WEAK SHOCK WAVES IN A THERMORELAXING MEDIUM\

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Abstract

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WEAK SHOCK WAVES IN A THERMORELAXING MEDIUM*

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It is known that if a shock wave propagates through a relaxing medium and the relaxation time is significantly greater than the time of molecular collisions, this leads to a substantial change in the structure of the shock wave and, in particular, to a considerable increase in the width of the transition layer. This question was first considered by Ya. B. Zel' dovich in work (^{1}), and later in other works as well (see, for example, (^{2}-^{6})).

For strong shock waves, the course of the parameters in the relaxation zone can be investigated by numerical integration of the hydrodynamic conservation equations together with the kinetic equation. An example of such a numerical calculation is given in work (^{7}). As for shock waves of small intensity, in a number of cases an analytical solution of the problem is possible here. Such a solution, for example, was given by S. P. D' yakov in work (^{8}) for the case of a weak difference between the limiting values of the speed of sound.

In the present work the structure of a weak shock wave in thermorelaxing media is investigated analytically. As such media, two-component mixtures are considered, for example an emulsion of one liquid in another or a gas and a suspension of solid particles in it. When the substance is compressed by a shock wave, a temperature difference arises between the components of such a medium, and this determines the relaxation process of heat exchange (thermorelaxation).

We shall not impose any restrictions on the limiting values of the speed of sound, and shall only assume that the state of the substance in the transition layer changes slowly in comparison with the establishment of equilibrium, which makes it possible to use the treatment of the relaxation process in the spirit of the method of L. I. Mandelstam and M. A. Leontovich (^{8}, ^{9}).

According to the latter, the presence of relaxation processes under sufficiently slow changes of state is equivalent to an anomalously large second coefficient of viscosity of the medium

$$\zeta = \tau\rho(u_\infty^2 - u_0^2), \tag{1}$$

where τ is the relaxation time, ρ is the density of the substance, u_∞ is the speed of sound at frequencies so high that the relaxation processes in the sound wave are completely "frozen," and u_0 is the speed of sound at frequencies so low that the medium in the sound wave has time to reach the state of complete thermodynamic equilibrium. In this case, obviously, one may use the usual formulas of the hydrodynamic theory of the transition layer (^{10}), in which thermal conductivity and the first viscosity should be neglected. Then for the course of the pressure in the transition layer we shall have

$$p = \tfrac{1}{2}(p_1 + p_0) + \tfrac{1}{2}(p_1 - p_0)\,\mathrm{th}(x/\delta), \tag{2}$$

where $p_1 - p_0 = \Delta p$ is the pressure jump in the shock wave, and δ is the width of the transition layer, equal to

$$\delta = 2V^3\zeta/u_0^3(\partial^2 V/\partial p^2)_s \Delta p. \tag{3}$$

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* In this work, thermorelaxing media are understood to be such media in which temperature relaxation takes place.

In (3), $V = 1/\rho$, while ξ is determined by formula (1).

Since these formulas are applicable when the state of the substance in the transition layer changes slowly in comparison with the establishment of equilibrium, this means that

$$\delta \gg u_0\tau, \tag{4}$$

whence, using (3) and (1), we shall have

$$\Delta p \ll 2V^2\,(u_\infty^2 - u_0^2)/u_0^4\,(\partial^2 V/\partial p^2)_s. \tag{5}$$

Here it is necessary to note that, upon substituting (3) and (1) into (4), τ turns out to be eliminated from (5), and since, of all the quantities entering into these relations, only τ depends on the thermal conductivity of the suspended particles, the condition of applicability of the concept of the second viscosity (5), evidently, should not depend on the thermal conductivity of these particles.

Let us represent the relaxing medium as a system consisting of a carrier (the first component) with particles distributed in it (the second component). To estimate the order of magnitude of the relaxation time τ, let r denote the radius of the particles suspended in the carrier, and consider, for example, the problem of the propagation of periodic temperature oscillations in a homogeneous half-space. This problem should be considered without initial conditions, since under repeated repetition of the temperature course at the surface the influence of the initial temperature of the medium may be neglected. As a result we arrive at the following problem (11):

$$\partial\Delta T/\partial t = a^2\partial^2\Delta T/\partial x^2 \quad (0 \le x < \infty,\ -\infty < t), \tag{6}$$

where ΔT is the temperature drop satisfying the condition $\Delta T(0,t) = A\cos\omega t$, a^2 is the coefficient of thermal diffusivity of the medium, and ω is the frequency of the temperature oscillation on its surface. The solution of this problem has the form (12)

