;$$

find

$$\min C = \Sigma f(B_1, B_2, \dots, B_n, B_{\text{m}}).$$

The procedure for solving such problems, which belong to problems of nonlinear programming with a separable objective function $C = \sum f(B_i)$, is sufficiently well known (2) and, for practical cases, is relatively easily implemented on a computer.

Thus, we obtain the possibility of quantitatively formulating requirements for the optimal reliability indices of the mechanisms and devices of a machine and of choosing, in accordance with them, the principle of operation, as well as the design or size type of these elements.

The reliability indices of the functional mechanisms and devices of a machine, which determine the reliability and, consequently, the productivity of the machine, are conditioned by many factors associated not only with the principle of operation of these mechanisms and devices, but also with the quality of manufacture and the specific nature of operating conditions.

Let us consider the relation between the failure-free operation and the accuracy of a functional mechanism, taking into account its possible states, characterized by a continuous change in the state parameter and the corresponding ability to perform the functions assigned to this mechanism. The probability of failure-free operation of the i-th mechanism, by the formula of total probability, is written as

$$P_i(t,\lambda) = \int_{-\infty}^{\infty} P(\Omega) f(\Omega, t, \lambda)\, d\Omega,$$

where $P(\Omega)$ is the probability that the mechanisms will perform the assigned functions for definite values of its parameter Ω; $f(\Omega, t, \lambda)$ is the probability density of the parameter Ω under operating condition λ.

To solve problems connected with the evaluation of accuracy and reliability (more precisely, failure-free operation), one should choose, in the appropriate way, the probabilistic effectiveness function $P(\Omega)$ and the parameter Ω (3).

Let us assume that the parameter Ω characterizes the accuracy of a functional mechanism. Next we introduce the effectiveness function $M(\Omega)$, by which we shall understand the probability that a functional mechanism will perform the assigned functions as a function of the deviation of the parameter Ω from the design values.

For the very frequently encountered normal distribution law of the probability density of the parameter Ω, with a bilateral tolerance for the indicated parameter, one may write (for a fixed time interval and given operating conditions)

$$P_i(t) = \Phi(a_\Omega - a_M)/\sqrt{\sigma_\Omega^2 + \sigma_M^2},$$

where $\Phi(\ )$ is the normalized Laplace function; a and σ^2 are, respectively, the mathematical expectation and variance of the random variable; the indices M and Ω correspond to the distribution law of the effectiveness function and the probability-density function of the parameter Ω. Bearing in mind that, first, the reliability coefficient of the machine is

$$\eta_{\text{rel}} = \left[1 + \sum_{i=1}^{n} m_i \frac{1 - \eta_{\text{rel}\ i}}{\eta_{\text{rel}\ i}}\right]^{-1},$$

where $\eta_{\text{rel } i}$ is the reliability coefficient of the machine' s i-th mechanism and device, and, second, that for a relatively short duration of stoppage-free operation of most technological machines (for example, during two shifts) there is numerical coincidence of the quantities $\eta_{\text{rel } i}$ and $P_i(t)$, one can find, as a function of the specified reliability coefficient of the i-th mechanism $\eta_{\text{rel } i}$, the permissible deviations of the accuracy parameter Ω of the functional mechanism from its mean value (the variance σ_Ω^2).

In order optimally to distribute the tolerance of the accuracy parameter of the entire functional mechanism among its constituent elements (elemen-

···tary mechanisms), minimizing the cost of manufacturing the parts of this mechanism, one can make use of the methods of dynamic programming.

If the accuracy parameter Ω of a functional mechanism is connected with the corresponding accuracy characteristics of the k elements of the mechanism by the function

$\Omega = \varphi(x_1, x_2, \dots, x_j, \dots, x_k)$,

and σ_j^2 is the variance, then the problem is formulated as follows:

$$\sum_{j=1}^{k} \left(\frac{\partial\varphi}{\partial x_j}\right)_0^2 \sigma_j^2 \leqslant \sigma_\Omega^2; \qquad \sigma_\Omega^2 \geqslant 0;$$

find

$$\min z = \sum_{j=1}^{k} f_j(\sigma_j^2).$$

In the latter expressions, $\partial\varphi/\partial x_j$ is the value of $\partial\varphi/\partial x_j$ calculated at the point a_j corresponding to the mean value of x_j, and $f_j(\sigma_j^2)$ is the cost function of the j-th element.

An analytical consideration of the interrelated factors according to the scheme: production cost and productivity of the machine—reliability of the machine—reliability of the functional mechanism—principle of action of the functional mechanism (or, further, accuracy of the functional mechanism—accuracy of the elementary mechanism) makes it possible, at the machine-design stage, to carry out a far-sighted search for design solutions, choosing those mechanisms whose principle of action and characteristics determine the required technical-and-economic efficiency of the machines.

State Scientific Research
Institute of Machine Science

All-Union Correspondence Institute
of the Textile and Light Industry

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REFERENCES

1. I. I. Artobolevskii, D. Ya. Il'inskii, I. I. Kapustin, DAN, **185**, No. 2 (1969).

2. D. Uaildi, *Nonlinear and Dynamic Programming*, Moscow, 1967.

3. A. D. Tsiphanov, *Reliability of Automatic Systems*, Moscow, 1964.

Note: Figure translations are in progress. See original paper for figures.