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ON THE PROBABILITY OF DOUBLE BETA DECAY OF NUCLEI IN REGIONS FAR FROM THE BAND OF BETA STABILITY

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The probability of double β-decay as a second-order process in the weak interaction is negligibly small. In the band of β-stability, when the mass difference of the isobars (Z, N) and $(Z \pm 2, N \mp 2)$, between which this process can occur, is small, the lifetime with respect to double β-decay exceeds 10^{20} yr ([1,2]).

In regions far from the band of β-stability, the mass difference $Q_{\beta\beta}$ reaches several tens of MeV ([3]), and, since the probability of double β-decay is a rapidly increasing function of this difference, one may expect a substantial increase in it. Moreover, in comparison with the region of β-stability, the number of final states that can be populated as a result of double β-decay increases many times over.

The total probability of double β-decay in regions far from the band of β-stability can be written in the form

$$\lambda^{\sigma}_{\beta\beta} = C^{\sigma}_1 \int_0^{Q_{\beta\beta}} f^{\sigma}_{\beta\beta}(Q_{\beta\beta} - E)\rho(E)|M|^2\, dE, \tag{1}$$

where $f^{\sigma}_{\beta\beta}(Q_{\beta\beta} - E)$ is the energy function of $\beta\beta$-decay to the level E of the daughter nucleus; $|M|^2$ is the square of the nuclear matrix element of the $\beta\beta$ transition; C^{σ}_1 is a coefficient including the dependence on the weak-interaction constant and the atomic number Z of the decaying nucleus. The index σ takes two values, corresponding to the two-neutrino and neutrinoless types of decay, which below will be denoted by $\nu\nu$ and 00, respectively.

Expressions for the functions $f^{\sigma}_{\beta\beta}$ and the coefficients C^{σ}_1 as applied to even nuclei in the region of β-stability and to allowed β-transitions were obtained by Primakoff and Rosen ([1]).

In the case of double β-decay in nuclei far from the band of β-stability, the condition $|\langle W_\nu - W_i\rangle| \gg \frac{1}{2}(\varepsilon_0 + 2)$ is, generally speaking, not fulfilled. (Here

ε_0 denotes the maximum kinetic energy carried away by leptons, $\langle W_\nu - W_i\rangle$ is the mean value of the energy difference of the virtual levels of the intermediate nucleus $(Z \pm 1, N \mp 1)$, which give the principal contribution to the transition probability, and the ground state of the decaying nucleus (Z, N).) Therefore the formula from work ([1]), obtained under the assumption that this inequality is fulfilled, cannot be used to estimate the probability of two-neutrino decay in nuclei far from the band of stability. In addition, in these nuclei, in contrast to β-stable ones, the values $\langle W_\nu - W_i\rangle$ may be negative.

Integration over lepton momenta under any assumption about the magnitude of $\langle W_\nu - W_i\rangle$ and its sign leads to an expression of the form

$$f^{\nu\nu}_{\beta\beta}(\varepsilon_0) = \frac{1}{8^9}\left\{\left[\frac{1}{\langle W_\nu - W_i\rangle + \frac{1}{2}(\varepsilon_0+1)}\sum_{m=0}^{1}\sum_{l=8}^{10} a_{lm}\chi^{l-m}\varepsilon_0^m + \sum_{m=0}^{1}\sum_{l=7}^{9} b_{lm}\chi^{l-m}\varepsilon_0^m\right] + \right.$$

$$\left. + \left[\frac{1}{\langle W_\nu - W_i\rangle + \frac{1}{2}(\varepsilon_0+1)}\sum_{m=0}^{2}\sum_{l=8}^{10} c_{lm}\chi^{l-m}\varepsilon_0^m + \sum_{m=0}^{2}\sum_{l=7}^{9} d_{lm}\chi^{l-m}\varepsilon_0^m\right]\ln\left|\frac{\chi+\varepsilon_0}{\chi}\right|\right\}, \tag{2}$$

where $\chi = \langle W_v - W_i\rangle + 1$, and a_{lm}, b_{lm}, c_{lm}, and d_{lm} are certain numbers. We give the values of the function $f^{\nu\nu}_{\beta\beta}(\varepsilon_0)$, approximated with respect to energy, as a function of the magnitude of the ratio $\omega = \chi/\varepsilon_0$ for $\varepsilon_0 \gg 1$:

ω	-0.48	-0.33	0.0	0.33
$f^{\nu\nu}_{\beta\beta}(\varepsilon_0)$	$\sim 0.02\,\varepsilon_0^{10} + 0.5\,\varepsilon_0^9$	$\sim 0.07\,\varepsilon_0^9$	$\sim 0.01\,\varepsilon_0^9$	$\sim 0.006\,\varepsilon_0^9$

It is seen that, over a broad interval of values of ω, the coefficient multiplying the energy factor changes by only 1-2 orders of magnitude. For values $|\chi| \gg \varepsilon_0$, formula (2) goes over into the Primakoff-Rosen formula (1).

In order to make numerical estimates by formula (1), one must specify an expression for the density of states of the daughter nucleus. It is most convenient to use the Fermi-gas density formula, according to which

$$\rho(E) = C_2(A)(E-\delta)^{-2}\exp(2\sqrt{a(E-\delta)}), \tag{3}$$

where $C_2(A)$ is a coefficient depending on the mass number A [[4]], $a \simeq \frac{1}{14}A(mc^2)^{-1}$, and δ is the pairing energy.