$$\Delta T(x,t) = A\big(\cos(\sqrt{\omega/2a^2}\,x - \omega t)\big)\exp(-\sqrt{\omega/2a^2}\,x), \tag{7}$$

from which it follows that the amplitude of the oscillations decreases exponentially with depth,

$$A(x) = A\exp(-\sqrt{\omega/2a^2}\,x). \tag{8}$$

From (8) one can determine the period of temperature oscillation at the surface for which the amplitude of the oscillations at distance r decreases by a factor of e. The magnitude of this period, evidently, may be set equal to the desired relaxation time. As a result we obtain

$$\tau = \pi\rho c r^2/\chi, \tag{9}$$

where χ is the coefficient of thermal conductivity of the medium, and c is its specific heat capacity.

Knowing the mechanical and thermodynamic constants of the components of the medium, one can easily determine the limiting values of the speed of sound. By subscripts we shall distinguish quantities referring to the first and second components, respectively; we shall omit them for quantities referring equally both to one and to the other component, and to the mixture as a whole. Using the known thermodynamic relations (13) for the adiabatic coefficient of compressibility and making use of the additivity of the quantities entering into it, as a result we obtain

$$\frac{u_\infty^{-2}}{\varepsilon\rho_1 + (1-\varepsilon)\rho_2} = -\left[\frac{\varepsilon\rho_1}{\gamma_1}\left(\frac{\partial V_1}{\partial p}\right)_T + \frac{(1-\varepsilon)\rho_2}{\gamma_2}\left(\frac{\partial V_2}{\partial p}\right)_T\right]; \tag{10}$$

$$\frac{u_0^{-2}}{\varepsilon\rho_1+(1-\varepsilon)\rho_2} = -\Bigg\{\varepsilon\rho_1\left(\frac{\partial V_1}{\partial p}\right)_T + (1-\varepsilon)\rho_2\left(\frac{\partial V_2}{\partial p}\right)_T + \frac{T}{\varepsilon\rho_1 c_{p1}+(1-\varepsilon)\rho_2 c_{p2}} \times \Bigg[\varepsilon\rho_1\left(\frac{\partial V_1}{\partial T}\right)_p + (1-\varepsilon)\rho_2 \Bigg(\qquad (11)$$

In (10) and (11), ε is the volume concentration of the first component; γ is the ratio of heat capacities; V_1 and V_2 are the specific volumes of the components, and c_{p1} and c_{p2} are their specific heats.

The present study is based on the assumption that the state of the medium in the transition layer changes slowly in comparison with the establishment of equilibrium. Consequently, the quantity $(\partial^2 V/\partial p^2)_s$ entering formulas (3) and (5) must be determined taking into account the fact that the temperature difference between the components of the medium has time to equalize during compression in the shock wave, i.e., macroscopically the process proceeds adiabatically, and microscopically—isothermally. Since the components of the mixture will then at all times be at the same pressure and temperature, in determining $(\partial^2 V/\partial p^2)_s$ it is necessary to choose p and T as the independent thermodynamic variables. For this purpose, let us express the quantity $(\partial^2 V/\partial p^2)_s$ for an individual component in terms of the variables p and T. Simple considerations show that

$$\left(\frac{\partial^2 V}{\partial p^2}\right)_s = \left(\frac{\partial^2 V}{\partial p^2}\right)_T + \frac{3T}{c_p}\left(\frac{\partial V}{\partial T}\right)_p \frac{\partial^2 V}{\partial p\,\partial T} + \frac{T}{c_p^2}\left(\frac{\partial V}{\partial T}\right)_p^2 \left[\left(\frac{\partial V}{\partial T}\right)_p + 2T\left(\frac{\partial^2 V}{\partial T^2}\right)_p\right], \qquad (12)$$

where V and c_p are either the specific volume and heat capacity, or the volume and heat capacity of an arbitrary mass. Taking into account the additivity of the quantities entering (12), for a unit mass of the mixture we obtain

