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Abstract

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CYBERNETICS AND CONTROL THEORY

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ON A PROPERTY OF PERIODIC MOTIONS OF A SINGLE-LOOP NONLINEAR IMPULSE SYSTEM WITH AN INTEGRATING ELEMENT

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Let us consider possible periodic motions of a control system whose dynamics is described by the differential equations

$$\dot{x} = Ax + b\varphi(\mu), \quad \dot{\mu} = \sum_{j=0}^{\infty} (g(t) + c^T x)\delta(t - j\tau), \quad (1)$$

where A is a constant nonsingular matrix ($n \times n$); b and c are constant column matrices ($n \times 1$); $x(t)$ is the same kind of column of unknown functions; $\mu(t)$, $\varphi(\mu)$, and $g(t)$ are, respectively, the unknown, the given, and the given periodic scalar functions with period a multiple of τ . Equations (1) are a particular case of equations (5) from (1) and represent the dynamics of a single-loop ideal impulse nonlinear control system, in whose scheme, in addition to the common feedback, there are connected in series a summing element, an ideal impulse element operating with repetition period τ , an ideal integrating element whose time constant is chosen as the time scale, a static nonlinear element with nonlinear characteristic $\varphi(\mu)$, and a linear element of order n . From a system with various real impulse elements it is usually possible to pass to a system with an ideal element (2-4).

The solution of system (1) for $x(-0) = x^0$ and $\mu(-0) = \mu^0$ can be written (5,6) in the form

$$x(t) = e^{At}x^0 + \int_0^t e^{A(t-s)}b\varphi(\mu(s)) ds, \quad \mu(t) = \mu^0 + \sum_{j=0}^{[t/\tau]} (g^j + c^T x^j)1(t - j\tau), \quad (2)$$

where $g^j \equiv g(j\tau - 0)$ and $x^j \equiv x(j\tau - 0)$, $j = 0, 1, 2, \dots$. Denoting further $\mu^j \equiv \mu(j\tau - 0)$ and passing, analogously to (1), to the point transformation in

phase space corresponding to the motion of system (1), carried out at each cycle of the system impulse element, we obtain, for $j = 1, 2, \dots$,

$$x^j = e^{A\tau}x^{j-1} + (e^{A\tau} - E)A^{-1}b\varphi(\mu^j), \quad (3)$$

$$\mu^j = \mu^{j-1} + g^{j-1} + c^T x^{j-1} = \mu^0 + \sum_{i=0}^{j-1} (g^i + c^T x^i). \quad (4)$$

Using the fact that the matrices A , A^{-1} , and $e^{A\tau}$ commute pairwise ⁽⁷⁾, it is not difficult to obtain the indicated point transformation, effected over the time $T = m\tau$ for arbitrary m , from (3) and (4) by induction in the form

$$x^m = e^{mA\tau}x^0 + (e^{A\tau} - E)A^{-1} \left(\sum_{j=1}^m \varphi(\mu^j)e^{(m-j)A\tau} \right) b, \quad (5)$$

$$\begin{aligned} \mu^m = \mu^0 + \sum_{j=0}^{m-1} g^j + c^T \left(\frac{e^{mA\tau} - E}{e^{A\tau} - E} x^0 - A^{-1}b \sum_{j=1}^m \varphi(\mu^j) + \right. \\ \left. + A^{-1} \left(\sum_{j=1}^m \varphi(\mu^j)e^{(m-j)A\tau} \right) b \right), \end{aligned} \quad (6)$$

where the quantities μ^j are defined by expression (4).

If now $T = m\tau$ is a multiple of the period of the control action $g(t)$, or $g(t) \equiv 0$, then for the existence in system (1) of a periodic motion with period $T = m\tau$, there must exist an invariant point of transformations (5) and (6) that is not a point of smaller multiplicity, i.e., the following must hold:

$$x^m = x^0 = x, \quad \mu^m = \mu^0 = \mu, \quad (7)$$

which, together with (5), (6), (3), and (4), determines the coordinates of the representative point on this periodic motion at the time instants $k\tau, (k+1)\tau, \dots, (k+m-1)\tau$ for arbitrary k . The stability of this periodic motion will be determined by whether the roots of the characteristic equation

$$\det \begin{pmatrix} \partial x^m / \partial x^0 - zE & \partial x^m / \partial \mu^0 \\ \partial \mu^m / \partial x^0 & \partial \mu^m / \partial \mu^0 - z \end{pmatrix} = 0, \quad (8)$$

in which the derivatives are taken at the invariant point, belong to the unit circle.

From (5) and (6) it is not difficult to see that the following property holds.

For periodic motions of the (impulse) nonlinear system (1) with period $T = m\tau$, under the condition $g(t + m\tau) = g(t)$, one has

$$\sum_{j=1}^m g(j\tau) = c^T A^{-1} b \sum_{j=1}^m \varphi(\mu^j), \quad (9)$$

and under the condition $g(t) \equiv 0$, i.e., for an autonomous system, one has

$$\sum_{j=1}^m \varphi(\mu^j) = 0. \quad (10)$$

This circumstance emphasizes the “symmetrizing” role of the integrating element in the system, which Ya. Z. Tsytkin pointed out as early as 1953 when discussing the theory of relay systems at the Second All-Union Conference on Automatic Control.

Moreover, the matrix of the transformation (5) and (6) linearized at the invariant point (7) is the product, in reverse order, of the matrices D_j of one-step transformations linearized at the same points (3) and (4). Therefore the constant term of the characteristic equation (8) is equal to $\det e^{mA\tau}$. Indeed, for $j = 1, 2, \dots, m$

$$\det D_j = \det \begin{pmatrix} e^{A\tau} + (e^{A\tau} - E)A^{-1}b\varphi'(\mu^{j*})c^T & (e^{A\tau} - E)A^{-1}b\varphi'(\mu^{j*}) \\ c^T & 1 \end{pmatrix} = \det e^{A\tau}, \quad (11)$$

as is easily verified by subtracting from the upper n rows of (11) the lower row multiplied on the left by the column $(e^{A\tau} - E)A^{-1}b\varphi'(\mu^{j*})$.

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Note: Figure translations are in progress. See original paper for figures.

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