Assuming that the mean value of the square of the nuclear matrix element is a constant quantity [[5]] and lies in the interval $|M|^2_{\beta\beta} =$

Fig. 1. A—dependence of the ratio $\lambda_{\beta\beta}^{\nu\nu}/\lambda_{\beta}^{\nu}$ on the mass difference of isobars between which two-neutrino double β-decay occurs. B—dependence of the ratio $\lambda_{\beta\beta}^{00}/\lambda_{\beta}^{\nu}$ on the mass difference of isobars between which neutrinoless β-decay occurs. 1—for $\xi = 10^2$; 2—$\xi = 1$; 3—$\eta = 10^4$; 4—$\eta = 1$.

Figure 1: Fig. 1. A—dependence of the ratio $\lambda_{\beta\beta}^{\nu\nu}/\lambda_{\beta}^{\nu}$ on the mass difference of isobars between which two-neutrino double β-decay occurs. B—dependence of the ratio $\lambda_{\beta\beta}^{00}/\lambda_{\beta}^{\nu}$ on the mass difference of isobars between which neutrinoless β-decay occurs. 1—for $\xi = 10^2$; 2—$\xi = 1$; 3—$\eta = 10^4$; 4—$\eta = 1$.

Fig. 1. *A*—dependence of the ratio $\lambda_{\beta\beta}^{\nu\nu}/\lambda_{\beta}^{\nu}$ on the mass difference of isobars between which two-neutrino double β-decay occurs.
B—dependence of the ratio $\lambda_{\beta\beta}^{00}/\lambda_{\beta}^{\nu}$ on the mass difference of isobars between which neutrinoless β-decay occurs. 1—for $\xi = 10^2$; 2—$\xi = 1$; 3—$\eta = 10^4$; 4—$\eta = 1$.

$= 0.01 \div 1$ [6], we obtain from formulas (1), (2), (3), for small values of Z, $A \simeq 60$, and $Q_{\beta\beta} \simeq 100\, mc^2$, the value $\lambda_{\beta\beta}^{\nu\nu} \simeq 0.01 \div 1$ yr^{-1}, which exceeds by many orders of magnitude the corresponding value for transitions in the stable region.

Writing, analogously to (1), an expression for the probability of single β-decay λ_{β}^{ν}, one can obtain the ratios $\lambda_{\beta\beta}^{\sigma}/\lambda_{\beta}^{\nu}$ ($\sigma = 00$ and $\nu\nu$), which are of interest for determining the experimental possibility of observing double β-decay.

In Fig. 1 (curves 1 and 2) the dependence of these ratios on the energy of double β-decay in neutron-rich nuclei for $A \simeq 60$ is shown, obtained using expression (3) for $\xi = |M|_{\beta\beta}^{2\sigma}/|M|_{\beta}^{2} = 10^2$ and 1.

The value of the ratio $Q_{\beta\beta}/Q_{\beta}$ was taken equal to 3. However, its choice is not critical, and for values of this ratio in the interval $2 \div 3$ comparable results are obtained.

Fig. 1 also shows the dependence of the ratios $\lambda_{\beta\beta}^{\sigma}/\lambda_{\beta}^{\nu}$ on the energy, calculated from (1) and (2) under the assumption that the strength function $M^2(E) \equiv |M|^2\rho(E)$ in expression (1) does not depend on energy, analogously to what occurs in single β decay (7). In the case of electron decay this approximation is reasonable, since in this case there is no population of analog states, where the strength function has a well-pronounced maximum (especially sharp for Fermi-type transitions (8)). The figure shows (curves 3 and 4) the results corresponding to two values of the parameter $\eta = M^2(E)_{\beta\beta}^{\sigma}/M^2(E)_{\beta}$, determined by the ratio of the mean values of the strength functions for double and single β decay. Values of η in the interval $1 \div 10^4$ can be obtained within the approximations of (1) (see formula (72)) by varying the quantities $|M|_{\beta}^{2}$ within the limits $0.01 \div 1$ and the number of intermediate states N_{ν} from 10 to 10^2.

According to preliminary data obtained in experiments under the ISOLDE program at CERN* for nuclei far from the β-stability band, the strength function is not a constant quantity but increases with energy. If this also occurs in double β decay, then the absolute values of the ratios $\lambda^{\sigma}_{\beta\beta}/\lambda^{\nu}_{\beta}$ will become larger than those calculated under the assumption of a constant strength function, and curves 3 and 4 in Fig. 1 will become steeper.

Let us note that, since only matrix elements of the Fermi type enter into the probability of allowed neutrinoless decay, the values of the parameters ξ and η for the neutrinoless and two-neutrino processes may be different for one and the same transition.

It should be emphasized that the results obtained are qualitative in character, and their error may amount to several orders of magnitude. However, despite the estimated nature of the results obtained, the following qualitative conclusions may be briefly formulated:

1. The periods of double β decays in regions far from the β-stability band, in comparison with β-stable nuclei, decrease substantially, reaching, for example, in the two-neutrino variant, values of only a few orders of years (for $A \simeq 60$ and neutron-rich nuclei).
2. The ratios $\lambda^{\sigma}_{\beta\beta}/\lambda^{\nu}_{\beta}$, as is seen from Fig. 1, increase with increasing decay energy (i.e., with increasing distance from the β-stability band).
3. In contrast to the β-stability band, in remote regions the probabilities of the two-neutrino and neutrinoless processes differ not very greatly, and allowance for a small deviation from the condition of two-component neutrino or for a nonzero neutrino mass will lead to suppression of the probability of the neutrinoless process in comparison with the two-neutrino one.

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Note: Figure translations are in progress. See original paper for figures.