$$\begin{aligned}\left(\frac{\partial^2 V}{\partial p^2}\right)_s &= n\left(\frac{\partial^2 V_1}{\partial p^2}\right)_T + (1-n)\left(\frac{\partial^2 V_2}{\partial p^2}\right)_T + \frac{3T}{nc_{p1}+(1-n)c_{p2}}\left[n\left(\frac{\partial V_1}{\partial T}\right)_p + (1-n)\left(\frac{\partial V_2}{\partial T}\right)_p\right] \\ &\times \left[n\frac{\partial^2 V_1}{\partial p\,\partial T} + (1-n)\frac{\partial^2 V_2}{\partial p\,\partial T}\right] + \frac{T}{[nc_{p1}+(1-n)c_{p2}]^2}\left[n\left(\frac{\partial V_1}{\partial T}\right)_p + (1-n)\left(\frac{\partial V_2}{\partial T}\right)_p\right]^2 \\ &\times \left\{n\left(\frac{\partial V_1}{\partial T}\right)_p + (1-n)\left(\frac{\partial V_2}{\partial T}\right)_p + 2T\left[n\left(\frac{\partial^2 V_1}{\partial T^2}\right)_p + (1-n)\left(\frac{\partial^2 V_2}{\partial T^2}\right)_p\right]\right\}.\end{aligned} \qquad (13)$$

In (13), V_1 and V_2 are the specific volumes of the components; c_{p1} and c_{p2} are their specific heats, and n is the mass concentration of the first component, related to ε by the relation $n = \varepsilon\rho_1/[\varepsilon\rho_1 + (1-\varepsilon)\rho_2]$.

If the first component of the mixture is a gas and the second consists of solid or liquid particles suspended in it, then for such systems one may neglect the compressibility of the second component. Then

$$u_\infty/u_1 = \{\rho_1/\varepsilon[\varepsilon\rho_1 + (1-\varepsilon)\rho_2]\}^{1/2},$$

$$u_0/u_1 = \{\rho_1[\varepsilon\rho_1 c_{p1} + (1-\varepsilon)\rho_2 c_{p2}]/\varepsilon[\varepsilon\rho_1 + (1-\varepsilon)\rho_2] \times [\varepsilon\rho_1 c_{p1} + (1-\varepsilon)\gamma_1\rho_2 c_{p2}]\}^{1/2}, \tag{14}$$

where u_1 is the speed of sound in the gas, and

$$(\partial^2 V/\partial p^2)_s = \varepsilon[\varepsilon\rho_1 c_{p1} + (1-\varepsilon)\gamma_1\rho_2 c_{p2}][\varepsilon(\gamma_1+1)\rho_1 c_{p1} + 2(1-\varepsilon)\gamma_1\rho_2 c_{p2}]/\gamma_1^2 p^2[\varepsilon\rho_1 + (1-\varepsilon)\rho_2][\varepsilon\rho_1 c_{p1} + (1-\varepsilon)\rho_2 c_{p2}]^2 \tag{15}$$

The condition for applicability of the second-viscosity concept is then written in the form

$$\Delta p/p \ll 2\gamma_1\rho_2 c_{p2}(\gamma_1 - 1)(1-\varepsilon)/[(\gamma_1+1)\rho_1 c_{p1}\varepsilon + 2\gamma_1\rho_2 c_{p2}(1-\varepsilon)]. \tag{16}$$

For $\varepsilon \leq 1 - [5\rho_1 c_{p1}(\gamma_1+1)/\gamma_1\rho_2 c_{p2}]$, the writing of this condition is substantially simplified: $\Delta p/p \ll \gamma_1 - 1$.

We present the results of comparing the width of the transition layer in air δ_0 and in the systems air-coal and air-water. It was assumed that the particle radius is $r = 10^{-4}$ cm, $p = 10^6$ dyn/cm^2, and $T = 288^\circ$ K.

δ/δ_0		$1-\varepsilon$: 10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
δ/δ_0	air-coal	$3.4 \cdot 10^1$	$9.2 \cdot 10^2$	$1.3 \cdot 10^2$	$3.7 \cdot 10^1$	4.4
δ/δ_0	air-water	$1 \cdot 10^3$	$2.9 \cdot 10^3$	$5.4 \cdot 10^3$	$2.8 \cdot 10^3$	$4.3 \cdot 10^2$

From these data it follows that in an air-coal mixture the width of the transition layer is more than an order of magnitude greater than in pure air. For the second mixture this difference is still greater and reaches three orders of magnitude. As the content of solid and liquid particles in the mixture decreases, the value of δ initially increases and, having reached a maximum, then begins to fall.

In those cases where the densities of the components of the medium differ substantially, i.e., when one of the components is a gas, the presence of diffusion and barodiffusion phenomena leads, as shown in ([14]), to a change in the concentration of the components in the transition layer of the shock wave and causes an increase in its width. Consequently, in such cases, in addition to the mechanism of thermal relaxation, it is also necessary to take into account this additional mechanism for increasing δ. It can be shown, however, that these two processes are practically independent, and the width of the transition layer will be determined by the larger of the two values of δ caused by each of these processes separately.

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Note: Figure translations are in progress. See original paper for figures.